Microeconomic Analysis of Base-Station Sharing in Green Cellular Networks

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***Abstract*—Cellular networks can be operated more energy-efficiently if operators agree to share base-stations during off-peak hours. We apply a micro-economic analysis for a single-cell two-operator scenario to investigate the conditions under which self-interested operators would agree to share resources in this manner. Our analysis yields a comprehensive treatment of the existence and number of Nash Equilibria. We consider the cases when the payment rates are exogenous, as well as when they can be set strategically by the operators. Through numerical solutions we examine the quality of the best and worst Nash Equilibria in comparison with the globally optimized solution. Our results show that there is often a sensitive dependence on key parameters such as energy price, capacity, load, revenues, penalties and payments.**

I. Introduction

With the growing demand for mobile data services, cellular wireless networks have been growing quite rapidly in recent years and are expected to continue to do so in the near future. The energy consumption of mobile telephony is significant and growing. It has been estimated [1] that cellular networks will consume so much energy that their CO_2 emissions equivalent will be 0.4 percent of the global value by 2020. The electricity bill forms a significant portion of the operating expenditure for a mobile service provider, and has been estimated globally to amount to about $22 billion in 2013. In case of increasing energy prices, this is expected to remain an important consideration for cellular system operators in the future.

It has been found that about 60 to 80 % of total energy consumption in cellular networks takes place in base stations (BS) [3]. Energy reduction in BSs can be achieved at many levels: from hardware design improvements to traffic-optimized deployment. Several researchers have focused on the problem of turning off base-stations during off-peak hours to save energy [4], [5]. While base-station activation for energy management is usually considered for a single operator, an additional promising solution was suggested and investigated quantitatively in [5]: multiple operators sharing base stations during off-hours.

Base-station sharing is predicated on the fact that there are typically multiple cellular providers in urban environments. And often in urban settings due to both a need for high-density deployment to provision peak traffic, and due to code constraints on where base stations can be deployed, operators often tend to have nearby, if not exactly collocated, base-stations. For instance, in [5], the authors examined a dataset from U.K., and noted that there were 5 different operators managing 139 base stations in 128 locations in a 3.5×3.5 km area of Manchester.

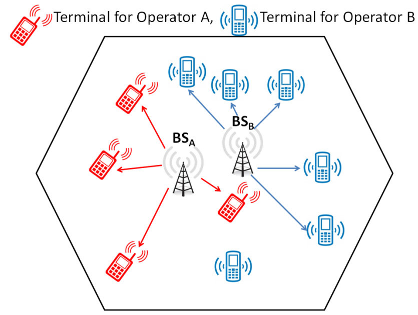
If two operators were able to enter into an agreement to share base stations during off-peak hours, they could potentially both benefit from the resulting energy savings. The authors of [5] estimated that such an approach could save up to 85% of total energy consumption during off-peak hours in an urban deployment, an additional 35% over what could be obtained if each operator acted in isolation.

However, just because it is globally efficient for operators to turn off base stations and share resources, it does not mean that they will do so. As self-interested entities, each operator is focused on maximizing only its own utility, which is a combination of revenue from customers, and cost of operation. If an operator were to shut one of its own base station and send its customers to a competing operator's base station, under a *micro-roaming* agreement[1], it would do so only if the additional cost of payments to the other operator and potential revenue loss from dissatisfaction of its customers were outweighed by the benefits of energy savings. Similarly, the other party would only agree to serve the first party's customers if the payments it received outweighed the loss in revenue due to potential dissatisfaction of its own customers.

In this paper we address the following questions: Under what conditions will self-centered operators be willing to enter into micro-roaming agreements? How does their utility compare to an idealized cooperative setting?

To do so, we build a simple but informative microeconomic model that examines the behavior of base station operators who are collocated in a single cell. The model takes into account the demand distributions for each customer, their energy costs, revenue from satisfied customers, loss of revenue from dissatisfied customers, service capacity, and payment rates (which we examine both as exogenous and endogenous parameters). Using a game theoretic formulation, whereby operators choose whether to turn base stations on or off, and if considered, also the payment rates to charge to the other operator, so that they operate at Nash Equilibria. Our analysis reveals conditions under which operators will clearly agree to share, but also reveals that there are conditions in which there are multiple equilibria and there is an additional problem of equilibria selection. And further, to see if their self-interest results in significant loss of performance with respect to the social welfare we examine through analysis and simulations the gap between the total utility at the global solution to that obtained at Nash equilibria for the formulated games.

[1] so named to distinguish it from traditional roaming agreements which are typically entered into by non-competing operators in geographically distinct areas such as different countries.



(a) (b)

Fig. 1. An example of the single cell layout, (a) Both BSs are on and serving their own demands, (b) BS_B is turned off and BS_A is serving both demand sets fairly.

Concretely, we make the following contributions in this paper:

- To our knowledge this is first study to examine the problem of base-station sharing from a game theoretic perspective.
- For exogenous payments, we show analytically that the equilibrium properties depend upon the payments, traffic parameters and the energy cost of operating a base station. Depending on the settings, there may be one or two pure Nash Equilibria, or in some cases, none (in these cases there are mixed Equilibria instead).
- For strategic payments, we find analytically there are generally always a multitude of pure Nash Equilibria. Thus, if reciprocal payments are to be strategically chosen by the operators, there is often a very challenging problem of equilibrium selection. Though we do not address this directly in our work, we identify this as a topic that needs to be addressed in future research by the community.
- Through numerical evaluations we further quantitatively demonstrate the conditions under which Base Station sharing emerges as a Nash Equilibrium, and the conditions under which equilibria result in good outcomes with respect to the globally optimized (cooperative) case.
- Interestingly and somewhat counter-intuitively, we show that in some cases with exogenous payments the pure Nash Equilibrium shifts from operator A turning on and operator B turning off to the reverse — operator A turning off and operator B turning on, if only the energy price common to both operators is increased (all other factors such as load, capacity, revenue and penalties being kept constant).
- Though the focus of this work is on turning off base stations to save energy, we also find incidentally from our modeling that when the operators have asymmetric traffic loads, it could be beneficial for both operators to enter into agreements to share their base stations even when both are turned on, so that the lower-loaded operator can help the higher-load operator in exchange for payment. Such agreements, it turns out, have recently been investigated by other researchers [17].

II. Related Work

Energy-efficient design of cellular wireless networks has recently received significant attention [3], [6], [7], [8], [9], [16], [10], [4], [5], [11], [13], [14]. One of the earliest works on green cellular networks is [6], where the idea of dynamic BS operation based on the traffic profile was proposed. Other similar studies studying the improvement in energy savings from turning base stations on and off, based on analytical models as well as a real traffic trace include [3], [7]. In addition, the BS operation concept considering cooperative network sharing among different hierarchical levels of networks, i.e. macro/macro, macro/micro, macro/femto-cells, are studied in [8], [9], [10]. Some concrete algorithms for base station operation are proposed in [11], [13], [14], [15]. In [11], [13], the authors researched about the energy efficient operation based on the cooperation transmission in multi-hop systems. Niu *et al.* proposed the cell zooming considering the BS cooperation and relaying in cellular systems [14]. In [16], the authors consider the use of dynamic voltage scaling which offers a finer-grained approach to energy minimization compared to completely turning off base stations, which relies on future hardware and software improvements. A common theme to all this prior work is the focus on dynamic base station operation by a single operator.

In [5], the authors evaluate the savings that can be obtained from base station switching using real data set of base station deployments from multiple operators. In particular, they propose and show that the sharing of base stations by different operators could be a way to significantly improve energy usage; their numerical evaluations show a 35% additional energy reduction from inter-operator cooperation above and beyond the benefit to a single operator. However, the concrete evaluation in that work assumes globally optimized inter-operator cooperation. Considering inter-operator cooperation issues as a challenging future direction for research into green cellular networks, the authors of [5] note that "this is also an interesting problem from a game theoretic perspective", and ask: "under what conditions would self-interested operators agree to cooperate with others? What kind of profit sharing agreements will provide an adequate incentive for all participants?" These are the very questions that we are taking a step towards addressing in this work.

A closely related recent work is [17], where the authors investigate the problem of cellular operators sharing excess spectrum with other operators to help them handle traffic overload. Similar to our work, they also assume that the demands for each operator are known statistically (i.e., as a distribution) but not exactly, and also consider a form of micro-roaming. However in that work, which is not focused on green cellular operation, there is no option to turn off base stations, and energy costs are not taken into account. The authors of [17] focus instead on revenue-sharing contracts that balance capacity sharing with customer satisfaction. In our work, we do consider similar spectrum sharing even when both base stations are turned on for the centralized global solution but not for the case of self-interested operators. That work also points to prior literature on analysis of roaming contracts such as [18], but indicates that most of the prior literature has been focused on investigating harmful collusive behavior under known demand conditions.

III. Problem Formulation

We consider a single cell with two available operators, called A and B, to serve the demands inside the cell. Assume that the set of demands D_i with the size of d_i is assigned to the operator $i \in \{A, B\}$. d_i is unknown but its distribution is assumed to be given. Serving each unit demand in D_i results in a revenue r_i and each unit of unserved demand in D_i results in a penalty q_i. Each operator has access to a Base Station with the available capacity of C and the operating energy cost of $\mathcal{E}$.

The utility of the operator i is equal to the total revenue collected by serving the demands in D_i minus the penalty related to the unserved demands and the energy consumption (when its BS is operating). The goal is to decide about turning on or off each BS in order to maximize the total utility which is equal to the summation of the utilities of two operators. The decision could be shown by a pair of Boolean variables (x_A, x_B) where $x_A = 0$ means the BS of operator A is off and $x_A = 1$ means it's on and serving the demands. This problem can be considered from two view points. One is the global solution where we assume that there exists a central decision-maker which has access to both Base stations to serve the demands in $D_A \cup D_B$ fairly. Another view point is based on game theory where we assume there is no central controller and two operators try to maximize their own utility. To save the energy one operator, namely A, may prefer to turn off its BS. Then the operator B could serve some demands in D_A by charging the operator A with some payment rate p_A. Therefore, the utility of operator A/B will decrease/increase with the amount of payments to B. In this case, two operators play a game and each one decides on turning on/off its own BS to maximize its utility. The payments p_A and p_B could be adjusted exogenously or be parts of strategy. In the later case, operator A could decide on the payment rate of p_B to charge the operator B in case of serving any demand in D_B. We explore the existence of Nash equilibra for different values of parameters, *e.g.* revenue, penalty, payment, etc.

In summary, the parameters corresponding to the operator $i \in \{A, B\}$ are as follows:

- d_i: demand assigned to the operator i with cumulative distribution function $F_i(.)$ and probability density function $f_i(.)$
- r_i: revenue rate for served demands
- p_i: payment rate made to the other operator
- q_i: penalty rate for unserved demands
- C: the assigned capacity to the BS_i
- $\mathcal{E}$: energy cost of operating (when BS_i is on)

Note that all of the above parameters are non-negative.

IV. Global Utility

In the global view point, there is a central controller who decides which BS should be on or off to maximize the total expected utility. For the central controller, both set of demands D_A and D_B have equal priority to be served. Therefore, the available capacity will be divided between two sets (even if the revenue or penalty rates are different). No payment is needed in the global solution and the total utility which is the summation of the utilities of two operators is given by:

$$\begin{aligned} u_G(x_A, x_B) = r_A s_A(x_A, x_B) + r_B s_B(x_A, x_B) - \mathcal{E}(x_A, x_B) \\ - q_A(d_A - s_A(x_A, x_B))^+ \\ - q_B(d_B - s_B(x_A, x_B))^+, \end{aligned} \tag{1}$$

where $(a)^+ = a$ if $a \geq 0$, otherwise $(a)^+ = 0$. As it is obvious from the above equation, the global utility is equal to the total revenue minus the energy cost and minus the total penalty. s_A is the served amount of demands in D_A, thus $r_A s_A$ is the total revenue earned by serving this amount of demands. $(d_A - s_A(x_A, x_B))^+$ is the unserved demands and $q_A(d_A - s_A(x_A, x_B))^+$ is the total penalty to be paid. $\mathcal{E}(x_A, x_B)$ is the total energy cost which is related to the number of active BSs as follows:

$$\mathcal{E}(x_A, x_B) = \begin{cases} 2\mathcal{E} & \text{if } (x_A, x_B) = (1, 1), \\ \mathcal{E} & \text{if } (x_A, x_B) = (1, 0) \text{ or } (0, 1). \end{cases} \tag{2}$$

The number of served demands, s_A which is always less than or equal to d_A, is given by:

$$s_A(1, 1) = \begin{cases} d_A & \text{if } d_A + d_B \leq 2C \\ \frac{d_A}{d_A + d_B} 2C & \text{if } d_A + d_B > 2C, \end{cases} \tag{3}$$

$$s_A(1, 0) = s_A(0, 1) = \begin{cases} d_A & \text{if } d_A + d_B \leq C \\ \frac{d_A}{d_A + d_B} C & \text{if } d_A + d_B > C, \end{cases} \tag{4}$$

which means all demands in D_A are served if the total demands do not exceed the available capacity. Otherwise, the available capacity is divided fairly between two sets of demands. Note that s_B is similarly computable with exchanging the indexes of A and B and other terms in (1) correspond to the operator B with the similar explanations. The goal here is to decide on the pair of (x_A, x_B) in order to maximize the expected global utility, computed in (5).

For the symmetry case of $r_A = r_B = r$ and $q_A = q_B = q$, the expected global utility can be simplified as given by (6) where $\bar{F}(.) = 1 - F(.)$ is the complementary cumulative distribution function of the total demand $d_A + d_B$. Then, the global solution can be obtained by:

$$(x_A, x_B) = \begin{cases} (1, 1), & \text{if } \mathcal{E} \leq t_2 \\ (1, 0), & \text{if } t_2 \leq \mathcal{E} \leq t_1 \\ (0, 0), & \text{if } \mathcal{E} \geq t_1 \end{cases} \tag{7}$$

where t_1 and t_2 are thresholds on the energy cost, computed from $u_G(0, 0) \geq u_G(1, 0)$ and $u_G(1, 1) \geq u_G(1, 0)$, respectively, by substituting the corresponding parameters given by

$$E[u_G(x_A,x_B)] = \begin{cases} (r_A+q_A)E[s_A(x_A,x_B)] + (r_B+q_B)E[s_B(x_A,x_B)] & \\ -\mathcal{E}(x_A,x_B) - q_A E[d_A] - q_B E[d_B], & \text{if } (x_A,x_B) \neq (0,0) \\ -q_A E[d_A] - q_B E[d_B], & \text{if } (x_A,x_B) = (0,0). \end{cases} \tag{5}$$

$$E[u_G(x_A,x_B)] = \begin{cases} 2C(r+q)\bar{F}(2C) + r\int_0^{2C} yf(y)dy - q\int_{2C}^{\infty} yf(y)dy - 2\mathcal{E} & \text{if } (x_A,x_B) = (1,1) \\ C(r+q)\bar{F}(C) + r\int_0^{C} yf(y)dy - q\int_{C}^{\infty} yf(y)dy - \mathcal{E} & \text{if } (x_A,x_B) = (1,0) \text{ or } (0,1), \\ -q\int_0^{\infty} yf(y)dy & \text{if } (x_A,x_B) = (0,0), \end{cases} \tag{6}$$

(2), (3), and (4) in (1). The results are given by:

$$t_1 = (q+r)\int_0^C yf(y)dy + C(q+r)\bar{F}(C)$$
$$t_2 = (q+r)\int_C^{2C} yf(y)dy + C(q+r)(\bar{F}(C) - 2\bar{F}(2C))$$

V. Game Theory

In the game theoretical view point, there is no central controller and each operator decides on turning its own BS on or off in order to maximize its utility. Therefore, two operators play a game and behave selfishly. We are going to explore the existence of Nash Equilibria, the decision pairs of (x_A, x_B) where if one of the operators changes its decision while the other one is fixed the utility will not increase. We will determine the possible Nash equilibria for different values of parameters including capacity, load, revenue, penalty and payment rates. We consider two payment cases: (i) the exogenous payment where the values of p_A and p_B are given and the players do not have any control on them, (ii) strategic payment where the values of p_A and p_B are parts of strategy and the players can select them, *i.e.* decide on how much to charge the other player in case of serving its demands.

A. Analysis of Nash Equilibria for Exogenous Payments

In the case of exogenous payment, the values of p_A and p_B is given and the players select their strategy based on the payment values and other parameters. We consider different conditions to see whether there exists a Nash equilibrium (NE) or not and in case of existence what it is. Let $u_i(x_A, x_B)$ indicates the utility function of operator $i \in \{A, B\}$ and if there exists a pure NE point, shown as (x_A^{NE}, x_B^{NE}), it must meet both conditions below:

$$E[u_A(x_A^{NE}, x_B^{NE})] \geq E[u_A(x_A, x_B^{NE})], \quad \forall x_A \tag{8}$$
$$E[u_B(x_A^{NE}, x_B^{NE})] \geq E[u_B(x_A^{NE}, x_B)], \quad \forall x_B. \tag{9}$$

The utility of operator A for different decision pairs of (x_A, x_B) is given by:

$$u_A(1,1) = -\mathcal{E} + r_A \min(d_A, C) - q_A(d_A - C)^+ \tag{10}$$
$$u_A(1,0) = -\mathcal{E} + r_A s_A - q_A(d_A - s_A)^+ + p_B s_B \tag{11}$$
$$u_A(0,1) = (r_A - p_A)s_A - q_A(d_A - s_A)^+ \tag{12}$$
$$u_A(0,0) = -q_A d_A \tag{13}$$

where, the served amount of demands s_A is similar to $s_A(1,0)$ in the global solution given by (4). Note that u_B and s_B can be computed similarly. From the inequalities (8) and (9), we find different conditions for each NE as follows, where for simplicity we use the notations $M_A = E[\min(d_A, C)]$, $M_B = E[\min(d_B, C)]$, $G_A = E[s_A]$, $G_B = E[s_B]$:

- (1,1) is a pure NE if

$$\mathcal{E} - p_A G_A \leq (r_A + q_A)(M_A - G_A) \triangleq L_A \tag{14}$$
$$\mathcal{E} - p_B G_B \leq (r_B + q_B)(M_B - G_B) \triangleq L_B \tag{15}$$

  Proof: To have (1,1) as a pure NE two conditions should be satisfied. First, by (8), the following equation should be greater than or equal to zero:

$$\begin{aligned} &E[u_A(1,1)] - E[u_A(0,1)] \\ &= -\mathcal{E} + r_A E[\min(d_A, C)] - q_A E(d_A - C)^+] \\ &- (r_A - p_A)E[s_A] + q_A E[(d_A - s_A)^+] \\ &= -\mathcal{E} + r_A M_A - (r_A - p_A)G_A \\ &+ q_A E[(d_A - s_A)^+ - (d_A - C)^+] \\ &= -\mathcal{E} + r_A M_A - (r_A - p_A)G_A \\ &+ q_A(M_A - G_A) \geq 0 \end{aligned}$$

  where $E[(d_A - C)^+ - (d_A - s_A)^+] = E[s_A - \min(d_A, C)] = G_A - M_A$. This results in (14) and (15) comes from $E[u_B(1,1)] - E[u_B(1,0)] \geq 0$ which is equivalent to (9).

  ■

- (0,0) is a pure NE if

$$\mathcal{E} - p_A G_A \geq (r_B + q_B)G_B \triangleq U_A$$
$$\mathcal{E} - p_B G_B \geq (r_A + q_A)G_A \triangleq U_B$$

- (1,0) is a pure NE if

$$\mathcal{E} - p_B G_B \geq (r_B + q_B)(M_B - G_B) = L_B$$
$$\mathcal{E} - p_B G_B \leq (r_A + q_A)G_A = U_B$$

- Similalry (0,1) is a pure NE if

$$\mathcal{E} - p_A G_A \geq L_A$$
$$\mathcal{E} - p_A G_A \leq U_A$$

The above three Nash equilibria are easily achievable from (8) and (9) with proofs similar to the proof of NE (1,1).

Fig. 2 shows the division of the two-dimensional space of $(\mathcal{E} - p_A G_A, \mathcal{E} - p_B G_B)$ to the regions where different (x_A, x_B)s are NE, for different values of L_A, U_A, L_B, U_B. For instance in Fig. 2-(a), at the region V, both (1,0) and (0,1) can be pure NE, and at the regions I and IX there is no pure NE. Figures 2-(b-e) show similar division for different relations between L_A, U_A and L_B, U_B. There are two more cases which could be achieved by exchanging the indexes of A and B in Fig.2-(b-c) and we skip them due to similarity. At the regions I and IX in Fig. 2-(a) where there is no NE, there can exist a mixed NE. Assume the mixed strategy assigns the probabilities $P(x_A = 0) = \alpha$ and $P(x_B = 0) = \beta$ where α can be computed from the indifference of player B to the decision of the player A, as follows:

$$\alpha E[u_B(0,0)] + (1-\alpha)E[u_B(1,0)] = \alpha E[u_B(0,1)] + (1-\alpha)E[u_B(1,1)] \quad (16)$$

Therefore,

$$\alpha = \frac{E[u_B(1,1)] - E[u_B(1,0)]}{E[u_B(0,0)] - E[u_B(1,0)] - E[u_B(0,1)] + E[u_B(1,1)]} = \frac{\mathcal{E} - p_B G_B - L_B}{U_A - L_B + \mathcal{E} - p_B G_B - (\mathcal{E} - p_A G_A)}, \quad (17)$$

and similarly,

$$\beta = \frac{\mathcal{E} - p_A G_A - L_A}{U_B - L_A + \mathcal{E} - p_A G_A - (\mathcal{E} - p_B G_B)}. \quad (18)$$

B. Analysis of Nash Equilibria for Strategic Payments

In this section, we consider the scenario where the player A (B) could decide about the price p_B (p_A), as well as about turning on or off its BS, x_A (x_B). Therefore, the strategy is shown by quadruple (x_A, x_B, p_A, p_B) and we are going to explore on what conditions, which quadruples could be NE. The payment rates p_A and p_B may be unconstrained or constrained. In case of unconstrained payment, there is no restriction on the values of p_A and p_B. But in case of constrained payment, the payment rates are restricted to the values such that the decision are prevented from changing the NE regions. In the later case, we restrict the payment rates such that if for instance the decision is $(0,1)$, the operator B is not allowed to charge A with very large values of payment; because in this case, A will prefer to turn on its own BS rather than paying too much to B. On the other hand, if the payment rate is too low, the operator B may prefer to turn off its BS and then strategy will switch to $(0,0)$. Therefore, the payments are restricted to the values corresponded to the borders of regions related to $(0,1)$ and $(1,0)$ Nash equilibria.

If there exists a pure NE for strategic payment shown as $(x_A^{NE}, x_B^{NE}, p_A^{NE}, p_B^{NE})$, it must meet both conditions below:

$$E[u_A(x_A^{NE}, x_B^{NE}, p_A^{NE}, p_B^{NE})] \geq E[u_A(x_A, x_B^{NE}, p_A^{NE}, p_B)], \quad \forall x_A, p_B \quad (19)$$

$$E[u_B(x_A^{NE}, x_B^{NE}, p_A^{NE}, p_B^{NE})] \geq E[u_B(x_A^{NE}, x_B, p_A, p_B^{NE})], \quad \forall x_B, p_A \quad (20)$$

1) Unconstrained strategic Payment: For unconstrained payment, the pure Nash equilibria are computed as follows:

- $(1, 1, p_A^{NE}, p_B^{NE})$ is a pure NE if

$$\mathcal{E} - p_A^{NE} G_A \leq L_A$$
$$\mathcal{E} - p_B^{NE} G_B \leq L_B$$

  which is exactly similar to the pure NE $(1,1)$ for the exogenous payment and means that all the points in the region VII in Fig. 2 can be pure NE for the strategic payment.
- To have a NE point $(0, 0, p_A^{NE}, p_B^{NE})$ the following inequalities should hold:

$$\mathcal{E} - p_A G_A \geq U_A, \ \forall p_A$$
$$\mathcal{E} - p_B G_B \geq U_B, \ \forall p_B$$

  which is impossible, because for some p_A and p_B the above inequalities are not valid. Therefore, this quadruple cannot be a pure NE.
- To have a point of $(1, 0, p_A^{NE}, p_B^{NE})$ as a pure NE, we get:

$$p_B^{NE} G_B \geq p_B G_B, \ \forall p_B \quad (21)$$
$$\mathcal{E} - p_B^{NE} G_B \leq U_B \quad (22)$$
$$\mathcal{E} - p_B^{NE} G_B \geq L_B \quad (23)$$

  Proof: The quadruple $(1, 0, p_A^{NE}, p_B^{NE})$ is a pure NE if three conditions hold. First, the following difference should be greater than or equal to zero:

$$E[u_A(1, 0, p_A^{NE}, p_B^{NE})] - E[u_A(1, 0, p_A^{NE}, p_B)] = (p_B^{NE} - p_B) G_B \geq 0$$

  which results in (21). Second,

$$E[u_A(1, 0, p_A^{NE}, p_B^{NE})] - E[u_A(0, 0, p_A^{NE}, p_B)] = -\mathcal{E} + r_A G_A - q_A E[d_A - s_A] + p_B^{NE} G_B + q_A E[d_A] = -\mathcal{E} + r_A G_A + q_A G_A + p_B^{NE} G_B \geq 0 \quad (24)$$

  which is equivalent to (22). Third,

$$E[u_B(1, 0, p_A^{NE}, p_B^{NE})] - E[u_B(1, 1, p_A, p_B^{NE})] = (r_B - p_B^{NE}) G_B - q_B E[d_B - s_B] + \mathcal{E} - r_B M_B + q_B E[(d_B - C)^+] = \mathcal{E} - p_B^{NE} G_B - (r_A + q_B)(M_B - G_B) \geq 0$$

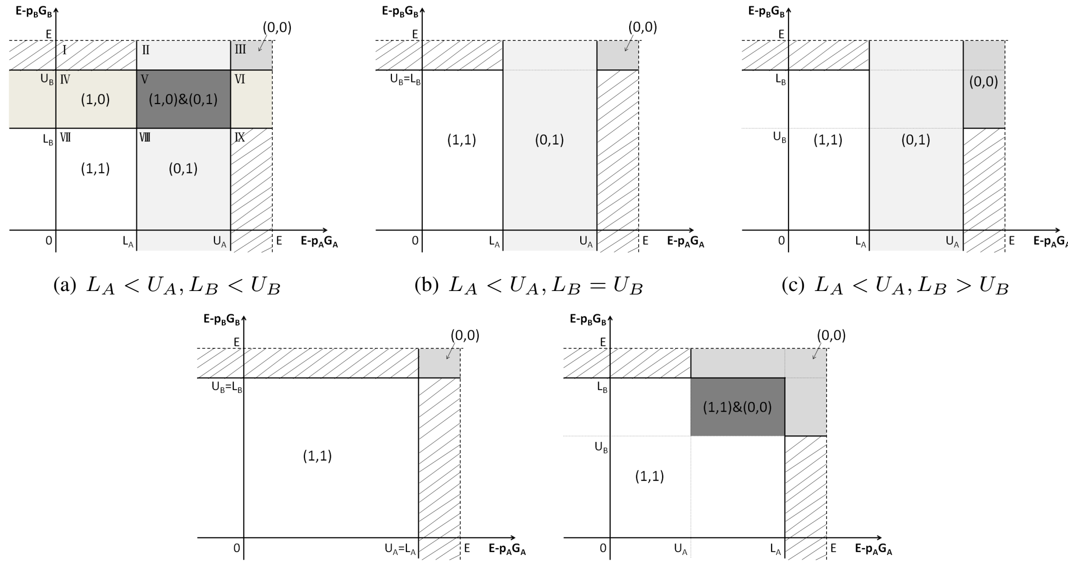
  which results in (23). For $p_B \geq (\mathcal{E} - L_B)/G_B$ the inequalities (21) and (23) are not satisfied, thus this case cannot be a pure NE. ■
- Similarly, $(1, 0, p_A^{NE}, p_B^{NE})$ cannot be a pure NE.

Fig. 3-(a) shows the division of the space of $(\mathcal{E} - p_A G_A, \mathcal{E} - p_B G_B)$ for the unconstrained strategic payments and the values of $L_A < U_A, L_B < U_B$. The region of $(1, 1, p_A, p_B)$ is similar to the region of $(1,1)$ for the case of exogenous prices shown in Fig. 2, and the rest of the regions do not have a pure NE anymore. Note that for other values of L_A, U_A, L_B, U_B, we have figures similar to Fig. 2-(b-e) with having only NE of $(1, 1, p_A, p_B)$ in the regions of $(1,1)$.

2) Constrained strategic Payment: Now lets consider the case where the payments are restricted to the values on the borders in Fig. 3-(a). For instance, in case of $(x_A, x_B) = (0,1)$, the player A is not allowed to select a price higher than $p_B^{max,10} = (\mathcal{E} - L_B)/G_B$ or lower than $p_B^{min,10} = (\mathcal{E} - U_B)/G_B$. Similarly, in the case of $(x_A, x_B) = (0,1)$, p_A should be limited to $p_A^{max,01} = (\mathcal{E} - L_A)/G_A$ and $p_A^{min,01} = (\mathcal{E} - U_A)/G_A$. Therefore, we will have additional conditions on the values of the payments and the NE will be computed as follows:

- The region for $(1, 1, p_A^{NE}, p_B^{NE})$ is similar to that for unconstrained case.
- In contrast to the unconstrained case, there is a NE of $(0, 0, p_A^{NE}, p_B^{NE})$ satisfying the following inequalities:

$$\mathcal{E} - p_A G_A \geq U_A, \ \forall p_A \leq p_A^{min,01}$$
$$\mathcal{E} - p_B G_B \geq U_B, \ \forall p_B \leq p_B^{min,10},$$



(a) $L_A < U_A, L_B < U_B$ (b) $L_A < U_A, L_B = U_B$ (c) $L_A < U_A, L_B > U_B$

(d) $L_A = U_A, L_B = U_B$ (e) $L_A > U_A, L_B > U_B$

Fig. 2. Division of the two-dimensional space of $(\mathcal{E}-p_AG_A, \mathcal{E}-p_BG_B)$ to the regions with different NE; a) $L_A < U_A, L_B < U_B$, b) $L_A < U_A, L_B = U_B$, c) $L_A < U_A, L_B > U_B$, d) $L_A = U_A, L_B = U_B$, e) $L_A > U_A, L_B > U_B$.

which is similar to the pure NE of (0,0) for the exogenous payment (*i.e.* all the points in the region III in Fig. 2 can be pure NE).

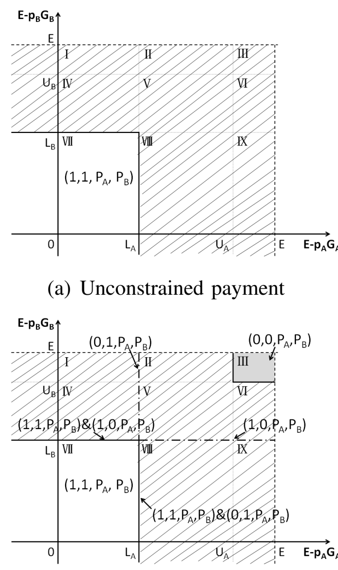
- To have a point $(1, 0, p_A^{NE}, p_B^{NE})$ as a pure NE, we obtain:

$$p_B \leq p_B^{NE}, \forall\ p_B^{min,10} \leq p_B \leq p_B^{max,10}$$
$$\mathcal{E} - p_B^{NE} G_B \leq U_B$$
$$\mathcal{E} - p_B^{NE} G_B \geq L_B.$$

$p_B^{NE} = p_B^{max,10}$ satisfies the above inequalities which means that on the line of $\mathcal{E} - p_B G_B = L_B$, $(1, 0, p_A, p_B^{max,10})$ is a pure NE.

- Similarly $(0, 1, p_A^{max,01}, p_B)$ could be a pure NE on the line of $\mathcal{E} - p_A G_A = L_A$.

Fig. 3-b shows the division of the space of $(\mathcal{E}-p_AG_A, \mathcal{E}-p_BG_B)$ for the constrained strategic payments and the values of $L_A < U_A, L_B < U_B$. The regions III and VII are similar to the exogenous case. On the above border of region VII both $(1, 1, p_A, p_B)$ and $(1, 0, p_A, p_B)$ could be NE and on its right border both $(1, 1, p_A, p_B)$ and $(0, 1, p_A, p_B)$ could be NE. At the intersection of these two borders all three strategies could be pure NE. Note that by the term (x_A, x_B, p_A, p_B) is a NE in a specific point/line/region, we mean that only the values of p_A and p_B corresponded to that point/line/region are valid.



(a) Unconstrained payment

(b) Constrained payment

Fig. 3. the two-dimensional space of $(\mathcal{E} - p_AG_A, \mathcal{E} - p_BG_B)$ for strategic payment (the case of $L_A < U_A, L_B < U_B$); a) Unconstrained payment, b) Constrained payment.

VI. Simulations

In the simulation part, the demands for both operators are considered to follow Poisson distributions with different expectations shown as λ_A and λ_B. The simulation parameters, except in the figures that their effect is considered, are fixed as follows: $C = 5$, $\mathcal{E} = 4$, $r_A = 1$, $r_B = 1.5$, $q_A = 0.5$, $q_B = 0.7$, $p_A = 0.6$, $p_B = 0.7$, $\lambda_A = 7$ and $\lambda_B = 5$. The parameters including $\mathcal{E}$, r_A, r_B, q_A, q_B, p_A and p_B are set only to indicate relative value with no practical significance. The value for C can be understood as the maximum number of users that one BS can serve. As for the Poisson distribution parameters, the average amount of users can be indicated. We compare the global and NE utilities in two cases, (i) the exogenous payments, and (ii) the strategic payments.

A. Exogenous Payment

In case of exogenous payments, the payment rates are given and the Nash equilibria are in the form of (x_A, x_B) pairs. Note that for the exogenous case in all regions but region V in Fig. 2, there is only one possible pair of NE and in the region V the total utilities of $(1, 0)$ and $(0, 1)$ are equal. Therefore in all regions the best and the worst Nash equilibria are the same.

Fig. 4 shows possible NE pairs for different values of λ_A and λ_B (Each region with different color corresponds to possible NE pairs as mentioned on the figure). The area related to $(0, 1)$ is larger than $(1, 0)$. Because $r_A + q_A < r_B + q_B$, the area of region VIII in Fig. 2-(a) will be larger than the area of region IV and therefore the chance of having NE of $(0, 1)$ is higher. For large values of λ_A and small values of λ_B, the decision pair will be in the region I of Fig. 2-(a) and therefore no pure NE will exist (which corresponds to the mixed NE indicated on the figure).

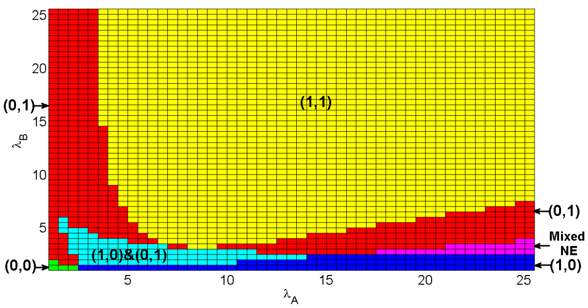


Fig. 4. The pairs of (x_A^{NE}, x_B^{NE}) for different values of λ_A and λ_B for exogenous payment.

Fig. 5 indicates the possible NE pairs for different values of C and $\mathcal{E}$.As it is obvious from the figure, for small values of $\mathcal{E}$, (1,1) is NE and for large values of $\mathcal{E}$, (0,0) is NE where the operators prefer to turn off their BSs to save the energy. Interestingly, for moderate values of $\mathcal{E}$ operators may move from turning off both BSs to sharing, as the capacity increases. Also, we see that which operator turns off and which one offers its BS for sharing can invert completely as $\mathcal{E}$ increases, which is rather counter-intuitive but is a consequence of asymmetry between the relative loads, revenues and payments in the investigated scenario.

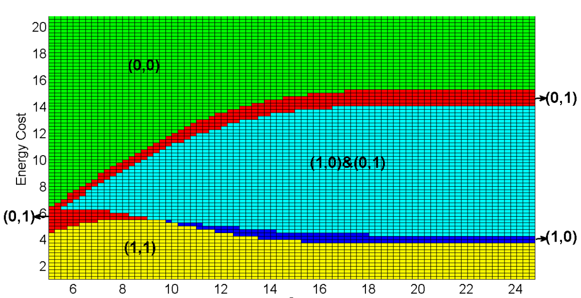


Fig. 5. The pairs of (x_A^{NE}, x_B^{NE}) for different values of C and $\mathcal{E}$ for exogenous payment.

Fig. 6 shows the global optimal utility and the NE utility versus the available capacity C, for exogenous payments. Both utilities increase by growing C. For small values of C, both BSs need to be on where the global utility is higher than the NE utility. Because in the NE solution, each BS only serve its corresponding demands and the chance of leaving some demands unserved is higher than the global solution. For large values of C, one of BSs could turn off and only one BS will be enough to serve all demands, thus the global and the NE utility will be equal.

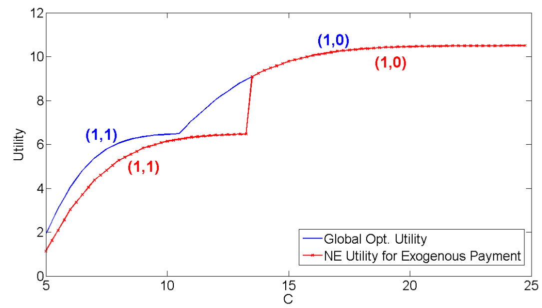


Fig. 6. The global optimal utility and the NE utility versus C for exogenous payments.

Fig. 7 shows the global optimal utility and the NE utility, versus the energy cost $\mathcal{E}$, for exogenous payments. By growing the energy cost, the utility decreases and the decision changes from $(1, 1)$ to $(0, 1)$ and then to $(0, 0)$. This change happens at smaller energy costs for the NE solution compared to the global solution which is becasue the NE solution leaves more unserved demands in case of $(1, 1)$ and it prefers to switch its decision earlier.

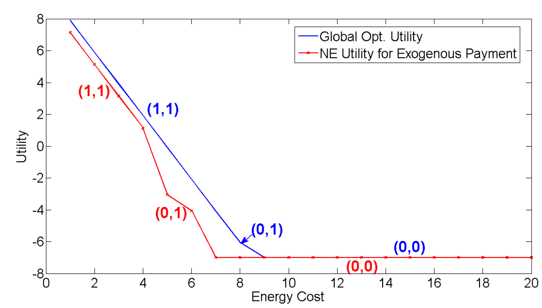


Fig. 7. The global optimal utility and the NE utility versus $\mathcal{E}$ for exogenous payments.

Fig. 8 shows the global optimal utility and the NE utility versus the Poisson distribution parameter λ_A, for exogenous payments. For small values of λ_A the number of demands, d_A, is small, so the decision for both the global and the NE solutions is $(0, 1)$. With increasing λ_A the number of demands d_A grows and the utility will decrease. So the decision switches to $(1, 1)$ to be able to serve more demands. This switching happens in the NE solution later than the global solution, because even in $(1, 1)$ decision BS_A will not serve the demands of B, and the NE utility will decrease more than the global utility.

Fig. 9 shows the gap between the global optimal utility and the NE utility versus λ_A and λ_B. The gap is lower when λ_A and λ_B are close and in case of increasing only one of them and decreasing the other one, the gap will increase. The reason is that in the NE solution, our model does not allow the operators to help the other's demands when both BSs are on. Therefore, when one of the operators has a heavy traffic the other one could not help it and the chance of having unserved demands are higher. This inefficiency can be alleviated by extending the roaming contract to include spectrum sharing even when both BSs are on, as considered in [17]. In addition, the gap for $\lambda_A \leq \lambda_B$ is larger than the

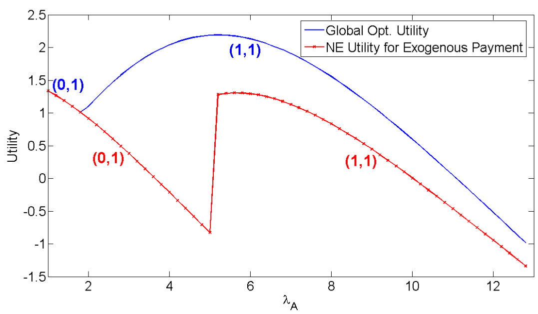


Fig. 8. The global optimal utility and the NE utility versus λ_A for exogenous payments.

gap for $\lambda_A \geq \lambda_B$ which is because of having asymmetric parameters of $r_A + q_A \leq r_B + q_B$.

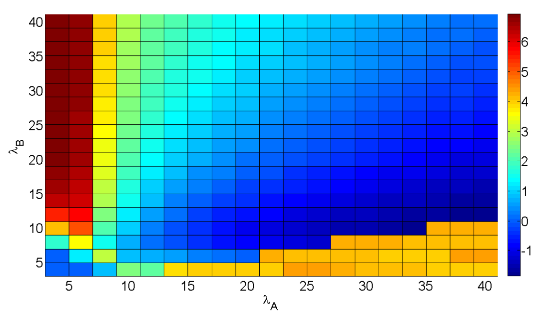


Fig. 9. The gap between the global optimal utility and the NE utility versus λ_A and λ_B for exogenous payments.

Fig. 10 shows the global optimal utility and the NE utility versus the revenue rate r_A, for exogenous payments. Both the utilities are increasing with respect to r_A. With decreasing the revenue r_A, A prefers to turn off its BS and the NE decision will change from $(1,1)$ to $(0,1)$. In other words, in Fig. 2-(a), with decreasing r_A we are moving the borders of L_A and U_A to the left and therefore a fixed point in the region VII will move to the region VIII. In this case the slope of the curve which is equal to s_A will decrease. In the global solution, because of using both BSs to serve d_A and d_B in $(1,1)$, decreasing the revenue of r_A can not affect the decision which stays in $(1,1)$ and thus the slope of the global utility versus r_A is fixed.

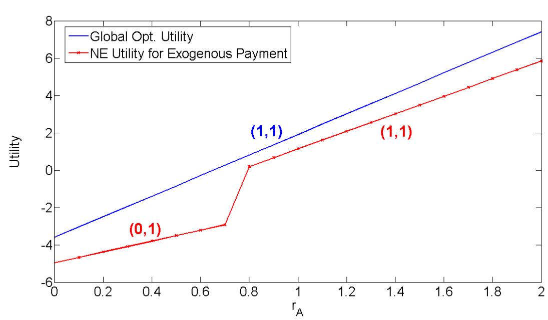


Fig. 10. The global optimal utility and the NE utility versus r_A for exogenous payments.

We skip the figure of the utilities versus q_A which follows same arguments as Fig. 10 with having decreasing behaviour with respect to q_A.

Fig. 11 shows the global optimal utility and the NE utility versus the payment rate p_A, for exogenous payments. We consider two different values for $\mathcal{E}$ and q_B: (i) $\mathcal{E} = 4$ and $q_B = 0.3$ and (ii) $\mathcal{E} = 8$ and $q_B = 0.3$. For the first (second) set of values, the NE utility will increasing (decreasing) by growing p_A and switching NE. But as far as the NE is not changed the total utility stays fixed, because of decreasing the utility of A and increasing the utility of B with the same amount.

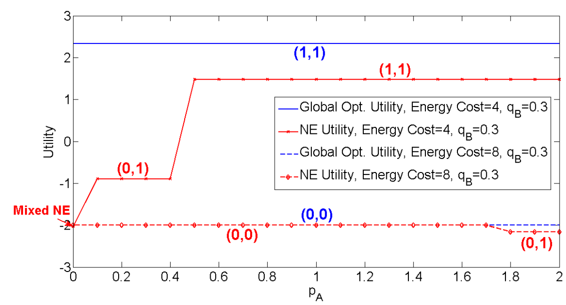


Fig. 11. The global optimal utility and the NE utility versus p_A for exogenous payments.

B. *Constrained Strategic Payment*

We present some simulation results for the constrained strategic payment. In this case the payment rates p_A and p_B are also parts of strategy which could move the NE point among all the regions (if there exists a NE) in Fig. 3. Therefore, the best and the worst Nash equilibria can be different and we will compare these two strategies with each other as well as with the global solution. In the strategic payment the selection of the NE is a more significant problem and the best NE achieves a lot better utility than the worst one. We skip the unconstrained strategic payment which only has one region of $(1,1)$ and the rest of the Nash equilibria do not exist and only consider the constrained payment. To have more clear figures we only show the decision parts of the strategies, as given by (x_A, x_B), and drop the p_A and p_B notations.

Fig. 12 and Fig. 13 show the global optimal utility and the best and the worst NE utilities versus C and $\mathcal{E}$, respectively, for constrained strategic payments. With increasing C, the worst NE which is $(0,0)$ is not changing and the best NE achieves better utility. With increasing $\mathcal{E}$ the worst NE changes from $(0,0)$ to $(1,1)$ which consumes a large amount of energy.

Fig. 14 shows the gap between the global optimal utility and the best NE utility versus λ_A and λ_B for the constrained strategic payments. We see similar results compared to Fig. 9.

Finally, we also examined how the global optimal utility and the best and the worst NE utilities vary versus r_A for constrained strategic payments, but we do not show the figure due to lack of space. Similar to the exogenous payment case, all utilities are increasing with respect to r_A. Similarly, though we do not show the figure, the utilities decrease with respect to q_A.

VII. Conclusion

The microeconomic model presented in this paper shows that inter-operator cooperation for energy-efficient operation

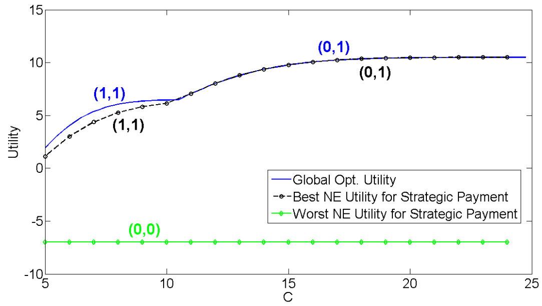


Fig. 12. The global optimal utility and the NE utility versus C for constrained strategic payments.

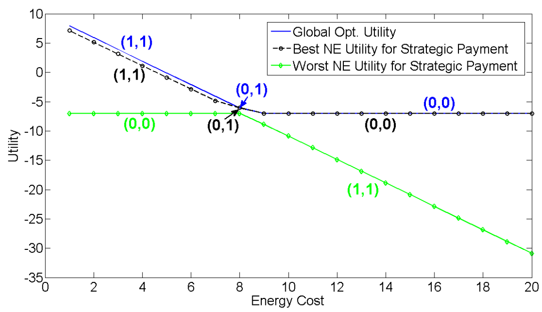


Fig. 13. The global optimal utility and the NE utility versus the enrgy cost, $\mathcal{E}$, for constrained strategic payments.

is a complex issue. It shows that the decision to cooperate or not depends sometimes sensitively on factors such as baseline energy consumption, capacity, traffic loads on each operator, and the revenues and penalties associated with satisfied and unsatisfied customers. We find that often there are multiple equilibria, and the gap between the best and worst equilibrium can be substantial, so that the question of equilibrium selection is a significant one in this domain, and needs to be addressed in future work.

We believe the work presented in this paper is a first step towards investigating these issues in more detail and there are many interesting open problems ahead. For instance, in our model, we have assumed that once operators agree to cooperate, they will treat both their own traffic and the traffic of the competing operators fairly; however, if they have different revenues and payments, higher utility may be obtainable by optimizing over the scheduling policy to favor one set of customers over another; this bears further investigation as well. Also, in the current model, equilibria were computed under the assumption of full-information. In practice the traffic of competing operators may be an unknown quantity, though it may be inferable from observations of the other operator's actions over repeated plays. Extending the model to address these issues would be another avenue for future work. Other future works could be considering time-varying energy cost/revenues, or asymmetric settings where the values of energy, capacity or revenues are different for different BSs.

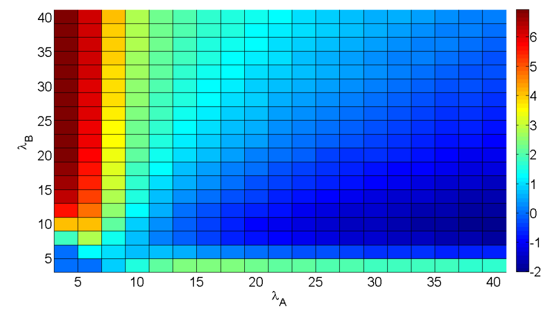


Fig. 14. The gap between the global optimal utility and the best NE utility versus λ_A and λ_B for constrained strategic payments.

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