Channel Selection in Multi-channel Opportunistic Spectrum Access Networks with Perfect Sensing

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***Abstract*— We study optimal transmission strategies in multi-channel opportunistic spectrum access networks where one secondary user (SU) opportunistically accesses multiple orthogonal channels that are owned and utilized by primary users (PU). In dynamic spectrum access networks, the protection of PU's is vital, since no PU would accommodate SU access to its own detriment. Therefore the objective of the problem we study is to maximize the SU throughput while protecting PUs on all channels. At a given time, the SU decides if it transmits and if so on which PU channel it should transmit on in order to protect PU performance. We use a constraint on the expected PU packet collision probability as the protection metric. We consider a general setting where the PUs are unslotted and may have different idle/busy time distributions and protection requirements. Under general idle time distributions, we determine the form of the SU optimal access policy. We also study the special case where PUs have independent, exponentially distributed idle time. For this case, we formulate a linear program that yields an optimal randomized strategy for the secondary user, and also present a tunable heuristic which allows for a tradeoff between complexity and throughput performance.**

I. Introduction

Cognitive Radio (CR) technology has great potential to alleviate spectrum scarcity in wireless communications. It allows secondary users (SUs) to opportunistically access spectrum licensed by primary users (PUs) while protecting PU activity. This paradigm is often referred to as *dynamic spectrum access* (DSA), where users in the system are divided into a multi-tiered hierarchy and primary users have priority of channel access over secondary users [1]. In this paradigm, because the protection of PU is vital, a design imperative for a SU opportunistic access strategy is to minimize the SUs' effect on PU transmissions. For example, in the DARPA XG project [2], one of the three major test criteria in a cognitive radio prototype field test is "to cause no harm" [3]. Predictably, this is also one of the main bottlenecks of SU performance.

We consider in this work the case where one SU senses multiple PU channels and can decide to transmit on at most one that is currently idle. The SU's goal is to maximize its own throughput while protecting PU performance on all channels. We consider packet collision probability as the PU protection requirement. Under this requirement, the SU must guarantee that the packet collision probability of a PU packet is less than a certain threshold specified *a priori* by the PU. This type of constraint has been widely considered in the literature [4], [5], [6], [7]. We assume that the PUs are unslotted and may have different idle/busy time distributions and protection requirements.

The principal contributions of this paper are two-fold. First, we determine the form of the SU's optimal access policy for general idle/busy time distributions. Second, for the special case where the idle/busy time distributions are exponential, we present a linear program that can compute the optimal SU strategy explicitly.

II. Related Works

In recent years, there has been an explosion of research in cognitive radio. A large portion of this research has been in spectrum overlay, where protocols are devised to maximize SU spectrum utility when PUs are idle and protect PU communication when they become busy. Within this paradigm, there are two focuses, spatial domain and temporal domain research [1].

In the former, SU activity is assumed to occur in a much faster timescale than the PU activity, and hence the spectral environment (i.e. PU channel occupancy) is treated as static. The main problem then becomes channel allocation among multiple SUs given certain topologies, different channel availabilities, and interference among SUs.

Our work falls into the latter temporal domain category, where PU activity varies quickly in time and SUs within interference range must devise sensing and access schemes in concert to avoid significantly harming PU communication. In this domain, PU protection is crucial. Two widely used PU protection metrics are interference temperature and collision probability. Several papers consider interference power. For example, in [8], the authors consider multiple SUs operating in a multi-PU system where each PU has an average rate requirement and outage probability constraint, both functions of the interference power caused by SUs. Power control for different states of PU activity is considered in [9].

In this paper, we use PU packet collision probability constraint. Researchers have developed medium access schemes for SUs under this protection requirement [4], [5], [6]. Partially observable Markov decision process (POMDP) has been widely used to formulate the sensing-transmission decision problem. In [10], [11], the authors consider a slotted PU network, and optimal sensing and access decisions are made by considering observation history. The POMDP framework for sensing and access has been extended to unslotted setting in [12]. In [13], [14], the authors consider a slotted system for a single SU with limited sensing, and identify the conditions under which a simple myopic policy is optimal for sensing and access, when the PU's channel occupancy can be modeled as i.i.d. Markov chains. The result is extended to the case of sensing multiple channels in [15]. In [16], the authors adopt the quickest change detection technique and establish a Bayesian formulation to decide which channel to access assuming geometrically distributed busy/idle times. In [7], the authors consider an overlay SU network on a multi-PU network with slotted structure. The authors use the Lyapunov optimization technique to design an online flow control, scheduling and resource allocation algorithm that meets the desired PU protection requirements. In comparison, these papers consider geometrically distributed (or exponentially distributed) idle/busy period while we consider general idle/busy time distributions.

In [17], the authors consider learning-based sensing schemes when the distribution of the primary user is unknown, but belongs to a parameterized family. In [18], an optimal sensing-transmission structure is proposed for a PU channel with a general idle time distribution. While these works focus on a single PU channel, we consider multiple PU channels. In [19], sensing of multiple general channels are considered. However, this work does not consider PU packet collision probability, which is a critical constraint in our case.

The most related works are [4], [20], where the authors propose optimal access strategy for a single PU channel with general idle/busy time distribution. In this work, we develop the results to multiple PU channels.

III. System Model

In this section we describe the system model used in this paper. We consider a single SU that operates within the interference range of multiple orthogonal PU channels.

A. Primary User Model

In our system model, we assume there are K orthogonal PU channels. At each PU channel, the PU transmits its data at will and exhibits an idle/busy usage pattern. The individual idle (X) and busy (Y) periods are independent. They follow distributions $f_i^x()$ and $f_i^y()$ with means $\bar{x}_i$ and $\bar{y}_i$, respectively, where index i represents PU channel i. The PU activity on different channels are independent.

Each PU i has a packet collision probability requirement denoted η_i, defined as the maximum allowable probability of collision for a packet of the ith PU. Assume that each PU packet has length 1. Over a time interval $[0, T]$, we denote the number of packets transmitted by the ith PU as $\mathcal{N}_i$, and the number of collisions experienced by that user as $\mathcal{N}_i^c$. The collision probability of the ith PUs' packets experiencing collision is denoted as

$$p_i^c = Pr[\text{packet collision of } i\text{th PU}],$$

where

$$p_i^c = \lim_{T\to\infty} \frac{N_i^c}{N_i},$$

and the PU protection requirement is thus

$$p_i^c \leq \eta_i, \quad i \in \{1, ..., K\}. \tag{1}$$

B. SU Model

In this paper, we consider a single SU. We assume that the SU has knowledge of the collision constraint, and the idle/busy time distribution, i.e. $f_i^x(\cdot)$, $f_i^y(\cdot)$, η_i for all i.

We assume perfect sensing by the SU, i.e. that the SU can always detect the presence or absence of a PU on a channel, and that sensing time is negligible. In addition, the SU has the capability to simultaneously sensing all channels. For instance, the SU may be equipped with multiple radios, with one dedicated for sensing. The sensing front-end has broadband sensing capability. Therefore, at any given time, the SU maintains a status vector, $\vec{t}$, for the PUs, where $\vec{t} = \{t_1, \cdots, t_K\}'$. When channel i is currently idle, t_i is the amount of time channel i has been idle since the last channel transition from busy to idle; and we set $t_i = -1$ if the channel is currently busy. This is illustrated in Figure 1. Channels 1 and 3 are idle, and t_1 and t_3 are the amount of time they have been idle. Channel 2 is busy, so $t_2 = -1$. We use s to denote a state that indicates which channels are idle. Let A_s be the set of available (i.e. idle) channels at state

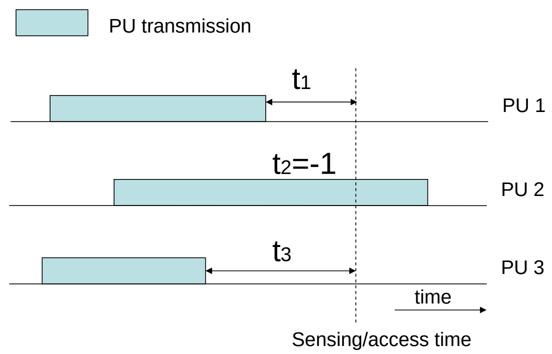

Fig. 1. Illustration of $\vec{t}$.

s. For instance, in the example illustrated in Figure 1, $A_s = \{1, 3\}$. Let π_s be the probability of state s. It can be calculated numerically using the average busy/idle times of the PUs. For example, for the situation shown in Figure 1, we have $\pi_s = (\bar{x}_1/(\bar{x}_1 + \bar{y}_1)) \times (\bar{x}_3/(\bar{x}_3 + \bar{y}_3)) \times (\bar{y}_2/(\bar{x}_2 + \bar{y}_2))$. Let π_k^s be the probability of state s given channel k is idle. We have

$$\pi_k^s = \begin{cases} \frac{\bar{x}_k + \bar{y}_k}{\bar{x}_k}\pi_s, & k \in A_s \\ 0, & k \notin A_s \end{cases}$$

where $\bar{x}_k/(\bar{x}_k + \bar{y}_k)$ is the probability that channel k is idle. Let $\vec{t_{\tilde{k}}} = \{t_1, \cdots, t_{k-1}, t_{k+1}, \cdots, t_K\}$. Abusing notations a little bit, we denote $\vec{t} = \{\vec{t_{\tilde{k}}}, t_k\}$.

We assume any overlapping transmission between the SU and the PU cause collision to both of them. This is a conservative assumption. In practice, collision depends on many other factors, such as distance between transmitter/receiver and transmission power. To address such possibilities, one can factor in capture effect on both SU and PU sides. Because of the perfect sensing assumption, collisions happen only in the following scenario: the SU started transmission when the PU channel is idle, and the PU returns before the SU finishes its transmission.

The decision variable of the SU is its transmission probability on a given channel, in particular, $p_s(k, \vec{t})$ is the probability of transmitting on channel k at state s given $\vec{t}$. Without loss of generality, we let

$$p_s(k, \vec{t}) = 0, \quad \text{if } s \notin A_s.$$

In other words, the SU only transmits on a channel if it is sensed idle.

The SU packet length is denoted Δ, and we assume that $\Delta < 1$ (1 is the length of a PU packet). Therefore, if a SU packet starts transmission at time t_k on channel k, its collision probability with the PU is

$$\int_{t_k}^{t_k+\Delta} f_k(t_k)dt_k.$$

In this paper, we study the extreme case where $\Delta \to 0$. We assume that even a very small portion of overlapping results in a collision with PU. This assumption allows us to maintain the quantification of the (negative) impact of SU transmission on the PU. Note that as Δ decrease, the collision probability of a single SU packet decreases. However, the SU needs to transmit more packets to achieve the same time capacity, and thus the collision probability remains the same. Having $\Delta \to 0$ alleviates tedious technical details while allowing us to provide insights on how a single SU should access multiple PU channels. In [4], it is shown that this results in the best SU capacity if no overhead is considered. With overhead, one can use similar techniques to decide the optimal packet length [4].

When SU uses transmission policy p, the average number of collisions PU k observes in each idle-busy period is

$$\sum_s \pi_k^s E_{\vec{t_{\tilde{k}}}} \left[\int_0^\infty p_s\left(k, \vec{t}\right) f_k\left(t_k\right) dt_k \right].$$

In the above equation, $\int_0^\infty p_s\left(k, \vec{t}\right) f_k\left(t_k\right) dt_k$ is the probability of collision in one idle-busy period. Because the policy depends on s and $\vec{t_{\tilde{k}}}$, the average number of collisions is taken expectation over s and $\vec{t_{\tilde{k}}}$. Note that collision can only happen in the beginning of the PU busy period and the SU transmission collides with at most one PU packet because of the perfect sensing assumption and that a SU packet is smaller than that of a PU packet. Normalized over the number of packets in each PU busy period $\bar{y}_k$, we have the PU collision probability. Equivalently, defining $\eta_k' = \eta_k \bar{y}_k$, the constraint on PU channel k is

$$\sum_s \pi_k^s E_{\vec{t_{\tilde{k}}}} \left[\int_0^\infty p_s\left(k, \vec{t}\right) f_k\left(t_k\right) dt_k \right] \le \eta_k'. \qquad (2)$$

The SU's performance metric is the *time capacity*, the percentage of time that the SU can transmit successfully.This metric is defined as:

$$C_s = \lim_{T\to\infty} \frac{\text{SU's successful access time in } [0, T]}{T}. \qquad (3)$$

To elaborate, if a SU packet takes δ unit of time to transmit and if the transmission of the SU packet has no overlapping with the transmission of a PU packet, we consider the SU to have successfully access the channel for δ unit of time. Taking $\delta \to zero$, under SU policy p, the SU's average transmission time in an idle-busy period on channel k is

$$\begin{aligned} &G_s(k,p) \\ &= \sum_s \pi_k^s E_{\vec{t}_{\bar{k}}} \left[\int_0^\infty \left(\int_0^\tau p_s\left(k,\vec{t}\right) dt_k \right) f_k(\tau) d\tau \right] \\ &= \sum_s \pi_k^s E_{\vec{t}_{\bar{k}}} \left[\int_0^\infty p_s\left(k,\vec{t}\right)(1-F_k(t_k)) dt_k \right]. \end{aligned} \quad (4)$$

Normalizing over the length of an idle-busy period, the time capacity (throughput), i.e., the percentage of SU transmission time, on channel k is thus

$$C_s(k,p) = \frac{G_s(k,p)}{\bar{x}_k + \bar{y}_k} \quad (5)$$

The time capacity on all channels is thus

$$C_s(p) = \sum_{k=1}^{K} C_s(k,p).$$

We note that there are idealized assumptions in this paper, including perfect sensing, infinitely small packet size, and no overhead. Under these assumptions, the SU performance is the benchmark performance and the (idealized) SU access scheme can provide insights/guidelines on the SU access for a more realistic scenario. We note that packet size and overhead can be addressed relatively easily, as in [4]. Assuming perfect sensing allows us to focus on investigating optimal capacity. Imperfect sensing was considered in [4], and similar approaches can be used here as well. The assumption that the SU has the perfect knowledge on all PU channels is the strongest assumption. There has been a significant amount of work in the literature that studies which channel to sense and access given history information (that does not provide complete current state information on channels) (e.g., in [13], [16]), mostly assuming geometric/exponential idle time distribution. It is our future work to generalize it to imperfect sensing and imperfect information on all channels.

Main notations used in the paper are summarized in Table I for easy reference.

TABLE I
NOTATIONS.

η_i	collision prob. constraint of channel i
η_i'	normalized collision prob. constraint
s	state
A_s	set of idle channels in state s
π_s	probability of state s
π_k^s	prob. of state s given channel k idle
$\vec{t}$	$\vec{t} = \{t_1, \cdots, t_K\}'$
$p_s(i,\vec{t})$	transmission probability
$\bar{x}_i$	avg. length of a PU idle period on chnl. i
$\bar{y}_i$	avg. length of a PU busy period on chnl. i

IV. Problem Formulation

The problem is formulated as maximizing the SU performance given the PU collision probability constraint. Formally, we have

$$\begin{aligned} \max_p \quad & \sum_k C_s(p) \\ \text{s.t.} \quad & \sum_s \pi_k^s E_{\vec{t}_{\bar{k}}} \left[\int_0^\infty p_s\left(k,\vec{t}\right) f_k\left(t_k\right) dt_k \right] \le \eta_k' \\ & k = 1,...,K, \\ & p_s\left(k,\vec{t}\right) \in \mathcal{E} \end{aligned} \quad (6)$$

where $\mathcal{E} : \left\{ p_s(k,\vec{t}) | \sum_{k=1}^K p_s(k,\vec{t}) \le 1, p_s(k,\vec{t}) \ge 0 \right\}$.

Proposition 1: The following policy is optimal:

$$p_s^*(k,\vec{t}) = \begin{cases} 1, & k = \operatorname*{argmax}_{i \in A_s} \left(\frac{1-F_k(t_k)}{\bar{x}_k+\bar{y}_k} - \lambda_k \frac{f_k(t_k)}{\bar{y}_k} \right), \\ & \quad \& \ \frac{1-F_k(t_k)}{\bar{x}_k+\bar{y}_k} \ge \lambda_k \frac{f_k(t_k)}{\bar{y}_k} \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

where $\vec{\lambda} = \{\lambda_1, \cdots, \lambda_K\}'$ is chosen such that, $\forall k$
i) $\lambda_k \ge 0$,
ii) $\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}} \left[\int_0^\infty p_s\left(k,\vec{t}\right) f_k\left(t_k\right) dt_k \right] \le \eta_k'$,
iii) If $\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}} \left[\int_0^\infty p_s\left(k,\vec{t}\right) f_k\left(t_k\right) dt_k \right] < \eta_k'$, then $\lambda_k = 0$.

Proof: Consider any other policy $\hat{p}$ that satisfies the collision probability constraints on all channels. We have

$$\begin{aligned}
&C_s(\hat{p})\\
=&\sum_k \frac{1}{\bar{x}_k+\bar{y}_k}\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}}\left[\int_0^\infty \hat{p}_s\left(k,\vec{t}\right)(1-F_k(t_k))dt_k\right]\\
\overset{(a)}{\leq}&\sum_k\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}}\left[\int_0^\infty \hat{p}_s\left(k,\vec{t}\right)\frac{1-F_k(t_k)}{\bar{x}_k+\bar{y}_k}dt_k\right]\\
&-\sum_{k=1}^K \lambda_k\times\left(\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}}\left[\int_0^\infty p_s\left(k,\vec{t}\right)f_k\left(t_k\right)dt_k\right]-\eta'_k\right)\\
=&\sum_k\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}}\left[\int_0^\infty \hat{p}_s\left(k,\vec{t}\right)\left(\frac{1-F_k(t_k)}{\bar{x}_k+\bar{y}_k}-\lambda_k f_k(t_k)\right)dt_k\right]\\
&+\sum_{k=1}^K\lambda_k\eta'_k\\
\overset{(b)}{\leq}&\sum_k\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}}\left[\int_0^\infty p_s^*\left(k,\vec{t}\right)\left(\frac{1-F_k(t_k)}{\bar{x}_k+\bar{y}_k}-\lambda_k f_k(t_k)\right)dt_k\right]\\
&+\sum_{k=1}^K\lambda_k\eta'_k\\
=&\sum_k\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}}\left[\int_0^\infty p_s^*\left(k,\vec{t}\right)\frac{1-F_k(t_k)}{\bar{x}_k+\bar{y}_k}dt_k\right]\\
&-\sum_{k=1}^K \lambda_k\times\left(\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}}\left[\int_0^\infty p_s^*\left(k,\vec{t}\right)f_k\left(t_k\right)dt_k\right]-\eta'_k\right)\\
\overset{(c)}{=}&\sum_k \frac{1}{\bar{x}_k+\bar{y}_k}\sum_s \pi_k^s E_{\vec{t}_{\bar{k}}}\left[\int_0^\infty p_s^*\left(k,\vec{t}\right)(1-F_k(t_k))dt_k\right]\\
=&C_s(p^*)
\end{aligned} \tag{8}$$

In the above equation, (a) holds because $\hat{p}$ is a feasible policy that satisfies the collision probability constraints on all channels, (b) holds by the definition of p^*, and (c) holds because of the complimentary slackness condition in (iii). ■

The intuition of the optimal policy p^* is as follows: the SU should choose a PU channel that has the smallest weighted conditional collision probability. Note that $f_k(t_k)/(1-F_k(t_k))$ can be considered as the conditional collision probability. And $\left(\frac{1-F_k(t_k)}{\bar{x}_k+\bar{y}_k}-\lambda_k\frac{f_k(t_k)}{\bar{y}_k}\right)$ is a weighted form reflecting conditional collision probability. The weight λ_k (the Lagrangian Multiplier for channel k's PU protection constraint) reflects the impact of PU k's collision probability constraint η_k. In addition, if none of the idle channels are good enough (i.e., $f_k(t_k)/(1-F_k(t_k)) \geq \bar{y}_k/(\bar{x}_k+\bar{y}_k)\lambda_k$ for all k), then the SU does not transmit. The policy clearly indicates the "opportunistic" nature of the SU transmission: SU transmits when the collision probability is relatively low both among users and across time.

Next, we consider the existence of the above policy. We define

$$\begin{aligned}
&P_{tie}\\
=&\sum_s \pi_s E_{\vec{t}}\left[P\left\{\left(\frac{1-F_k(t_k)}{\bar{x}_k+\bar{y}_k}-\lambda_k\frac{f_k(t_k)}{\bar{y}_k}=z\right)\right.\right.\\
&\qquad\cap\left(\frac{1-F_j(t_j)}{\bar{x}_j+\bar{y}_j}-\lambda_j\frac{f_j(t_j)}{\bar{y}_j}=z\right)\\
&\qquad\cap\cap k\neq j\}],
\end{aligned}$$

where

$$z=\min_i\left(\frac{1-F_i(t_i)}{\bar{x}_i+\bar{y}_i}-\lambda_i\frac{f_i(t_i)}{\bar{y}_i}\right).$$

In other words, p_{tie} is the probability of a tie in choosing the minimum value in policy p^*.

Proposition 2: The policy defined in Proposition 1 exists if $P_{tie}=0$.

Proof: The existence of the Lagrangian multiplier vector follows the Strong Duality Theorem (e.g., Proposition 5.3.1 in [21]). Note here Lagrangian multiplier is defined as follows: a vector is said to be a Lagrangian multiplier for the primal problem if it is nonnegative and the minimum value of the primal problem equals to the infimum of the corresponding Lagrangian function.

The problem defined in Eq. (6) is a convex optimization problem with a finite value of the objective function (≤ 1). In addition, if $p\equiv 0$, all collision probabilities are equal to zero, and therefore satisfy the constraint η'_i with strict inequality, i.e. the interior point condition (Slater) is satisfied. Therefore, by the Strong Duality Theorem, there exists at least one Lagrangian multiplier vector, denoted as $\vec{\lambda}$.

By the definition of the Lagrangian multiplier, optimal solutions can be obtained in minimizing the Lagrangian function. Since the probability of tie is zero, p^* minimizes the Lagrangian function and thus is an optimal solution of the primal problem. Because (λ, p^*) is a Lagrangian-primal pair, the three conditions are satisfied. ■

Note: At least one Lagrangian multiplier exists for the problem presented in the problem defined in Eq. (6). In addition, at least one optimal policy exists that minimizes the Lagrangian. However, for a given Lagrangian multiplier, there could be multiple solutions ($\vec{p}$) that minimizes the Lagrange function and not all of them are optimal solutions. Therefore, when there are ties in Eq. (7) (with non-zero measure), we need to decide which one is optimal that satisfies the collision probability constraints. When there are ties, randomized policies may be necessary. Exponential distribution is one such example, as discussed in detail next.

V. Exponential Case

In this section, we study a special case of the general problem where the PUs' have exponentially distributed idle times. In this case, ties always happen and one cannot rely on the Lagrange duality-based approach described in the previous section to find an optimal policy.

Because of the memoryless property of the exponential distribution [22], time becomes irrelevant. Due to the memoryless property, it also suffices to find a stationary transmit probability for each idle channel that depends only on the current state. We denote this time-independent policy by the simple notation $p_s(k)$.

The general optimization then simplifies naturally to the following linear program (LP):

$$\begin{aligned}\max_{p} \quad & \sum_{s} \pi_s \sum_{k=1}^{K} p_s\left(k\right) \\ \text{s.t.} \quad & \sum_{s} \pi^s_k p_s\left(k\right) \leq \eta'_k, \quad k=1,...,K, \\ & \sum_{k=1}^{K} p_s(k) \leq 1, \quad \forall s \\ & p_s(k) \geq 0, \quad \forall s,k.\end{aligned} \tag{9}$$

We explicitly point out that $p_s(k) = 0$ if $k \notin A_s$. We note that exponential distribution results in the worst case SU capacity among all idle time distributions assuming the same average idle time and collision probability constraint. The explanation is as follows. The problem defined in Eq. (9) provides an optimal solution on SU capacity assuming exponentially distribution idle time. If we apply this (randomized) policy to a general idle time distribution, the policy results in the same SU capacity as in the exponential case. However, the policy defined in Prop. 1 is an optimal policy under the general idle time distribution, which, by definition, results in higher or equal SU capacity compared to the randomized policy. Therefore, exponential distribution results in the worst case SU capacity among all idle time distributions. Last, we note that we can use the randomized policy defined in Eq. (9) as a feasible randomized policy when we do not have knowledge on PU distributions or $\vec{t}$.

While any linear program can be solved in polynomial time with respect to the input size, in this case the number of states, and therefore the number of variables $p_s(k)$, is exponential in the number of channels. To handle settings with larger numbers of channels, therefore, we propose a heuristic algorithm that reduces the computational complexity of the LP formulation.

The idea behind the heuristic we propose is simple. When the number of idle PU channels is small, the resource is scarce and the SU doesn't have many alternative choices. In this case, we can solve the exact LP problem to obtain the optimal policy. When the number of idle PU channels is large, the resource is plentiful and the SU has relatively greater flexibility on channel selection. In this case, we can handle the policy more coarsely to reduce the number of variables. Formally, the heuristic algorithm can be expressed as follows.

Let $1 \leq G \leq K$ represent a pre-defined granularity parameter for the heuristic. Let $\bar{b}_s$ denote the number of idle channels at state s.

- For the currently busy channel k, $p_s(k) = 0$.
- For currently idle channel k, there are two subcases.
  - For the states s where $\bar{b}_s > G$,
  $$p_s(k) = min(\frac{1}{\bar{b}_s}, \eta'_k)$$
  - For the states s where $\bar{b}_s \leq G$,
  $$p_s(k) = argmax \sum_{s} \pi_s \sum_{k=1}^{K} p_s(k)$$
  subject to the same constraints as shown in (9).

Note that compared to $K2^K$ variables in the linear program given in (9) that yields the optimal solution, this heuristic solves a smaller LP with the number of variables reduced to $\sum_{i=1}^{G} i \cdot C(K,i)$. When G is small compared to K, this reduction in complexity could be substantial (albeit at the expense of potentially worse performance due to the decreased granularity).

VI. Simulation Results

The heuristic algorithm reduces the computational complexity of the original linear programming at the expense of throughput performance. However, we have found empirically through various simulations that the heuristic algorithm can generally obtain expected throughput close to optimal for reasonable low values of G. In this section, we present some representative simulation data for 4 channels and $G = 2$.

We use the *bpmpd* solver at the NEOS optimization server [23] to compute both the optimization solution for the linear programming and the sub-optimal solution for the heuristic algorithm. We present the expected throughput comparison for ten representative cases in table II. We can see that in most cases, the heuristic algorithm performs close to the optimal solution in terms of expected throughput. The cases where it gives poor performance, such as case 8, have the feature that there are multiple channels that have both a high probability of being idle and strict constraints.

While we have evaluated the heuristic only for a relatively small number of channels, we should note that unless primary user occupancy and the interference constraint are also scaled to be correspondingly stricter,

TABLE II

COMPARISON OF THROUGHPUT OBTAINED BY THE OPTIMAL AND HEURISTIC SCHEMES

Case Number	$\frac{\bar{x}_i}{\bar{x}_i+\bar{y}_i}$ $i = \{1, 2, 3, 4\}$	$(\eta'_1, \eta'_2, \eta'_3, \eta'_4)$	Optimal	Heuristic	Ratio
1	$(1/3, 2/3, 2/3, 1/3)$	$(0.2, 0.2, 0.2, 0.2)$	0.4	0.4	1
2	$(1/3, 2/3, 2/3, 1/3)$	$(0.2, 0.2, 0.2, 0.25)$	0.42	0.42	1
3	$(1/3, 2/3, 2/3, 1/3)$	$(0.2, 0.2, 0.25, 0.25)$	0.45	0.45	1
4	$(1/3, 2/3, 2/3, 1/3)$	$(0.2, 0.25, 0.25, 0.25)$	0.48	0.48	1
5	$(1/3, 2/3, 2/3, 1/3)$	$(0.25, 0.25, 0.25, 0.25)$	0.5	0.5	1
6	$(1/9, 8/9, 8/9, 1/9)$	$(0.1, 0.8, 0.8, 0.1)$	0.99	0.95	0.96
7	$(1/9, 8/9, 8/9, 1/9)$	$(0.1, 0.8, 0.1, 0.8)$	0.9	0.86	0.96
8	$(1/9, 8/9, 8/9, 1/9)$	$(0.8, 0.1, 0.1, 0.8)$	0.36	0.28	0.78
9	$(1/9, 8/9, 8/9, 1/9)$	$(0.8, 0.8, 0.8, 0.1)$	0.99	0.97	0.98
10	$(1/9, 8/9, 8/9, 1/9)$	$(0.1, 0.1, 0.1, 0.8)$	0.28	0.24	0.86

the throughput performance of the heuristic will in fact improve as more channels become available.

VII. DISCUSSIONS

Throughout the paper, we have assumed that there exists one SU. In fact, all results in the paper can be extended to the case with M coordinated SUs and the case that one SU can transmit on M channels simultaneously. In particular, we simply need to change the constraint to $\sum_{k=1}^{K} p_s(k, \vec{t}) \leq 1$ to $\sum_{k=1}^{K} p_s(k, \vec{t}) \leq M$, and include an additional constraint $0 \leq p_s(k, \vec{t}) \leq 1$. We avoid the detailed formulation due to notation complexity.

We also note that in the case of general distributions, although a threshold policy is presented in a clear form. It is somewhat difficult to find the policy numerically, because it depends not only on the state s, but also on $\vec{t}$. Even when the optimal policy is found, its application depends on the perfect information on $\vec{t}$. Therefore, it will be of practical value to find good heuristics to implement this policy. There are following options.

The first possibility is to degenerate the optimal policy that depends on $\vec{t}$ to randomized policies as those developed in the exponential case. The randomized policies do not depend on the values of $\vec{t}$ and thus are much easier in practice. In addition, it does not require information on idle/busy distribution. The only information needed is the average idle/busy time, which is much easier to obtain. The randomized policy balances the transmission among multiple channels, and thus is opportunistic among multiple channels. On the other hand, the randomized policy is not opportunistic across time (i.e., to select relatively good time to transmit) because it is not a function of time.

The second option is to simplify the selection among channels. Consider the single channel case (i.e., $K = 1$). It is shown in [20], as well as in Prop. 1 by having $K = 1$, that the optimal policy is a threshold policy. In other words, if the conditional collision probability is smaller than a threshold, the SU should transmit. Otherwise, the SU should not transmit. The threshold can be found using simple stochastic approximation techniques, even in the case without prior knowledge of PU idle/busy distributions [20]. To extend the results to multiple PU channels, we can allow each PU channel to have its own threshold. A channel is considered favorable if the conditional transmission probability is smaller than the threshold. The SU can then randomly choose a favorable PU channel to transmit if available. Note that this threshold depends on the number of PU channels. To obtain the threshold for each channel, one can again apply stochastic approximation techniques. Intuitively, if the collision probability observed on the channel is higher than the PU constraint, one increases the threshold for the channel. If the collision probability is lower, one can decrease the threshold.

VIII. CONCLUSION

In this paper, we study optimal transmission strategies in multi-channel cognitive radio networks where one secondary user (SU) opportunistically accesses multiple orthogonal channels. We derived optimal access policies for a single SU under general PU idle-busy time distributions. The optimal policy clearly indicates the “opportunistic” nature of the SU transmission: SU transmits when the collision probability is relatively low both among users and across time. On the other hand, when PU channels have independent exponentially distributed idle time, the opportunistic nature of the SU transmission disappears because all opportunities are equal. For this case, we formulate a linear program that yields an optimal randomized strategy for the secondary user as well as a tunable-granularity heuristic. We note that exponential distribution results in the worst-case SU capacity among all distributions assuming the same average idle time and collision probability constraint.

We note that there are idealized assumptions in this paper, including perfect sensing, infinitely small packet size, and no overhead. Under these assumptions, the SU performance is the benchmark performance and the (idealized) SU access scheme can provide insights/guidelines on the SU access for a more realistic scenario. Certain assumptions are easy to address, including overhead and packet size. The assumption that the SU has the perfect knowledge on all PU channels is the most challenging assumption. It is our future work to generalize it to imperfect sensing and imperfect information on all channels.

REFERENCES

[1] Q. Zhao and B. Sadler, "A survey of dynamic spectrum access: Signal processing, networking, and regulatory policy," *IEEE Signal Processing Magazine*, vol. 55, no. 5, pp. 2294–2309, 2007.

[2] Shared spectrum company: DARPA XG program information. [Online]. Available: http://www.sharedspectrum.com/technology/darpaxg.html

[3] F. W. Seelig, "A description of the august 2006 xg demonstrations at fort a.p. hill," in *Second IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, 2007, pp. 1–12.

[4] S. Huang, X. Liu, and Z. Ding, "Opportunistic spectrum access in cognitive radio networks," *IEEE INFOCOM 2008, Phoenix, AZ, USA, April*, 2008. [Online]. Available: "http://www.ece.ucdavis.edu/˜senhua/OSAinfocom08.pdf"

[5] Q. Zhao, S. Geirhofer, L. Tong, and B. M. Sadler, "Optimal dynamic spectrum access via periodic channel sensing," in *Proc. Wireless Communications and Networking Conference (WCNC)*, 2007.

[6] T. Shu, S. Cui, and M. Krunz, "Medium access control for multi-channel parallel transmission in cognitive radio networks," *Proceedings of the IEEE GLOBECOM 2006 Conference, San Francisco, CA, Dec.*, 2006.

[7] R. Urgaonkar and M. J. Neely, "Opportunistic scheduling with reliability guarantees in cognitive radio networks," *IEEE INFOCOM 2008, Phoenix, AZ, USA, Apr.*, 2008.

[8] D. I. Kim, L. B. Le, and E. Hoossain, "Joint rate and power allocation for cognitive radios in dynamic spectrum access environment," *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 5517–5527, 2008.

[9] Y. Chen, G. Yu, Z. Zhang, H.-H. Chen, and P. Qiu, "On cognitive radio networks with opportunistic power control strategies in fading channels," *IEEE Transactions on Wireless Communications*, vol. 7, no. 7, pp. 2751–2761, 2008.

[10] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 3, pp. 589–600, 2007.

[11] Y. Chen, Q. Zhao, and A. Swami, "Joint design and separation principle for opportunistic spectrum access in the presence of sensing errors," *Proc. of IEEE Asilomar Conference on Signals, Systems, and Computers, November,*, 2006.

[12] Q. Zhao and K. Liu, "Detecting, tracking, and exploiting spectrum opportunities in unslotted primary systems," *Proc. of IEEE Radio and Wireless Symposium (RWS), Jan.*, 2008.

[13] Q. Zhao, B. Krishnamachari, and K. Liu, "On myopic sensing for multi-channel opportunistic access: Structure, optimality, and performance," *IEEE Transactions on Wireless Communications*, 2008.

[14] S. Ahmad, M. Liu, T. Javidi, Q. Zhao, and B. Krishnamachari, "Optimality of myopic sensing in multi-channel opportunistic access," *IEEE Transactions on Information Theory*, 2009.

[15] S. H. A. Ahmad and M. Liu, "Multi-channel opportunistic access: A case of restless bandits with multiple plays," 2009.

[16] J. Ye and Q. Zhao, "Quickest change detection in multiple on-off processes: Switching with memory," 2009.

[17] J. Unnikrishnan and V. V. Veeravalli, "Dynamic spectrum access with learning for cognitive radio," 2008.

[18] S. Huang, X. Liu, and Z. Ding, "Optimal sensing-transmission structure for dynamic spectrum access," in *IEEE Infocom*. IEEE, 2009.

[19] H. Kim and S. K. G., "Efficient discovery of spectrum opportunities with MAC-layer sensing in cognitive radio networks," *IEEE Transactions on Mobile Computing*, vol. 7, no. 5, pp. 533–545, May 2008.

[20] S. Huang, X. Liu, and Z. Ding, "Optimization of transmission strategies for opportunistic access in cognitive radio networks," *IEEE Transactions on Mobile Computing,*, vol. 8, no. 12, Dec. 2009.

[21] D. Bertsekas, *Nonlinear Programming*. Athena Scientific, 1999.

[22] S. M. Ross, *Introduction to Probability Models, Ninth Edition*. Orlando, FL, USA: Academic Press, Inc., 2006.

[23] The neos server for optimization. [Online]. Available: http://www-neos.mcs.anl.gov/