Optimal Information Extraction in Energy-Limited Wireless Sensor Networks

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***Abstract*—The current practice in wireless sensor networks is to develop functional system designs and protocols for information extraction using intuition and heuristics, and validate them through simulations and implementations. We address the need for a complementary formal methodology by developing non-linear optimization models of static WSN that yield fundamental performance bounds and optimal designs. We present models both for maximizing the total information gathered subject to energy constraints (on sensing, transmission and reception), and for minimizing the energy usage subject to information constraints. Other constraints in these models correspond to fairness and channel capacity (assuming noise but no interference). We also discuss extensions of these models that can handle data aggregation, interference and even node mobility. We present results and illustrations from computational experiments using these models that show how the optimal solution varies as a function of the energy/information constraints, network size, fairness constraints, and reception power. We also compare the performance of some simple heuristics with respect to the optimal solutions.**

Keywords: wireless sensor networks, network flow optimization, maximum information extraction.

I. INTRODUCTION

Wireless sensor networks (WSN) are an emerging technology which seem ready to revolutionize the availability and quality of information in a wide array of application areas. This new technology has come about due to the rapid advances in embedded microprocessors, wireless communications, and MEMS sensors over the past decade.

As we set out to design and implement these kinds of systems, however, one fact becomes clear. *In the area of WSN there is a significant gap between practice and formal understanding: proposed system designs and protocols are rapidly out-pacing analysis.* There are very few formal models for analyzing the fundamental performance of information routing in wireless sensor networks.

Such models are necessary to understand the theoretical bounds on performance and how they are affected by different design parameters such as topology, number of nodes, energy levels, and fairness. We take an optimization approach in this paper. Due to the underlying equations that describe the capacity of physical channels, we will rely on convex non-linear programming techniques. We present models both for maximizing the total information gathered subject to energy constraints (on sensing, transmission and reception) and for minimizing the energy usage subject to information constraints. Fairness constraints are also modeled. We will also discuss extensions of these models that can handle data aggregation, interference and even node mobility.

Optimization models can aid us in two complementary ways. The first involves designing a WSN for a given application. The best network configuration for an application is often difficult to determine due to the variability in problem parameters that characterize the diverse applications to which this technology can be applied. These parameters include the quality of information requested, the energy cost of sensing and receiving information, and node positions. The most appropriate network parameters for the application in question can be determined by comparing the optimal performance for different parameter settings.

The second way in which optimization models can improve our understanding of WSN concerns the operation of a sensor network. Here an optimization model provides the means to evaluate proposed protocols for information routing. Much of the current literature in

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sensor and ad-hoc wireless networks consists of practical proposals for new protocols for information routing. Typically, simulation results are used to examine the impact of various parameters on the effectiveness of the protocol. Comparisons are usually performed with respect to some baseline heuristic strategies or with alternative protocols. Iterated over time, this procedure yields practical, implementable protocols with successively better performance characteristics. However, if we do not know the fundamental bounds imposed by the underlying problem structure, then it will not be clear how the implemented protocol differs from optimal performance. It is important to know the fundamental performance bounds and optimal solutions to determine if there is room for additional improvement of a given protocol.

The rest of the paper is organized as follows. In section II, we discuss some recent papers on information routing in wireless sensor networks and optimization models of wireless networks to place our work in context. In section III, we define our notation and present a basic operational optimization model. We expand this model in section IV to a more general pair of optimization problems that can be used for WSN design problems and discuss extensions. We present computational results based on these models in section V, and conclude with a discussion in section VI.

II. Related Work

Wireless sensor networks, consisting of large numbers of unattended devices capable of communication, computation and sensing are a subject of significant ongoing research [1]. In most of the prior work, however, the performance of proposed querying and routing mechanisms is validated through simulations or implementation, without any notion of fundamental performance bounds or reference to an optimum benchmark solution.

Most closely related to our work presented here are papers relating to optimization models of general multi-hop wireless networks as well as wireless sensor networks (which can be considered a special case of the former with a many-to-one data flow instead of arbitrary communication between pairs of nodes). The most important work in this area in recent years has been the work by Toumpis and Goldsmith on capacity regions for wireless networks [2], [3]. Using a linear-programming optimization based formulation (similar in spirit to our work), the authors study the characteristics of the maximum information throughput that can be obtained in a network with arbitrary topology. One key difference from our work is that Toumpis and Goldsmith focus on general-purpose wireless networks and do not incorporate energy or fairness constraints in their modeling. They also do not use constraints corresponding to the non-linear channel capacity.

The non-linear physical channel constraints are considered in the optimization models discussed in [10], [11]. In these works the authors consider a similar model to ours (jointly optimizing the routing as well as power control and bandwidth allocation). They also treat the constraints imposed by interference in their models. Again a significant difference between our work and these models is that they do not focus on sensor networks where energy and fairness constraints are important.

Optimization models have also been used to study maximum lifetime conditions for ad hoc and sensor networks. Bhardwaj and Chandrakasan [6] develop upper bounds on the lifetime of networks based on optimum role assignments to sensors (e.g. whether they should act as routers or aggregators). Kalpakis *et al.* [7] formulate a linear programming problem to schedule flows within the network in such a way as to maximize the network lifetime. Chang and Tassiulas [8] also formulate a flow-based linear programming problem and related heuristics to maximize the network lifetime. Our work incorporates and investigates a number of different constraints from these, such as the non-linear physical channel constraint (which allows for joint optimization of power control and routing) and fairness constraints. This work also builds on our some of our previous analysis concerning the impact of fairness constraints [5].

III. Notation and Preliminary Model

Our first optimization model considers the problem of operating an existing WSN in the most efficient manner. Assume we have placed n sensor nodes in fixed locations, each with a limited energy supply E_i, and let d_{ij} denote the physical distance between nodes i and j. The purpose of this network is to extract as much information as possible to a sink node (node $n+1$ with unlimited energy resources – a reasonable assumption if the sink is “plugged in”). Each node consumes C units of energy per-bit received and β units of energy per-bit sensed.

We assume that the sensor nodes can adjust both the information flow rate and the transmission power, which are denoted f_{ij} and P_{ij} for the link between nodes i and j, respectively. The relation between the flow rate and transmission power on a link is given by Shannon’s capacity equation for an AWGN channel, assuming a square-law signal decay d_{ij}^{-2}, a noise of η on the communication channel, and that all transmissions are scheduled (either through time or frequency division multiplexing) such that they are non-interfering.

The objective is to find the coordinated operation of all nodes by setting transmission powers and flow rates in order to maximize the amount of information that reaches the sink. We assume that there is no data aggregation in this model, and additionally we guarantee end-to-end fairness of our solution by explicitly enforcing that each node sends at most a fraction α_i of the total information that reaches the sink. The total energy consumed at node i, which we denote ε_i, is the sum of the energy consumed sensing, transmitting and receiving. There is a constraint that this energy should not exceed the available energy E_i for each sensor node.

Under these assumptions, the problem is expressed by the following non-linear program

$$
\begin{aligned}
&\max \sum_{j=1}^{n} f_{jn+1} \\
&\text{s.t.} \\
&\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \geq 0 \\
&\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \leq \alpha_i \sum_{j=1}^{n} f_{jn+1} \\
&\beta \left(\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \right) + \sum_{j=1}^{n+1} P_{ij} + \sum_{j=1}^{n} C f_{ji} \leq E_i \\
&f_{ij} \leq \log \left(1 + \frac{P_{ij} d_{ij}^{-2}}{\eta} \right) \\
&f_{ij} \geq 0, P_{ij} \geq 0
\end{aligned}
\tag{1}
$$

IV. Design Models

Consider now the problem of designing a WSN for a given application. In this context we now have the liberty to determine the position of the nodes and the amount of energy to place at each node. The problem of actively optimizing the location of the nodes poses serious difficulties, as for example the non-linear inequality representing the Shannon capacity bound becomes non-convex. We avoid this problem in the current work.

The problem of deciding how to distribute a given overall amount of energy is easier to implement. Adding the consumption of energy for every node i we obtain the following expression for the total energy consumed by the sensor nodes of the WSN:

$$
\begin{aligned}
&\sum_{i=1}^{n} \varepsilon_i = \\
&\quad = \sum_{i=1}^{n} \left(\beta \left(\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \right) + \sum_{j=1}^{n+1} P_{ij} + \sum_{j=1}^{n} C f_{ji} \right) \\
&\quad = \sum_{i=1}^{n} \left(\beta f_{in+1} + P_{in+1} \right) + \sum_{i=1}^{n} \sum_{j=1}^{n} \left(C f_{ij} + P_{ij} \right) \ .
\end{aligned}
$$

Note that this expression for the total energy consumed has a nice interpretation: the first term represents the cost of sensing the information that is sent to the sink and the cost of transmitting it to the sink, while the second term represents simply the cost of transmitting information among all pairs of nodes $i - j$.

We assume that there is an overall energy budget of $E_{\max}$ to distribute among the sensor nodes, which bounds the total energy consumption. Replacing the energy constraints in Problem (1) by this global energy constraint leads to the following non-linear optimization problem, which automatically determines the optimal energy distribution.

$$
\begin{aligned}
&\max \sum_{j=1}^{n} f_{jn+1} \\
&\text{s.t.} \\
&\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \geq 0 \\
&\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \leq \alpha_i \sum_{j=1}^{n} f_{jn+1} \\
&\sum_{i=1}^{n} \left(\beta f_{in+1} + P_{in+1} \right) + \sum_{i=1}^{n} \sum_{j=1}^{n} \left(C f_{ij} + P_{ij} \right) \leq E_{\max} \\
&f_{ij} \leq \log \left(1 + \frac{P_{ij} d_{ij}^{-2}}{\eta} \right) \\
&f_{ij} \geq 0, P_{ij} \geq 0
\end{aligned}
\tag{2}
$$

The maximum information that can be obtained from the WSN is in essence bounded by the total energy available. We now present a related problem that also pertains to the trade-off between minimum energy requirements and maximum information possible for a given network topology. This problem considers minimizing the total energy usage, while guaranteeing at least $f_{\min}$ information to the sink. This problem can be stated as

$$
\begin{aligned}
&\min \sum_{i=1}^{n} (\beta f_{in+1} + P_{in+1}) + \sum_{i=1}^{n}\sum_{j=1}^{n} (C f_{ij} + P_{ij}) \\
&\text{s.t.} \\
&\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \geq 0 \\
&\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \leq \alpha_i \sum_{j=1}^{n} f_{jn+1} \\
&\sum_{j=1}^{n} f_{jn+1} \geq f_{\min} \\
&f_{ij} \leq \log\left(1 + \frac{P_{ij} d_{ij}^{-2}}{\eta}\right) \\
&f_{ij} \geq 0, P_{ij} \geq 0
\end{aligned}
\tag{3}
$$

The remainder of this section discusses relations between these two models, some properties, and possible generalizations.

A. Relation between design problems

The relationship between Problems (2) and (3) above is the subject of the following proposition[1]. In it we denote by $(f, P) = (f_{12}, \ldots, f_{nn+1}, P_{12}, \ldots, P_{nn+1})$ the vector of flow rate and transmission power variables.

Proposition 1: If (f^*, P^*) is the optimal solution to Problem (3) with $f_{\min}$, then (f^*, P^*) is the optimal solution to Problem (2) with

$$E_{\max} = \sum_{i=1}^{n} \left(\beta f^*_{in+1} + P^*_{in+1}\right) + \sum_{i=1}^{n}\sum_{j=1}^{n} \left(C f^*_{ij} + P^*_{ij}\right) .$$

Conversely, if (f^*, P^*) is the optimal solution to Problem (2) with $E_{\max}$, then (f^*, P^*) is the optimal solution to Problem (3) with $f_{\min} = \sum_{j=1}^{n} f^*_{jn+1}$.

The relationship between Problems (2) and (3) can also be observed computationally. In Figure 1 we plot both the maximal information extracted as a function of the energy bound and minimum energy needed as a function of the information bound. The experiments that originated these results considered the same WSN with all nodes placed in a straight line, the sink node at one end, 10 sensor nodes uniformly distributed from a distance 1 to 10 of the sink, and the following values for other problem parameters: $\beta = 0.00001$, $C = 0.00005$, $\eta = 0.0001$, and $\alpha_i = 0.2$ for all i. The minimum information bound was varied from $f_{\min} = 1$, to $f_{\min} = 20$ when solving Problem (3), and the maximum energy bound was varied from $E_{\max} = 0.01$ to $E_{\max} = 0.2$ when solving Problem (2).

[1]The proof of this and other propositions in the paper are omitted due to space considerations. Please see [4] for the proofs.

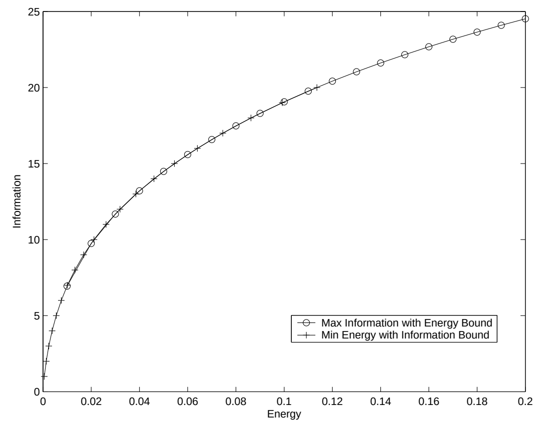

Fig. 1. Optimal energy versus information

However there is one striking difference between these two optimization problems, Problem (3) exhibits consistently a faster convergence than Problem (2). This is illustrated by the following table which summarizes the number of interior point method (IPM) iterations it takes to solve each problem, as we increase the number of nodes. Here we used the same problem parameters as in the prior experiment, except $\alpha_i = 0.25$ for all i and we varied the number of sensor nodes uniformly distributed from a distance 1 to 10 of the sink. The minimum information bound was 10 for Problem (3) and the maximum energy bound was 0.01 for Problem (2).

TABLE I

IPM CONVERGENCE FOR PROBLEMS (2) AND (3)

No. nodes	Problem (2) IPM iterations	Problem (3) IPM iterations
4	19	19
7	21	19
10	19	20
15	22	21
20	22	27
25	24	22
30	116	27
40	447	39
50	272	31
60	89	39
70	371	49
80	347	34

B. Problem properties

The discussion in this subsection highlights some important properties of Problem (3), which can be reduced to a much simpler form. This simplification, which can't be translated to Problem (2), provides insight into the difference in the observed computational convergence.

To simplify our presentation we will use the arc-incidence matrix, as it is used in the Network Flows literature; see for example [9]. For a network with $n+1$ nodes and m arcs, the arc-incidence matrix, usually denoted by N, is a $n+1$ by m matrix with coefficients equal to 0, 1 or -1. The matrix is defined by

$$N_{i(k,l)} = \begin{cases} 1 & \text{if } i = k \\ -1 & \text{if } i = l \\ 0 & \text{otherwise} . \end{cases}$$

We can write the flow constraints (the first two constraints in Problems (2) and (3)) using matrix N as

$$0 \le Nf \le \alpha \sum_{j=1}^{n} f_{jn+1} \ .$$

Proposition 2: Define $\kappa_j = C$ if $j = 1 : n$ and $\kappa_{n+1} = \beta$. Problem (3) obtains the same optimal solution as Problem (4) below:

$$\begin{array}{l} \min \sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_j f_{ij} + \eta d_{ij}^2 \left(e^{f_{ij}} - 1 \right) \\ \text{s.t.} \\ 0 \le Nf \le \alpha f_{\min} \\ \sum_{j=1}^{n} f_{jn+1} = f_{\min} \\ f_{ij} \ge 0 \end{array} \tag{4}$$

For f a vector of flow rates, let us denote by $z(f)$ the objective function of Problem (4),

$$z(f) = \sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_j f_{ij} + d_{ij}^2 \left(e^{f_{ij}} - 1 \right) \ ,$$

note that $z(f)$ is a convex function. We also denote by $S(y)$ the feasible region of Problem (4) and $\phi(y)$ the optimal objective function value of Problem (4), both as a function of the information bound $f_{\min} = y$. Therefore Problem (4) can be rewritten as

$$\begin{array}{rll} \phi(f_{\min}) = & \min & z(f) \\ & \text{s.t.} & f \in S(f_{\min}) \end{array} \tag{5}$$

Proposition 3: The function $\phi(y)$ is convex for all $y \ge 0$

C. Model variations

Below we discuss some possible variations on the models considered above:

- We can formulate the problem only in terms of variables f_{ij} representing the number of bits transmitted from i to j (rather than the bit rate). This implies that each sensor node transmits with a fixed power, and therefore at a fixed rate. More bits are transmitted by taking a longer period of time. The resulting problem is a simpler linear program. This bit model is obtained by replacing upper bound on the flow rate by $f_{ij} \le B$ and the energy bound by
$$\beta \left(\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \right) + \sum_{j=1}^{n+1} T_{ij} f_{ij} + \sum_{j=1}^{n} C f_{ji} \le E_i \ ,$$
where T_{ij} is the per-bit transmission energy cost. The upper bound on the total bits effectively limits the length of a round. An interesting open question is to quantify the loss of efficiency on the network by considering this more constrained model.
- An alternate objective can be to maximize a weighted fairness. This could be achieved by replacing the objective in Problem (1) by $\max \sum_{i=1}^{n} w_i \alpha_i$ and by replacing the flow constraints by
$$\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \ge \alpha_i \ .$$
- Additional requirements that can be modeled by incorporating a simple linear constraint are: a per node limit on the total energy available to each node and a limit on the amount of information that can be sensed by each node during a round.
- Multiple time periods can be easily represented in these models. The only constraint that links different time periods is the energy constraint The total energy available has to be distributed across all time periods.
- The problems above do not allow for data aggregation, i.e. all data that is sensed must leave the network through the sink. Data aggregation can be accommodated using multiple flows in the same network to represent separately the flow of data and the usage of a communication channel, and to identify which data can be aggregated together.

The variations on the problem above are fairly straightforward and the only potential complications in solving the new models are that considering multiple time periods or multi-commodity flows makes the problem larger. Below are a pair of variations on the model that create non-convex optimization problems, and are therefore much harder to solve. We mention them here to show possible future research directions.

- The models above assume that the network has scheduled communications on all links (using

TDMA or FDMA). For a CDMA like environment, interference poses a non-convex constraint. There are some techniques that can be used to handle such constraints approximately [11].
- Models can also consider the possibility of mobile nodes, in which locations and therefore inter-node distances can be varied as a design parameter at the expense of some energy for motion. However, this also introduces a non-convex constraint.

V. Computational Experiments

Throughout our computational experiments we have considered two different types of network topologies that are easily scalable: the *line topology* and the *square topology*. In the line topology we considered WSN where all sensor nodes lie uniformly distributed in a line of length L, with the sink node placed at one end. The square topology considers $n = k^2$ nodes uniformly distributed on a square grid with sides of length L ($[0, L] \times [0, L]$) with the sink located outside that square. We performed our computational experiments with the non-linear solver LOQO 6.02 called from AMPL scripts. We used the NEOS server for optimization to perform our computations; see [12].

Our computational experiments illustrate different possible uses of optimization models for WSN. We first show how to use Problem (4) to determine the optimal amount of information that can be extracted from a given WSN. We then point out that optimization models give benchmarks for routing heuristics, we present comparisons against two simple heuristics for illustration. Our last three studies investigate the effect of different problem parameters on the performance of the sensor network. We study the effect of the fairness pattern on the minimum energy required, the effect of the fairness patterns on optimal energy distribution and routing patterns, and the effect of the reception cost on the routing behavior.

A. Optimal level of information extraction

From Figure 1 above we note that, for that particular WSN, each extra unit of information demands an increasing amount of energy. We now address the question of whether for any WSN each extra unit of information demands an increasing amount of energy. The answer is yes and we show this by proving that for any WSN the subgradient of $\phi(y)$, $\partial\phi(y)$, is an increasing positive multifunction. The fact that $\partial\phi(y)$ is monotonic increasing is due to the convexity of $\phi(y)$ (Proposition 3), so we simply have to show that it is positive. We work with subgradients because $\phi(y)$ could be a continuous piece-wise convex function.

Proposition 4: Assume that either $\beta > 0$ or $d_{in+1} > 0$ for all $i = 1 : n$. Then for any $y > 0$, all subgradients of $\phi(y)$ are positive.

Given the increasing energy cost of additional information for any WSN, it natural to look for the optimal amount of information to extract from a given WSN. In a commercial setting it is reasonable to assume that there is some monetary value for information from a WSN, i.e. a dollars per unit of information, and a monetary value for the cost of energy, i.e. b dollars per unit of energy. Thus we can explicitly compare the trade-off between information extraction and energy consumption by maximizing the net return function $V(y) = ay - b\phi(y)$, where y is the amount of information extracted. The maximum level of net return is obtained for the solution y^* that satisfies $0 \in \partial V(y^*)$, since $\phi(y)$ is convex. This implies that the optimal information level satisfies $a/b \in \partial\phi(y^*)$. Optimization solvers also provide subgradient values of $\partial\phi(y)$ as the dual variable on the information constraint. The values in $\partial\phi(y)$ quantify the change in the objective function with respect to changes in y. Because of the monotonicity of the subgradient, we can perform a binary search to obtain the information level y^* which has $a/b \in \partial\phi(y^*)$ and thus gives the optimal level of return. This leads to an approach where to obtain a solution $\hat{y}$ such that $|y^* - \hat{y}| \le \varepsilon$, we solve a polynomial number ($O(\log(\frac{1}{\varepsilon}))$ of Problems (4).

B. Comparison of Optimal Performance v.s. Heuristics

In this subsection we explore how the optimal performance given by Problem (4) compares to two very simple heuristics for assigning energy to nodes and distributing the information. The purpose of this comparison is only to illustrate how an optimization model can provide benchmarks. Benchmarking efficient heuristics, which have been proposed, requires unifying the assumptions of the optimization model with those made by the heuristics and is part of our future work.

In our first heuristic we only allow transmissions from nodes directly to the sink. Problem (4) under this assumption is to minimize $\sum_{i=1}^{n} \left(\beta f_i + \eta d_i^2 e^{f_i} - \eta d_i^2\right)$, subject to $\sum_{i=1}^{n} f_i = f_{\min}$, $f_i \le \alpha_i f_{\min}$, and $f_i \ge 0$, where f_i is the flow rate from i to the sink. A seemingly efficient solution to this problem is to assign as much information as possible to the nodes with smallest objective function contribution. We achieve this solution by the following heuristic, which we denote the Direct

Heuristic: (1) sort the nodes according to their distance to the sink, that is $d_1 \leq d_2 \leq \ldots \leq d_n$, (2) set the flow from $i = 1$ to n to $f_i = \alpha_i f_{\min}$ until $\sum_{i=1}^{n} f_i = f_{\min}$, and (3) set all remaining flows to zero.

Our second heuristic, which we denote the Hop Heuristic, routes all information from a node to the closest node in the direction of the sink. This heuristic can be generally described as follows, Hop Heuristic: (1) sort the nodes according to their distance to the sink, (2) set the amount of flow generated at i from $i = 1$ to n to $\alpha_i f_{\min}$ until $\sum_{i=1}^{n} f_i = f_{\min}$, (3) determine the shortest path from every node providing information to the sink, (4) send all the information from i to the next node on the shortest path from i to the sink.

In Figures 2 and 3 below we present how the optimal energy levels compare with the energy levels obtained from the heuristic procedures. The experiments considered linear and square topologies, and present the different energy levels as we increase the number of nodes in the network. For the both types of problems we considered the following problem parameters $\beta = 0.00001$, $C = 0.00005$, $\eta = 0.0001$, and $f_{\min} = 10$. The linear topologies example considered from 4 to 80 sensor nodes placed in a line uniformly distributed a distance 1 to 10 from the sink. The square topologies considered from 4 to 81 nodes uniformly distributed on a grid in the square $[0, 100] \times [0, 100]$ with the sink located at $(-30, 50)$. For both types of problems we considered a uniform fairness pattern with $\alpha_i = 2/n$ for all i.

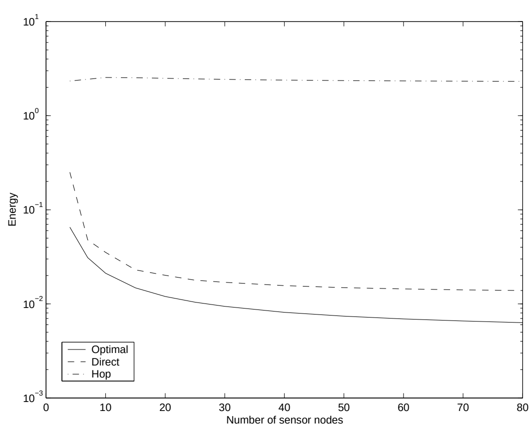

Fig. 2. Minimal Energy and Heuristics, for variable fairness $2/n$ as a function of the number of nodes, linear topology

A striking observation from Figure 2 is the poor performance of the Hop Heuristic. The reason for this is that in the line topology all the information is routed through the node that is closest to the sink, this node then is forced to spend a significant amount of energy to transmit it to the sink. In the Direct Heuristic and in the optimal solution no node transmits all the information, so the transmission powers can be much smaller. The performance of the Hop Heuristic for the square topology is better, as can be noted in Figure 3. In this example, all the information is first routed to the nodes that are on the face of the square next to the sink, from where they can be transmitted directly. Therefore there is no “bottleneck” node.

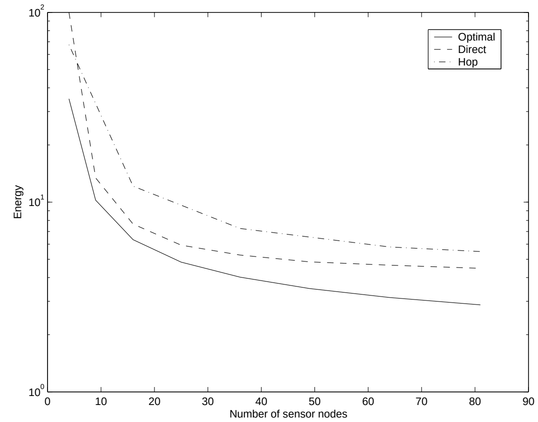

Fig. 3. Minimal Energy and Heuristics, for variable fairness $2/n$ as a function of the number of nodes, square topology on $[0, 100]^2$

Note that the energy is plotted in a log axis, therefore although in general the differences persist, the Direct Heuristic provides a reasonable approximation to the optimal solution for large n. This statement however should be taken with a grain of salt, as the topology of the problem and problem parameters do influence the proximity of the heuristic to the optimal solution.

C. Effect of fairness on optimal energy

From Problem (4) we note that the energy needed to extract a certain amount of information, depends exponentially on the amount of information to extract. Consider a WSN with n sensor nodes, a fixed topology, energy cost coefficients β and C, and on a communication channel with noise η. We now obtain an upper bound on the energy needed to extract an information level $f_{\min}$ with any fairness pattern α, such that $\alpha_i \geq \frac{1}{n}$ for all i. The optimal energy level E^* for the WSN will be less than or equal to $\widehat{E}$, the energy needed to route the information for a completely fair WSN, that is when $\alpha_i = \frac{1}{n}$ for all i. Analogously $\widehat{E}$ will be less than or equal to the energy needed to have each node route $\frac{1}{n} f_{\min}$ directly to the sink, as this is a feasible way of routing that information.

This means that

$$\widehat{E} \quad \leq \quad \beta f_{\min} + \sum_{i=1}^{n} d_{in+1}^{2} \eta \left(e^{\frac{1}{n} f_{min}} - 1 \right) \ .$$

This provides an upper bound on the energy needed to extract $f_{\min}$ from a WSN with any fairness pattern α. We plot this upper bound, as well as the energy needed for a WSN with uniform alpha patterns (we considered $\alpha_i = 0.1, 0.2$ and 1 on every sensor node) in Figure 4. The experiment considered a network with 11 nodes, equally spaced on a line a distance .1 to 1.1 from the sink, and other problem parameters at $C = 0.00005$, $\beta = 0.00001$, and $\eta = 0.001$. We present the minimal energy needed for different information bounds. A lower bound is trickier, as the optimal solution for the completely unfair WSN (i.e. $\alpha_i = 1$ for all i) still uses the shortest route and source to obtain its information. Simple strategies, such as all the information from the node closest to the sink, which has a cost of $\beta f_{\min} + \eta(\min\{d_{1n+1}, \ldots, d_{nn+1}\})^2 \left(e^{f_{min}} - 1\right)$, do not provide bounds and in fact increase sharply.

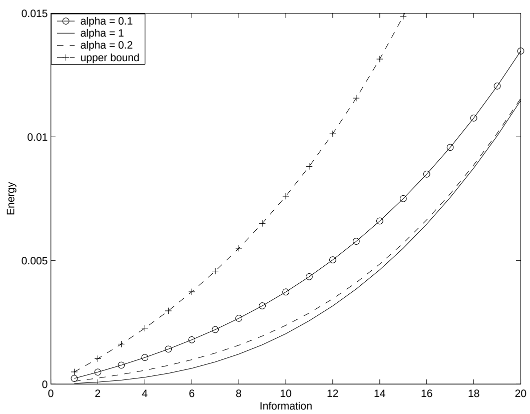

Fig. 4. Information versus energy for different α

D. Effect of fairness patterns on performance

In this subsection we study how different fairness patterns affect the optimal energy distribution in the WSN and also the form of the optimal information routing.

We consider an example with 25 sensor nodes uniformly distributed on a grid $[0, 10] \times [0, 10]$ and a sink node located at $(-3, 5)$. Other problem parameters were set at $\beta = 0.00001$, $C = 0.00005$, $\eta = 0.0001$, and $f_{\min} = 10$. Below we present two examples with different fairness pattern on the network. The first pattern considered is a totally unfair network, where every node could potentially send all the information, $\alpha_i = 1$ for all i, The optimal energy distribution and flow rates are presented in Figure 5. We note that although all the information could potentially originate from a single node, the optimal is to use several nodes, the ones that are within a certain radius from the sink to obtain all the information. Also note that in this solution the information is routed directly to the sink.

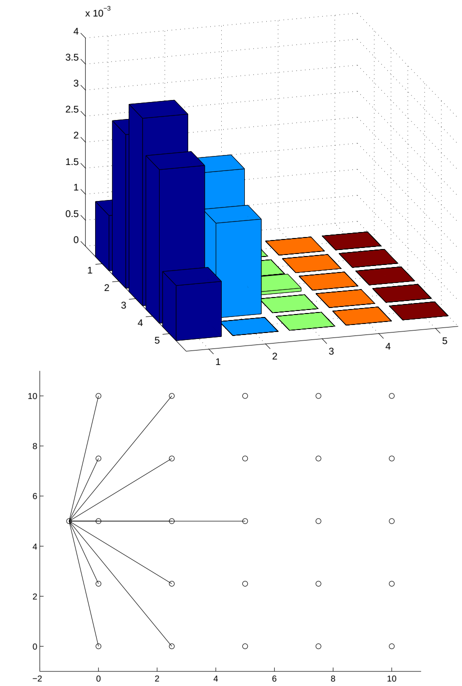

Fig. 5. Optimal energy distribution and flow rates, $\alpha_i = 1$

Our second experiment considers a more restrictive uniform fairness pattern. We set $\alpha_i = 0.05$ for all i. Although this is not a totally fair system, now every node can send at most 5% of the total information. We present the optimal energy distribution and flow rates in Figure 6. Note that some information originates far enough that it is beneficial to route the information through other nodes, for example information that originates in nodes on the 4th column is not routed directly to the sink.

Both, the reason for not sending all the information from a single node in the first example and deciding to route the information in the second, are decisions that minimize the transmission cost, which from the objective function of (4) is $\eta d_i(e^{f_{ij}} - 1)$. The reason to send information from nodes that are further away than available capacity is not to increase the exponent f_{ij} too much, and the reason to route the information that is far away is to keep at zero the contribution of terms which have a big d_{ij}.

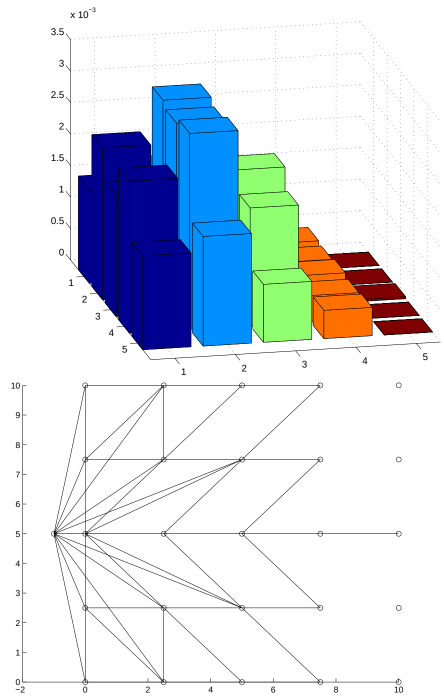

Fig. 6. Optimal energy distribution and flow rates, $\alpha_i = 0.05$

E. Types of solution

In this section we investigate when the optimal solution prefers to route the information directly and when it is more efficient to hop through a different node. An alternative is to use the optimization model to study how the hopping behavior of the optimal routing is affected for different parameter values. Here we take a different approach, we plan to study the effect of the reception cost C on the hopping behavior for a very simple example that allows an analytical solution which can be verified computationally.

We consider a problem with two sensor nodes with all the information to be extracted on the one furthest, distance 1, from the sink (i.e. $\alpha_1 = 0$, $\alpha_2 = 1$). The question is when node 2 prefers to send the information directly to the sink and when it prefers to route it through node 1. For simplicity we place node 1 exactly mid-way between node 2 and the sink. Assume also that $f_{\min} = 1$.

Here we are simply comparing the solution in which we route all the information directly, at a cost $h_s = \beta + \eta(e-1)$, with the case in which we send f_1 from node 2 to node 1 and then to the sink, and $f_2 = 1 - f_1$ directly form node 2 to the sink, at a cost $h_c(f_1) = \beta + Cf_1 + \frac{1}{2}\eta(e^{f_1} - 1) + \eta(e^{1-f_1} - 1)$. The amount of information that will be routed will be the minimizer of $h_c(f_1)$ on the domain $[0, 1]$.

We now determine the value of C for which $H_C(f_1) = h_c(f_1) - h_s \geq 0$ for all $f_1 \in [0, 1]$. These are the values of C for which any amount of hopping is more expensive than routing directly. It is easy to show that $H_C(f_1)$ is a convex function and $H_C(0) = 0$, therefore to guarantee that $H_C(f_1) \geq 0$ for all $f_1 \geq 0$ it is sufficient to show that $H_C'(f_1) \geq 0$. This is equivalent to $C \geq \eta(e - \frac{1}{2})$.

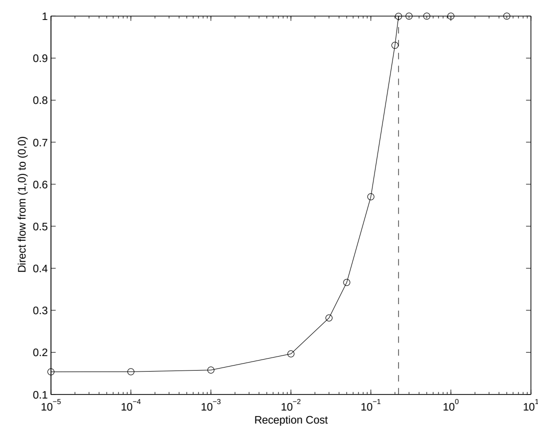

Fig. 7. Direct flow to the sink as a function of the reception cost C

In Figure 7 we plot the total value of flow that is sent directly to the sink from node 2 for different reception cost values. This computational example has $\beta = 0.00001$ and $\eta = 0.1$. The critical value for the reception cost is $C = 0.2218$, which is plotted as a vertical line in Figure 7. Note that the computational experiment validates this critical value, and that for any value of C node 2 sends information directly to sink.

VI. Conclusions

In this paper we addressed the need for a systematic methodology by developing formal non-linear optimization models of static WSN that yield fundamental performance bounds and optimal designs. We presented models for two problems: 1. maximizing the total information gathered subject to energy constraints (on sensing, transmission and reception) and 2. minimizing the energy usage subject to information constraints. Other constraints in these models correspond to fairness and channel capacity (assuming noise without interference). We showed that the two problems are in fact equivalent to each other in terms of a correspondence between optimal solutions and constraints. However, we showed that the second model is computationally more efficient.

We then conducted several studies with these optimization models. (1) We discussed how the dual variable of the information constraint can be used to determine the optimal trade-off between information extraction and energy expenditure. (2) We presented computational results of how simple heuristics (sending directly to sink and shortest multi-hop paths) compare to the optimal solution as network size increases. While these were illustrative examples, in our future work we would like to compare more sophisticated heuristics similar to those presented in [8]. (3) We investigated the effect of fairness pattern on minimum energy requirements, optimal energy distribution, and flow patterns. One interesting result is that when there are no fairness constraints the optimal way for all nodes to send directly to the sink is for them to send information in such a way that their contributions to the objective function are all equal. (4)Our final result pertains to the effect of reception cost on whether the the optimal routing solution sends its information directly to the sink. We identified a threshold for the reception cost beyond which the optimal solution routes all information directly to the sink.

There are a number of natural extensions of this work we plan to undertake in the future. Many of these involve the model variations we mentioned in section IV-C — in particular enriching our models to incorporate data aggregation, mobility and interference (which would be meaningful in a CDMA environment as opposed to the interference-free TDMA/FDMA scheduled access assumed in this paper).

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