On Hardness of Multiflow Transmission in Delay Constrained Cooperative Wireless Networks

Marjan Baghaie†, Dorit S. Hochbaum‡, Bhaskar Krishnamachari†
†Dept. of Electrical Engineering, University of Southern California, Los Angeles
‡Dept. of Industrial Engineering & Operations Research, University of California, Berkeley
Email: {baghaiea, bkrishna}@usc.edu, hochbaum@ieor.berkeley.edu

***Abstract*—We consider the problem of energy-efficient transmission in multi-flow multihop cooperative wireless networks. Although the performance gains of cooperative approaches are well known, the combinatorial nature of these schemes makes it difficult to design efficient polynomial-time algorithms for joint routing, scheduling and power control. This becomes more so when there is more than one flow in the network. It has been conjectured by many authors, in the literature, that the multiflow problem in cooperative networks is an NP-hard problem. In this paper, we formulate the problem, as a combinatorial optimization problem, for a general setting of k-flows, and formally prove that the problem not only NP-hard but it is $o(n^{1/7-\epsilon})$ inapproxmiable. To our knowledge, the results in this paper provide the first such inapproxmiablity proof in the context of multiflow cooperative wireless networks. We further prove that for a special case of k = 1 the solution is a simple path, and offer a polynomial time algorithm for jointly optimizing routing, scheduling and power control.**

I. Introduction

In a wireless network, a transmit signal intended for one node is received not only by that node but also by other nodes. In a traditional point-to-point system, where there is only one intended recipient, this innate property of the wireless propagation channel can be a drawback, as the signal constitutes undesired interference in all nodes but the intended recipient. However, this effect also implies that a packet *can* be transmitted to multiple nodes simultaneously without additional energy expenditure. Exploiting this broadcast advantage, broadcast, multicast and multihop unicast systems can be designed to work cooperatively and thereby achieve potential performance gains. As such, cooperative transmission in wireless networks has attracted a lot of interest not only from the research community in recent years [1], [2], [4], [5], [6], [7], [8] but also from industry in the form of first practical cooperative mobile ad-hoc network systems [9]. The majority of the work in the cooperative literature has so far focused on the single flow problem, though recently there has been an increased interest in considering multiflow settings in cooperative networks [12], [13], [14], [15], [16].

A key problem in such cooperative networks is routing and resource allocation, i.e., the question of which nodes should participate in the transmission of data, and when, and with how much power they should be transmitting. The situation is further complicated by the fact that the routing and resource allocation depends on the type of cooperation and other details of the transmission/reception strategies of the nodes. We consider in this paper a time-slotted system in which the nodes that have received and decoded the packet are allowed to re-transmit it in future slots. During reception, nodes add up the signal power (energy accumulation, EA) received from multiple sources. Details of EA, and possible implementations have been extensively discussed in prior work [2], [4], [5], [13].

We focus on the problem of minimum-energy multiflow cooperative transmission in this work, where there are k source-destination pairs, with the source node wanting to send a packet to its respective destination nodes, in a multihop wireless network. Other nodes in the network, that are neither the source nor the destination, may act as relays to help pass on the message through multiple hops. The transmission is completed when all the destination nodes have successfully received their corresponding messages. It has been noted in the literature ([8], [17]) that a key tradeoff in cooperative settings is between the total energy consumption[1] and the total delay measured in terms of the number of slots needed for all destination nodes in the network to receive the message. Therefore, we take delay into consideration and focus on the case where there is a delay constraint, whereby the destination node(s) should

This research was sponsored in part by the U.S. Army Research Laboratory under the Network Science Collaborative Technology Alliance, Agreement Number W911NF-09-2-0053, by NSF award No. DMI-0620677 and CBET-0736232, and by NSF awards CNS-0627028 and CNS-1049541.

[1]As we consider fixed time slot durations, we use the words energy and power interchangeably in this paper.

receive the message within some pre-specified delay constraint. We therefore formulate the problem of performing this transmission in such a way that the total transmission energy over all transmitting nodes is minimized, while meeting a desired delay constraint on the maximum number of slots that may be used to complete the transmission. The design variables in this problem determine which nodes should transmit, when, and with what power.

We furthermore assume that the nodes are memoryless, i.e., accumulation at the receiver is restricted to transmissions from multiple nodes in the present time slot, while signals from previous timeslots are discarded. This assumption is justified ([8], [17]) by the limited storage capability of nodes in ad-hoc networks, as well as the additional energy consumption nodes have to expand in order to stay in an active reception mode when they overhear weak signals in preceding timeslots.

The main contribution of our work is as follows: It has been *conjectured* in the literature that *the problem of jointly computing schedules, routing, and power allocation for multiple flows in cooperative networks* is NP-hard [15], [16], [14]. In this paper we formulate the joint problem of scheduling, routing and power allocation in a multiflow cooperative network setting and formally prove that not only it is NP-hard, but it is also $o(n^{1/7-\epsilon})$ inapproximable. (i.e. unless $P = NP$, it is not possible to develop a polynomial time algorithm for this problem that can obtain a solution that is strictly better than a logarithmic-factor of the optimum in all cases). We are not aware of prior work on multiflow cooperative networks that shows such inapproximability results. We further prove that for a special case of $k = 1$, the solution is a simple path and offer an optimal polynomial time algorithm for joint routing, scheduling and power control.

The rest of this paper is organized as follows: In section II we provide a mathematical formulation of the problem. In section III we consider the special case of $k = 1$ and prove the solution is a simple path and can be found optimally in polynomial time. The inapproximablity results are presented in section IV using reduction from minimum graph coloring problem. The paper is concluded in section V.

II. Problem Formulation

Consider a network, G, with a total of n nodes, $I = \{1, .., n\}$. Assume we have r source nodes, labeled $\mathcal{S} = \{s_1, s_2, ..., s_r\}$, and r corresponding destination nodes, $\mathcal{D} = \{d_1, d_2, ..., d_r\}$. The source-destination nodes can be thought of as pairs, $\{(s_k, d_k)\}_{k=1}^{r}$, all with the same delay constraint T. The goal is to deliver a unicast message from each source to its corresponding destination, possibly using other nodes in the network as relays. The objective is to do so using the minimum amount of sum transmit power and within the delay constraint.

We consider a cooperative wireless setting with EA and consider signal-to-intereference-plus-noise (SINR) threshold model, [13], [2], [11], [10]. That is, in order for node i to be able to decode message k at time t, the following inequality needs to be satisfied:

$$\frac{\sum\limits_{j \in s_k(t)} p_{jt} h_{ji}}{\sum\limits_{u \notin s_k(t)} p_{ut} h_{ui} + N} \geq \theta. \tag{1}$$

Here $s_k(t)$ is the set of nodes transmitting the message k at time t, h_{ij} is a constant between 0 and 1 representing the channel gain between node i and j, and N and θ are constants representing the noise and the decoding threshold respectively.

Equation (1) can be re-written as

$$\sum_{j=1}^{n} h_{ji} p_{jt}^{k} - \theta \sum_{\substack{q=1 \\ q \neq k}}^{r} \sum_{u=1}^{n} h_{ui} p_{ut}^{q} - \theta N \geq 0, \tag{2}$$

where p_{it}^{k} is the power used by node i at time t to transmit message k.

The system is memoryless, meaning although we are allowed to accumulate the same message from multiple sources during each time slot, we cannot accumulate over time. The relays are half-duplex, meaning they cannot transmit and receive simultaneously. The relays cannot transmit more than one message at the same time either.

In order to apply ideas driven by the rich literature on multicommodity flows [18] to our problem, we need to somehow introduce the notion of delay constraint into the multicommodity setting. What follows is a transformation of our network graph that would allow for the multicommodity flow technique to be applied, while observing the delay constraint: For a delay constraint T, map the given network to a layered graph with T layers as shown in Figure 1. Place a copy of all the nodes in the network on each of the layers. Connect each node, on each layer, to its corresponding copy on its neighboring layers with an edge weight of 0. Also create directed edges between each node, on each layer k, and the nodes on the next layer $k + 1$, with edge weights representing the amount of power required to transmit the message from the node on the top level to the node on the bottom level, as a whole. Notice that there is no edge between the nodes on the same level. Call the new graph G'. Assign the nodes corresponding to the source nodes of G on level 1 of G' as source nodes in G' and the destination nodes on level T of G', corresponding to destination nodes in G, as destinations in G', as shown in the figure. Similar transformations have been used in the literature in the context of multiflow transmission [15].

Without loss of generality, we assume unit length time slots. The nodes who want to transmit are to do so at the beginning of each time slot, and the decoding (by nodes who receive enough information during that time slot) will happen by the end of that time slot. Let z_{it}^k be an indicator binary variable that indicates whether or not node i decodes the message k during time slot t, as per inequality in equation (1). In other words, we define z_{it}^k to be 1, if node i decodes message k during time slot t, and 0 otherwise. Let p_{it}^k be the transmit power used by node i at each time t to transmit message k. We define another binary variable x_{it}^k, that is 1 if node i is *allowed* to transmit message k at time t, and 0 otherwise. A node is *allowed* to transmit during a particular time slot, if it has already decoded that message in previous time slots, and it's not receiving or transmitting any other messages during that time slot. Notice that being allowed to transmit does not necessarily mean that a transmission actually occurs. To take care of actual transmissions, let us define v_{it}^k to be a binary variable that is 1 if node i transmits message k at time t, and 0 otherwise. The problem can then be formalized as a combinatorial optimization problem:

$$
\begin{array}{lll}
\min & P_{total} = \sum_{t=1}^{T} \sum_{i=1}^{n} \sum_{k=1}^{r} p_{it}^k & (3) \\
 & 1. \quad p_{it}^k \geq 0, \ \forall i, t, k & \\
\text{s.t.} & 2. \quad x_{d_k T+1}^k = 1, \ \forall k & \\
 & 3. \quad x_{it+1}^k \leq z_{it}^k + x_{it}^k, \ \forall i, t & \\
 & 4. \quad (-M)(1 - z_{it}^k) \leq y_{it}^k, \ \forall i, t & \\
 & 5. \quad p_{it}^k \leq M v_{it}^k, \ \forall i, t & \\
 & 6. \quad \sum_{k=1}^{r} \left(v_{it}^k + z_{it}^k \right) \leq 1, \ \forall i, t & \\
 & 7. \quad v_{it}^k \leq x_{it}^k, \ \forall i, t, k & \\
 & 8. \quad x_{s_k 1}^k = z_{s_k 1}^k = 1, \forall k & \\
 & 9. \quad x_{i1}^k = z_{i1}^k = 0, \forall i \in I \backslash \{s_k\} & \\
 & 10. \quad x_{it}^k \in \{0, 1\} & \\
 & 11. \quad z_{it}^k \in \{0, 1\} & \\
 & 12. \quad v_{it}^k \in \{0, 1\}. &
\end{array}
$$

Here $y_{it}^k = \sum_{j=1}^{n} h_{ji} p_{jt}^k - \theta \sum_{\substack{q=1 \\ q \neq k}}^{r} \sum_{u=1}^{n} h_{ui} p_{ut}^q - \theta N$, M is a large positive constant, and the constraints have the following interpretations:

1) No negative power is allowed.
2) Every node in the destination set is required to have decoded the data by the end of time slot T.
3) If a node has not decoded a message by the end of time slot t, that node is not allowed to transmit that message at time $t+1$.
4) z_{ti}^k is forced to be 0 if message k is not decoded in time slot t.
5) p_{it}^k is forced to be 0, if node i is not transmitting message k at time t (i.e. if $v_{it}^k = 0$).
6) A node cannot transmit and receive at the same time and can only transmit or receive a single message at each time slot.
7) v_{it}^k is forced to be 0, node i is not allowed to transmit message k at time t (i.e. if $x_{it}^k = 0$).
8) Only sources have the message at the beginning.
9) No one else has the message at the beginning.
10) x, z and v are binary variables.

We call this optimization problem MCUE, for multiflow cooperative unicast with Energy Accumulation.

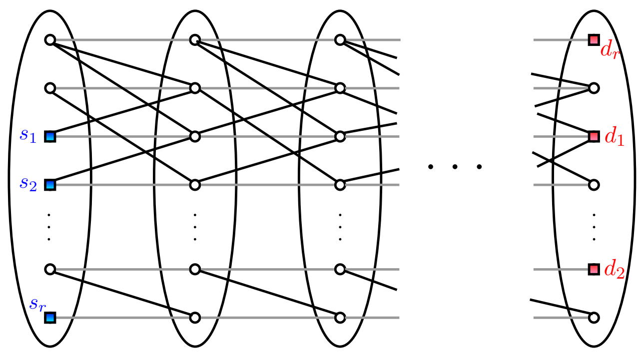

Fig. 1: Applying the multicommodity flow technique for unicast cast

III. Special Case of $k = 1$

In this section we consider MCUE for the special case of $k = 1$.

Theorem III.1 *The optimal solution for MCUE is a simple path for* $k = 1$, *but not necessarily so for* $k > 2$.

Proof: The claim can be proved by induction on T: For delay $T = 1$, the claim is trivially true, as the optimal solution is direct transmission from the source, s, to the given destination, d. Let us assume the claim is true for $T = t - 1$. To complete the proof, we need to show the claim holds for $T = t$. Pick any node in the network as the desired destination d. If the message can be transmitted from source s to d with minimum energy in a time frame less than t, then an optimal simple path exists by the induction assumption. So consider the case when it takes exactly $T = t$ steps to turn on d. The system is memoryless, so d must decode by accumulating the energy transmitted from a set of nodes, $\mathbf{v}$, at time t. This can be represented as $\sum_{v_i \in \mathbf{v}} p_{v_i t} h_{dv_i} \geq \theta$. We observe that there must exist a node $v_o \in \mathbf{v}$ whose channel to d is equal or better than all the other nodes in $\mathbf{v}$. Therefore, given $h_{dv_o} \geq h_{dv_i}, \forall v_i \in \mathbf{v} \backslash \{v_o\}$ then $\sum_{v_i \in \mathbf{v}} p_{v_i t} h_{dv_o} \geq \sum_{v_i \in \mathbf{v}} p_{v_i t} h_{dv_i} \geq \theta$. In other words, if we add the power from all nodes in $\mathbf{v}$ and transmit instead from v_o, our solution cannot be worse. v_o must have received the message by time $t-1$, to be able to transmit the message to d at time t. We know by the induction assumption that the optimal simple path solution exists from source to any node to deliver the message within $t-1$ time frame. Thus, for $T = t$, there exists a simple path solution between s and d, which is optimum. ∎

Considering the above theorem, the MCUE problem formulation (for the special case of $k = 1$) reduces to:

$$
\begin{array}{lll}
\min P_{total} = \sum_{t=1}^{T} \sum_{i=1}^{n} p_{it} & & (4) \\
\text{s.t.} & 1. & p_{it} \geq 0, \ \forall i, t \\
 & 2. & x_{dT+1} = 1 \\
 & 3. & -M(1 - x_{it+1}) \leq \sum_{j=1}^{n} h_{ji} p_{jt} - \theta N, \ \forall i, t \\
 & 4. & p_{it} \leq M x_{it}, \ \forall i, t \\
 & 5. & x_{s1} = 1 \\
 & 6. & x_{i1} = 0, \forall i \neq s \\
 & 7. & x_{it} \in \{0, 1\}
\end{array}
$$

This can be solved optimally in polynomial time using dynamic programming. Let $C(i, t)$ be the minimum cost it takes for source node s to turn on i, possibly using relays, within at most t time slots. Then we can write:

$$C(i, t) = \min_{j \in Nr(i)} [C(j, t-1) + w_{ji}] \qquad (5)$$

with $C(s, t) = 0$, for all t and $C(i, 1) = w_{si}$, where $Nr(i)$ is the set that contains i and its neighboring nodes that have a non-zero channel to i, w_{ji} represents the power it takes for j to turn on i using direct transmission. Thus the solution to (4) is given by $C(d, T)$ and its computation incurs a running time of $O(n^3)$.

IV. Hardness

For $k = 1$, we proved in Theorem III.1, that the optimal solution is a simple path. For $k > 2$, we can consider the following counter-example to argue that the solution is not necessarily a single-path. Consider the scenario shown in Figure 2, where $T = 3$, where the edge weights are equal and the edges shown in gray show strong interference. The red nodes cannot by themselves transmit the message to d_2, as it causes interference for d_1 and d_3 preventing them from being able to decode the data. However, they can cooperate with each other, by each sending with half power to get the message to d_2 without causing too much interference for the other destinations.

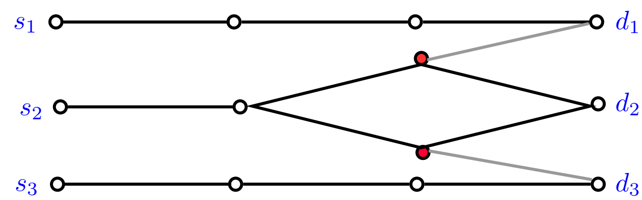

Fig. 2: An example of $k > 2$, with $T = 3$, where the optimal solution is not a single path.

To investigate the complexity of MCUE, let us start by looking at a sub-problem. Imagine a one hop setting of k source nodes and their corresponding k destination nodes, with no relay nodes. Due to interference, not all sources can transmit simultaneously. The task is to schedule the sources appropriately, so that everyone can get their message delivered to their corresponding destination within a time delay T. The problem is to find the minimum such T. Let us call this problem MOSP, for multi-source one-hop scheduling problem[2]. It is important to note that MCUE is at least as hard as MOSP. Thus, any hardness results obtained for MOSP imply hardness of MCUE.

In this section, we derive inapproximablity results for MOSP by showing that any instance of *minimum graph coloring problem* [18] can be reduced to an instance of MOSP.

Lemma IV.1 *MOSP is $o(n^{1/7-\epsilon})$ inapproximable, for any $\epsilon > 0$.*

Proof: Given an instance $G(V, E)$, $|V| = n$, of the minimum graph coloring, we construct a bipartite graph G', with the bi-partition X and Y with $|X| = |Y| = n$. For each node $v_i \in G$, we place two nodes $u_i \in X$ and $u'_i \in Y$ and connect them with an edge (u_i, u'_i). Also for every edge in G, $e_{ij} = \{v_i, v_j\}$, place two edges (u_i, u'_j) and (u_j, u'_i) in G'. We assign u_i and u'_i to be a source and destination pair respectively for all i. We set equal edge weights for all the edges in G' and set $\theta > 1$ to get an instance of MOSP.

A simple example is shown in Figure 3. Notice that the gray edges in the figure represent interference, and by setting $\theta > 1$, a message can be successfully decoded if and only if there is no interference at that node.

This in turn means two sources in G' can simultaneously transmit if and only if there is no edge in between

[2]This is essentially the problem considered in [20], though no proof of complexity is given in that paper.

them in G. Thus, the set of nodes that are transmitting simultaneously in G' correspond to an independent set in G. Consequently, the optimal solution to MOSP is equal to the minimum graph coloring of G, which is known to be $o(n^{1/7-\epsilon})$ inapproximable [19].

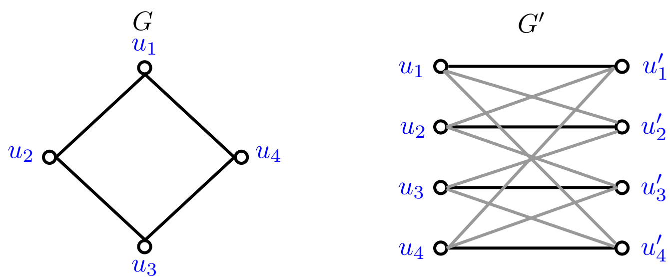

Fig. 3: Example construction of G', for a given G.

■

The following theorem follows by noticing that MOSP is a special case of MCUE.

Theorem IV.1 *MCUE is $o(n^{1/7-\epsilon})$ inapproximable,for any $\epsilon > 0$.*

Notice that the inapproximability result, given by Theorem IV.1, is stronger than, and implies, the NP-hardness result. In other words, it implies that not only finding the optimal solution is NP-hard but finding a polynomial time approximation algorithm that approximates the optimal solution to MCUE with a factor of $o(n^{1/7-\epsilon})$ is also NP-hard.

V. Conclusion

We formulated the problem of minimum energy cooperative transmission in a delay constrained multiflow multihop wireless network, as a combinatorial optimization problem, for a general setting of k-flows and formally proved that the problem not only NP-hard but it is $o(n^{1/7-\epsilon})$ inapproxmiable. To our knowledge, the results in this paper provide the first such inapproxmiablity proof in the context of multiflow cooperative wireless networks. We further proved that for a special case of $k = 1$, the solution is a simple path and offered an optimal polynomial time algorithm for joint routing, scheduling and power control. It is interesting to note that although the minimum graph coloring problem is NP-hard, the fractional graph coloring can be solved in polynomial time. That presents an interesting venue for future work and for designing efficient heuristics for this problem.

References

[1] A. Khandani, J. Abounadi, E. Modiano, L. Zhang, "Cooperative Routing in Wireless Networks," *Allerton Conference on Communications, Control and Computing*, October, 2003.

[2] I. Maric and R. D. Yates, Cooperative Multihop Broadcast for Wireless Networks, *IEEE JSAC*, 2004.

[3] M. Janani, A. Hedayat, T. Hunter, and A. Nosratinia, "Coded Cooperation in Wireless Communications: Space-time transmission and iterative decoding," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 362-371, February 2004.

[4] B. Sirkeci Mergen, A. Scaglione, G. Mergen, "Asymptotic Analysis of Multi-Stage Cooperative Broadcast in Wireless Networks," *Joint special issue of the IEEE Transactions on Information Theory and IEEE/ACM Trans. On Networking*, Vol. 52, No. 6, June 2006.

[5] B. Sirkeci Mergen, A. Scaglione "On the power efficiency of cooperative broadcast in dense wireless networks," *IEEE Journal on Selected Areas in Communications (JSAC), Volume 25, Issue 2,* February 2007.

[6] G. Jakllari, S. V. Krishnamurthy, M. Faloutsos and P. Krishnamurthy, "On Broadcasting with Cooperative Diversity in Multi-hop Wireless Networks," *IEEE JSAC, Special Issue on Cooperative Communications and Networking*, Vol. 25, No. 2, February 2007.

[7] M. Baghaie, B. Krishnamachari, "Fast Flooding using Cooperative Transmissions in Wireless Networks", *IEEE ICC 2009.*

[8] M. Baghaie, B. Krishnamachari, "Delay Constrained Minimum Energy Broadcast in Cooperative Wireless Networks", *INFOCOM 2011.*

[9] T. Halford, K. Chugg, "Barrage Relay Networks," *UCSD ITA Workshop*, San Diego, 2010.

[10] R. Madan, D. Shah, O. Leveque, "Product Multicommodity Flow in Wireless Networks" IEEE Trans. Info Theory, April 2008

[11] S. Kirti, A Scaglione, B. Krishnamachari, "Cooperative Broadcast in Dense Wireless Networks," CRISP-TR-May10, 2010

[12] J. Zhang, Q. Zhang, "Cooperative Routing in Multi-Source Multi-Destination Multi-Hop Wireless Networks," *INFOCOM 2008.*

[13] M. Dehghan and M. Ghaderi, "Energy efficient cooperative routing in wireless networks," Tech. Report 2009-930-09, Uni. of Calgary, 2009.

[14] M. Dehghan, M. Ghaderi and D. Goeckel, "Cooperative diversity routing in wireless networks," WiOpt 2010

[15] G. Middleton, B. Aazhang, "Relay selection for joint scheduling, routing and power allocation in multiflow wireless networks ," *ISCCSP 2010*

[16] G. Middleton, B. Aazhang, "Polynomial-Time Resource Allocation in Large Multiflow Wireless Networks with Cooperative Links ," *IZS 2010*

[17] M. Baghaie, B. Krishnamachari, and A. Molisch, "A Generalized Algorithmic Formulation of Energy and Mutual Information Accumulation in Cooperative Multihop Wireless Networks," arXiv:1102.2825

[18] D. S. Hochbaum, *Approximation Algorithms for NP-Hard Problems*, PWS Publishing Company, 1997

[19] M. Bellare, O. Goldreich, M. Sudan,"Free bits, PCPs and non-approximability - towards tight results", *SIAM J. Comp.* 27, 804-915, 1998.

[20] T. ElBatt and A. Ephremides, "Joint Scheduling and Power Control for Wireless Ad Hoc Networks," *IEEE Transactions on Wireless Communications*, vol. 3, no. 1, Jan. 2004.