# Randomized Strategies for Multi-User Multi-Channel Opportunity Sensing

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*Abstract*— **We show how the expected network throughput of contending secondary users in an opportunistic spectrum access network can be optimized by making appropriate sensing decisions. We consider both uncoordinated symmetric users that see the same primary user behavior, and also a more general coordinated asymmetric setting. For the uncoordinated symmetric case, we show that when the number of users exceeds the number of channels, the optimal strategy is independent of primary user behavior. For the coordinated asymmetric case, we show that at the optimal operation point each user can adopt a pure strategy. Furthermore, the optimal solution can be obtained by using the Hungarian algorithm for bipartite maximum weight matching.**

## I. Introduction

Cognitive radios are gaining attention due to their promise of alleviating the fundamental challenges associated with limited bandwidth. A set of approaches focuses on opportunistic spectrum access, where secondary users proble/sense channels to determine if they may safely send packets without interfering with primary users. In case of multiple channels, a key component of these approaches is determining which channel to sense. Most prior work has focused on the sensing decision from the perspective of a single secondary user. When there are multiple users contending for opportunities, however, the sensing decision must take into account the possibility that good channels may also be desired by other secondary users.

We consider in this paper how the expected total throughput of multiple secondary users can be maximized by treating the sensing policy as a multi-channel randomized multi-access decision. We formulate and solve two problems under this framework, both pertaining to making sensing decisions for a single stage of a time-slotted system in which primary user occupancy probabilities are known forehand.

In the first problem, we consider a symmetric setting in which contending secondary users experience a common set of known channel occupancy probabilities for primary users. The goal is to obtain a randomized sensing policy whereby each channel is selected for sensing with some given probability. We show that the optimal policy randomizes over all channels whose primary occupancy probability is strictly less than one. Somewhat surprisingly, we find that when the number of users $n$ exceeds the number of channels $m$, then the optimal strategy for each user is to sense each channel with probability $\frac{1}{n}$ regardless of the primary occupancy distribution; in this case, there can be some incentive for users to ascribe some nonzero probability to desist from sensing any channel at all.

In the second problem, we consider a more general coordinated case where the contending secondary users may each experience different primary channel occupancy probabilities. In this asymmetric case, we assume that primary user occupancy probabilities for all channels are known *a priori* to each contending secondary user. The goal is to maximize the total network throughput in a coordinated manner. We show that in this case, there exists an optimal operating point where each secondary user chooses a pure strategy. At this point, each user either picks a particular channel to sense with probability one or none at all. Furthermore, we map this problem to a maximum weight matching problem on bipartite graphs. This combinatorial problem can then be solved efficiently by applying the well-known Hungarian algorithm [12].

The rest of the paper is organized as follows. We briefly review some related work in section 2. We formulate and solve the symmetric problem in section 3. We then consider the more general asymmetric case

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in section 4. We conclude with some discussion and thoughts on future directions in section 5.

## II. Related Works

There are several related lines of research on opportunistic spectrum access. Several studies (e.g., [2], [3], [4], [6]) consider multiple rounds of probing/sensing before a channel access decision is made. These are typically formulated as stopping time problems. Other studies, such as [1], consider the setting in which sensing takes place on only one channel at the beginning of each time slot; in this case a key question is that of deciding which channel to sense at each stage. This is formulated as a partially observable Markov decision process (POMDP). However, all these works focus on deciding sensing strategies from the perspective of a single user. Our focus in this paper is to consider the sensing decision in the case of multiple users. We follow the approach of [1] in treating a time-slotted system in which only one channel can be sensed before each transmission, but our modeling differs from that work in that it considers the primary users behavior not as a Markov process but in the form of a single-stage a priori probability of channel occupancy by primary users.

Some studies focusing on multiple users have proposed the use of separate coordination channels or control messages ([5], [11]) to prevent overlapping transmission of secondary users by allowing them to reserve the channel. Others have proposed scheduling methods for distributed channel allocation ([7], [8]). In these approaches, generally the multi-access problem is considered separately from channel sensing. In contrast, we focus here on a light-weight random access approach based on Slotted Aloha, and take contention into account as part of sensing decision itself.

There is some prior literature on multichannel slotted Aloha (e.g. [9], [10]), but these generally consider homogeneous channels from which one is selected with equal probability. We explicitly consider channels with different qualities in this work, as defined by different primary user occupancy probabilities which result in a potentially different sensing probability for each channel. Further, our focus on multichannel slotted Aloha for making throughput-optimal sensing decisions in a cognitive radio context is quite different.

## III. The Symmetric Uncoordinated Problem

### A. Problem Formulation

We consider a system with $N = \{1, 2, ..., n\}$ secondary users. They can sense $M = \{1, 2, ..., m\}$ accessible channels. For user $i$, each channel $j$ has a probability $p_{i,j}$ that it will be available for use (i.e., sensing it would reveal that it's free[1]). In other words, $1 - p_{i,j}$ is the probability that channel $j$ is occupied by a primary user that can be sensed by the user $i$. The strategy for each secondary user is to decide with what probability to sense each channel so that they can minimize collision with all other secondary users (and avoid colliding with primary user) in order to maximize the total expected throughput. Mathematically, the strategy for user $i$ is a probability vector $(q_{i,1}, q_{i,2}, ..., q_{i,m})$ where $\sum_{k=1}^{m} q_{i,k} \leq 1$ and $q_{i,k} \geq 0$ for all $k \in M$. Notice that we allow the case that a secondary user avoids choosing any channel to sense in a given round.

In this section, we consider the case where all the secondary users have the same evaluation of the availability of each channel. That is, $p_{i,j} = p_{k,j}$ for all $i, k \in N$. To simplify the notation, in this section we denote $p_{i,j}$ as $p_j$. In this case, in the absence of explicit coordination, it is clear that the secondary users should have symmetric strategies. Therefore, we denote the symmetric strategy for each secondary user as vector $(q_1, q_2, ..., q_m)$. We assume that channel $k$ provides bandwidth $B_k$.

The problem now can be formed mathematically as follows:

$$\max \quad n \sum_{k=1}^{m} q_k (1 - q_k)^{n-1} p_k B_k \tag{1}$$

$$s.t \quad \sum_{k=1}^{m} q_k \leq 1 \tag{2}$$

Since each user is identical in this symmetric setting, we can simplify the objective function to the per-user expected throughput:

$$\max \sum_{k=1}^{m} q_k (1 - q_k)^{n-1} p_k B_k \tag{3}$$

### B. Analysis of the Optimal Solution

In this subsection, we use Lagrangian multipliers and the KKT (Karush-Kuhn-Tucker) conditions to solve the above constrained optimization problem.

Introduce Lagrangian multiplier $\lambda$ to the optimization problem with simplified objective function, we get:

$$L(q_k, \lambda) = \sum_{k=1}^{m} q_k (1 - q_k)^{n-1} p_k B_k - \lambda(\sum_{k=1}^{m} q_k - 1) \tag{4}$$

Apply KKT conditions, we need to solve two cases as follows: a) when $\lambda = 0$ and $\sum_{k=1}^{m} q_k < 1$; b)$\lambda \neq 0$ and $\sum_{k=1}^{m} q_k = 1$. We solve the two cases in following.

[1] We assume perfect sensing in this work.

Case a): We substitute the condition $\lambda = 0$ back to equation (4) and take first order derivative regarding to $q_k$. We have

$$\frac{\partial L}{\partial q_k} = (1-q_k)^{n-1} p_k B_k - q_k(n-1)(1-q_k)^{n-2} p_k B_k \quad (5)$$

The optimal value holds when $\frac{\partial L}{\partial q_k} = 0$. Hence we get $q_k = \frac{1}{n}$. Now we need to check the Hessian matrix of the objective function to verify that the critical point we get is the maximum point. Notice that $\frac{\partial^2 L}{\partial q_k \partial q_j} = 0$ for all $k \neq j$. Therefore Hessian matrix of this function is a diagonal matrix. There exist $m$ eigenvalues for this Hessian matrix. They are $\frac{\partial^2 L}{\partial^2 q_k}$ for all $k$. We can verify that whenever $q_k \leq \frac{2}{n}$, $\frac{\partial^2 L}{\partial^2 q_k} \leq 0$. That is, when condition $q_k \leq \frac{2}{n}$ is satisfied for all $k$, the Hessian matrix is negative definite. Hence the critical point is the maximum point.

Notice that we also have to satisfy the condition $\sum_{k=1}^{m} q_k < 1$, we know that this case only holds when $\frac{m}{n} < 1$, i.e. $m < n$. This means that when the number of the secondary user is larger than the accessible channel number, the best strategy for each secondary user is just to choose $\frac{1}{n}$ probability on sensing each channel, no matter what the channel availability distribution is. This is a somewhat counter-intuitive result at first glance, but essentially it says that in this over-crowded scenario, each user should operate at an secondary-interference-limited optimal point on each channel.

When $n \geq m$ holds, the total optimal throughput for the network is

$$n \sum_{k=1}^{m} \frac{1}{n}(1-\frac{1}{n})^{n-1} p_k B_k$$

Let $n \to \propto$, we have

$$\lim_{n \to \propto} n \sum_{k=1}^{m} \frac{1}{n}(1-\frac{1}{n})^{n-1} p_k B_k = \frac{1}{e} \sum_{k=1}^{m} B_k p_k$$

This is the network throughput with relatively large number users sharing limited channels.Note that the $\frac{1}{e}$ loss in efficiency compared to perfect scheduling here is the same as in traditional slotted Aloha.

Case b): This is the case where $m \geq n$. We take the first order derivative of equation (4),

$$\frac{\partial L}{\partial q_k} = B_k p_k[(1-q_k)^{n-1} - q_k(n-1)(1-q_k)^{n-2}] - \lambda \quad (6)$$

Again, a critical value holds when $\frac{\partial L}{\partial q_k} = 0$. This critical value is the maximum value when Hessian matrix is negative definite. That is, for all $k = 1, 2, ..., m$, $q_k \leq \frac{2}{n}$. For arbitrary distribution of $p_k$, the first order derivative equation is a high-order polynomial equation of $q_k$ that can be solved numerically.

Below, we give an exact solution when $n = 2$ (when we have linear equations).

Substitute $n = 2$ back into equation (6) and considering condition $\sum_{k=1}^{m} q_k = 1$, we can solve the group of resulting linear equations and get

$$\lambda = \frac{m-2}{\sum_{k=1}^{m} \frac{1}{p_k B_k}}$$

and

$$q_k = \frac{1}{2} - \frac{\lambda}{2 p_k B_k} = \frac{1}{2} - \frac{m-2}{2 p_k B_k \sum_{k=1}^{m} \frac{1}{p_k B_k}}$$

Note that in two users case, $\frac{2}{n} = 1$ and the probability to access each channel is no greater than 1. Hence the Hessian is negative-definite in this case. This results shows us that in the two secondary users case, no user would like to put more than $\frac{1}{2}$ probability to a channel, no matter what the channel conditions are.

### C. Illustrative Example

We give an illustration of the solution for the case where the channel availability probability distribution is exponential. We suppose that all the channels are indexed by numbers $1, 2, ..., m$ and that channel $k$'s availability probability is represented as $p_k = c^{-k}$ where $c$ is a constant. Table 1 illustrates the best strategies for a secondary user with the exponential constant $c$ changes when there are totally 2 users and 3 channels accessible.

TABLE I

NUMERICAL RESULTS OF $q_k$ FOR A 2-USER SYSTEM WITH 3 ACCESSIBLE CHANNELS, WHEN CHANGING EXPONENTIAL CONSTANT $c$

| | $c=2$ | $c=3$ | $c=4$ | $c=5$ | $c=6$ | $c=7$ |
|---|---|---|---|---|---|---|
| $p_1 = \frac{1}{c}$ | 0.43 | 0.46 | 0.48 | 0.48 | 0.49 | 0.49 |
| $p_2 = \frac{1}{c^2}$ | 0.36 | 0.39 | 0.40 | 0.42 | 0.43 | 0.44 |
| $p_3 = \frac{1}{c^3}$ | 0.21 | 0.15 | 0.12 | 0.10 | 0.08 | 0.07 |

## IV. THE ASYMMETRIC COORDINATED PROBLEM

In this section, we generalize the previous formulation to a more complex case. We now consider that each secondary user has a different view of the channel availability probabilities. Figure 1 illustrates how this situation could happen. In Figure 1, user P1 and P2 are primary users who are relatively far from each other so that they can use the channel without affecting each other. User S1 and S2 are two secondary users who are located between primal user P1 and P2. User S1 can collide with both user P1 or user S2, but will not collide with P2. Similarly, user S2 interferes with user P2 and

S1, but not P1. According to the different behavior of primary users P1 and P2, secondary users S1 and S2 have different views of the availability probabilities of this particular channel.

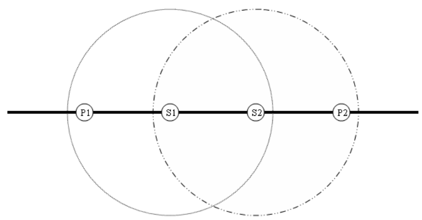


Fig. 1. Illusion for two secondary users have different views on channel availability

In this case, we use the original notation to formulate the problem. Notice that the discussion in previous section is a special case of this more generalized case. We also assume that each secondary user knows all the other secondary user's view for the channel availability. That is, we assume each second user has a prior global information on the channel availability. Now the optimization problem is[2]:

$$\max \quad \sum_{i=1}^{n}\sum_{k=1}^{m}(q_{i,k}\cdot p_{i,k}B_k \prod_{j\in N, j\neq i}(1-q_{j,k})) \tag{7}$$

$$such\ that \qquad \sum_{k=1}^{m} q_{i,k} \leq 1 \qquad \forall i \in N \tag{8}$$

In the rest of this paper, the bandwidth $B_k$ is removed from the original objective function for clarity (replacing $p_{i,k}$ with $p_{i,k}B_k$ everywhere would easily correct for this omission). We also denote the objective function as $f$ in the following.

In this asymmetric case, applying KKT condition can also derive a critical point. However, we found that the Hessian matrix of objective function for asymmetric case is NOT negative-definite. Hence, the critical point derived from Lagrangian relaxation and KKT condition is not the maximum point.

We define a *pure strategy* for a secondary user $i$ as a strategy which user $i$ allocates sensing probability for at most one channel as 1 and all other channels as 0. Mathematically, user $i$'s strategy is called a pure strategy when $q_{i,k} = 1$ or $q_{i,k} = 0$ for all $k \in M$ while satisfying $\sum_{k=1}^{m} q_{i,k} \leq 1$. A strategy profile $S$ defines all secondary users' strategies. A pure strategy profile is a strategy profile where all secondary users choose pure strategies. Each strategy profile $S$ is associated with a performance value $f(S)$, where $f$ is the objective function. In the following, we will prove that in asymmetric case, the optimal operation point is associated with a pure strategy profile.

[2]We mention briefly that this problem formulation as well as the symmetric case studied before can be generalized easily to handle realistic SINR-based channel capture effects. It is not essential to the formulation that simultaneous transmission must necessarily result in collision for both secondary users.

**Lemma 1:** Given a strategy profile $S$, there always exists a pure strategy profile $\tilde{S}$ such that $f(S) \leq f(\tilde{S})$.

Due to the page limitation, we briefly sketch the proof of this lemma here. In order to prove this lemma, we claim that: when all other secondary users' strategies are kept fixed, we can always keep the total objective the same or higher by making a particular user $i$'s strategy a pure strategy. Without loss of generality, let us focus on user 1. Since we fix all other secondary users' strategies, they can be treated as constants. Now the objective function becomes a function of $q_{1,k}$. Actually, it is a linear combination of $q_{1,k}$. We can rewrite the objective function as:

$$\max \sum_{k=1}^{m} a_{1,k}q_{1,k} + b_{1,k}$$

with constraint $\sum_{k=1}^{m} q_{1,k} \leq 1$, where $a_{1,k}$ and $b_{1,k}$ are constants.

It is not hard to verify that the above function's maximal value holds when user 1 adopts a pure strategy. Specifically, if $a_{1,k} \leq 0$ for all $k \in M$, then $q_{1,k} = 0$ for all $k$. Otherwise, get the index of maximal $a_{1,k}$ and set the corresponding $q_{1,k} = 1$, $q_{1,j} = 0$ for all $j \neq k$. Since this newly constructed pure strategy maximizes the value of the objective function, it performs at least the same as the given strategy in terms of secondary user 1. Hence the claim is proved.

When this claim holds, we can improve the value of the objective function by changing user strategies one by one. Start from the first user's strategy in $S$; after $n$ iterations, we will end up with a pure strategy profile $\tilde{S}$ such that $f(S) \leq f(\tilde{S})$. The following theorem follows directly from Lemma 1.

**Theorem 1:** The maximal value of the objective function is associated with a pure strategy profile.

Note that at the optimal operating point, at most one user will sense any given channel. Collision among secondary users will reduce total network throughput. Applying this theorem to the objective function, we can reduce the asymmetric case to a maximum weighted matching problem in a bipartite graph. Theorem 1 indicates that the maximal value of the objective function is the summation of a sequence of $p_{i,k}$. Now we construct a weighted undirected bipartite graph $G_b(V_1, V_2; E)$, where $V_1 = N$ and $V_2 = M$. The weight of the corresponding edge $e_{i,j} \in E$ is $p_{i,k}$. A matching pair $(i, j)$ in the solution means $q_{i,j} = 1$. The maximal value of the objective function is the total weight for the maximum weighted matching in this bipartite graph $G_b$.

The maximum weighted matching problem in bipartite graph can be solved using Hungarian algorithm [12] in polynomial time. Specifically, if $m \leq n$, it can be solved in $O(nm^2)$ time; if $m \geq n$, it can be solved in $O(mn^2)$ time for our asymmetric problem. Note that when $m \leq m$, i.e. secondary user number is larger than the channel number, some secondary users will be unmatched, which means these secondary users will not sense any channel.

## V. Conclusion and Future Works

In the context of sensing-based opportunistic spectrum access, multiple secondary users may contend with each other either during the sensing-decision phase or during transmission. When secondary users transmit data after each sensing decision, as we have considered in this work, the contention must be handled to some extent in the sensing-decision phase itself. The question then becomes fundamentally one of deciding how to randomize between the available channels in deciding which one to sense. We have formulated this problem under both symmetric and (more general) asymmetric cases.

In the symmetric case, we found in general that there is always some incentive to sense any channel that may be potentially free, with the better channels more preferred in general. However, when the number of users is greater than the number of channels, we showed that the optimal strategy that maximizes the total throughput is to pick each channel with a probability that is the inverse of the number of users. This is somewhat surprising as this implies that the optimal strategy is then oblivious to the primary user occupancy behavior of the channels.

In the asymmetric case, we observe that the maximal value of the objective function occurs on when all the secondary users choose pure strategy. This observation simplifies the problem to a maximum weight matching in a bipartite graph, which can be solved in polynomial time by the Hungarian algorithm. However, in this case, when the number of users is larger than the number of channels, in order to get total network throughput optimized, some secondary users will never get a chance sense a channel, which leads to severe unfairness.

There are many interesting future directions that can be considered. One question is whether a generalization is possible that extends the single-stage optimization problem presented here to a multi-stage optimization which takes into account both future and immediate rewards, based on richer models of primary user behavior (e.g. a Markov process, as considered in [1]). In such a case, users may have incomplete or even flawed information regarding the primary user occupancy probability for each channel. We would also like to generalize the asymmetric case to a multi-hop setting where not all secondary users share a common medium, where interference could be depicted, for instance, via a conflict graph. Determining equilibrium strategies for selfish users is also of interest; this will address, in part, the fairness problem of current solution. Another approach to address this fairness problem would be to change the objective function to maximize the minimum throughput among all secondary users.

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