ALGORITHMIC ASPECTS OF ENERGY EFFICIENT TRANSMISSION IN MULTIHOP COOPERATIVE WIRELESS NETWORKS

by

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Dedication

To my dear Maman and Baba, whom I love and admire.

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Abstract

We consider the problem of energy-efficient transmission in cooperative multihop wireless networks. Although the performance gains of cooperative approaches are well known, because of the combinatorial nature of these schemes, designing efficient polynomial-time algorithms to decide which nodes should take part in cooperation, and when and with what power they should transmit, has remained a challenge. We propose to tackle this problem in this dissertation.

We provide a generalized algorithmic formulation of the problem that encompasses the two main cooperative approaches, namely: energy accumulation and mutual Information accumulation. We investigate the similarities and differences of these two approaches under our generalized formulation, focusing in particular on the scenario where a delay constraint is present. We prove that the broadcast and multicast problems are, in general, $o(\log(n))$ inapproximable. We break this NP hard problem into three parts: ordering, scheduling and power control and propose a generalized novel algorithm that, given an ordering, can optimally solve the joint power allocation and scheduling problems simultaneously in polynomial time. We further show empirically that this algorithm used in conjunction with an ordering derived heuristically using the Dijkstra's shortest

path algorithm yields near-optimal performance in typical settings. In the unicast case, we prove that although the problem remains hard with mutual information accumulation, it can be solved optimally and in polynomial time when energy accumulation is used. We use our algorithm to study, numerically, the trade-off between delay and power-efficiency in cooperative broadcast and compare the performance of energy accumulation vs mutual information accumulation as well as the performance of our cooperative algorithm with a smart non-cooperative algorithm in a broadcast setting. We also provide an $O(T \log^2(n))$ approximation algorithm for the broadcast case where energy accumulation is used.

We further formulate the problem of minimum energy cooperative transmission in a delay constrained multiflow multihop wireless network, as a combinatorial optimization problem, for a general setting of k-flows and formally prove that the problem is not only NP-hard but it is $o(n^{1/7-\epsilon})$ inapproxmiable. To our knowledge, the results in this dissertation provides the first such inapproxmiablity proof in the context of multiflow cooperative wireless networks. We show that for a special case of $k = 1$, the solution is a simple path and develop an optimal polynomial time algorithm for joint routing, scheduling and power control. We then use this algorithm to establish analytical upper and lower bounds for the optimal performance for the general case of k flows. Furthermore, we propose a polynomial time heuristic for calculating the solution for the general case and evaluate the performance of this heuristic under different channel conditions and against the analytical upper and lower bounds.

Chapter 1

Introduction

In a wireless network, a transmit signal intended for one node is received not only by that node but also by other nodes. In a traditional point-to-point system, where there is only one intended recipient, this innate property of the wireless propagation channel can be a drawback, as the signal constitutes undesired interference in all nodes but the intended recipient. However, this effect also implies that a packet *can* be transmitted to multiple nodes simultaneously without additional energy expenditure. Exploiting this “broadcast advantage”, broadcast, multicast and multihop unicast systems can be designed to work cooperatively and thereby achieve potential performance gains. As such, cooperative transmission in wireless networks has attracted a lot of interest not only from the research community in recent years [36, 37, 39, 41–44] but also from industry in the form of practical cooperative mobile ad-hoc network systems [49].

1.1 Thesis Focus

We focus on the problem of cooperative transmission in this work, starting with a case where a single node is sending a packet to either the entire network (broadcast), a single destination node (unicast) or more than one destination node (multicast), in a multihop wireless network. Other nodes in the network, that are neither the source nor the destination, may act as relays to help pass on the message through multiple hops. The transmission is completed when all the destination nodes have successfully received the message. We particularly focus on the case where there is a delay constraint, whereby the destination node(s) should receive the message within the delay constraint, however, we also discuss how our results apply to the unconstrained case. We also look at the case where there are more than one sources, each trying to send their message (possibly through relays) to their corresponding destinations.

A key problem in such cooperative networks is routing and resource allocation, i.e., the question which nodes should participate in the transmission of data, and when, and with how much power, they should be transmitting. The situation is further complicated by the fact that the routing and resource allocation depends on the type of cooperation and other details of the transmission/reception strategies of the nodes. We consider a time-slotted system in which the nodes that have received and decoded the packet are allowed to re-transmit it in future slots. During reception, nodes perceive added up signal power (energy accumulation, EA), or the added up mutual information (mutual information accumulation, MIA) received from multiple sources. EA, which has been discussed in prior work [37,39,41,42], can be implemented by using maximal ratio combining (MRC) of orthogonal signals from source nodes that use orthogonal time/frequency channels, or spreading codes, or distributed space-time codes. MIA can be achieved using rateless codes [44,45]. Although these techniques are often treated separately in the literature, we shall see how our formulation of the problem encompasses both approaches and allows many of the results to be extended to both.

We furthermore assume that the nodes are memoryless, i.e., accumulation at the receiver is restricted to transmissions from multiple nodes in the present time slot, while signals from previous timeslots are discarded. This assumption is justified by the limited storage capability of nodes in ad-hoc networks, as well as the additional energy consumption nodes have to expand in order to stay in an active reception mode when they overhear weak signals in preceding timeslots. Note that much of the literature cited above has used the assumption of nodes with memory, so that their results are not directly comparable to ours.

A key tradeoff is between the total energy consumption[1] and the total delay measured in terms of the number of slots needed for all destination nodes in the network to receive the message. At one extreme, if we wish to minimize delay, each transmitting node should transmit at the highest power possible so that the maximum number of receivers can decode the message at each step (indeed, if there is no power constraint, then the source node could transmit at a sufficiently high power to reach all destination nodes in

[1]As we consider fixed time slot durations, we use the words energy and power interchangeably throughout.

the first slot itself). On the other hand, reducing transmit power levels to save energy, may result in fewer nodes decoding the signal at each step, and therefore in a longer time to complete the transmission. We therefore formulate the problem of performing this transmission in such a way that the total transmission energy over all transmitting nodes is minimized, while meeting a desired delay constraint on the maximum number of slots that may be used to complete the transmission. The design variable in this problem is to decide which nodes should transmit, when, and with what power.

1.2 Main Contributions

The key contributions of this dissertation are as follows:

- We formulate the problem of minimum energy transmission in cooperative networks. Although the prior literature have focused on either EA ([37, 42]) or MIA ([44, 45]) and have treated them separately, our generalized formulation can treat both methods as variations of the same problem.

- Our formulation of delay-constrained minimum energy broadcast in cooperative networks, goes beyond the prior work in the literature on cooperative broadcast which has focused either on minimizing energy without delay constraints [37, 42], or on delay analysis without energy minimization [67]. Our extended problem formulation allows us to expose and investigate the energy-delay tradeoffs inherent in cooperative networking.

- We not only prove that the delay constrained minimum energy cooperative broadcast (DMECB) and multicast (DMECM) problems are NP-complete[2], but also that they are $o(\log(n))$-inapproximable (i.e., unless $P = NP$, it is not possible to develop a polynomial time algorithm for this problem that can obtain a solution that is strictly better than a logarithmic-factor of the optimum in all cases). We are not aware of prior work on cooperative broadcast or multicast that shows such inapproximability results.

- We show that the delay constrained minimum energy cooperative unicast (DMECU) problem is solvable in polynomial time using EA but is NP-complete using MIA. We are unaware of any hardness results on unicast approaches using mutual information accumulation.

- For the cases where we prove the transmission problem to be hard, we are able to show that for any given *ordering* of the transmissions (which dictates that a node later in the ordering may not transmit before the nodes earlier in the ordering have decoded successfully), then the problem of joint scheduling and power allocation can in fact be solved optimally in polynomial time using a combination of dynamic programming for the scheduling and convex optimization or linear programming for the power allocation.

[2]Throughout the thesis, the terms NP-complete and NP-hard might be used interchangably when referring to the hardness of the same problem. The reader should note that the former is referring to the decision version of the problem and the latter to the optimization version of the same problem. Chapter 2 provides a background on these concepts.

- For small network instances, we compute the optimal solution through exhaustive search, and show empirically through simulations that our proposed joint scheduling and power control method works near-optimally in typical cases when used in conjunction with an ordering provided by the Dijkstra tree construction.

- We also show through simulations the delay-energy tradeoffs and minimum energy performance for larger networks and demonstrate the significant improvements that can be achieved by our solution compared to non-cooperative broadcast. We further compare the performance of our proposed broadcast algorithm under MIA and EA approaches.

- For DMECB where EA is used, we present a reduction that would allow for a polynomial time algorithm for the joint ordering-scheduling-power control problem that is provably guaranteed to offer a $O(n^{\epsilon})$ approximation, for any $\epsilon > 0$. This algorithm is based on the current best-known algorithm for the bounded diameter directed Steiner tree problem [54]. Using the same reduction, we can also get an approximation factor of $O(T \log^2(n))$ for a fixed delay constraint T. Given that DMECB is $o(\log(n))$ inapproximable for any $T > 2$, this provides a fairly tight approximation, especially for small T.

- We further formulate the problem of minimum energy cooperative transmission in a delay constrained multiflow multihop wireless network, as a combinatorial optimization problem, for a general setting of k-flows and formally prove that the problem not only NP-hard but it is $o(n^{1/7-\epsilon})$ inapproxmiable. To our knowledge, the results in this dissertation provide the first such inapproxmiablity proof in the context of multiflow cooperative wireless networks.

- We observe that for a special case of $k = 1$, the solution is a simple path and offered an optimal polynomial time algorithm for joint routing, scheduling and power control. We then use this algorithm to establish analytical upper and lower bounds for the optimal performance for the general case of k flows.

- Furthermore, we propose a polynomial time heuristic scheme to address the problem of minimum energy cooperative transmission in a delay constrained multiflow multihop wireless network for the general case and evaluate the performance of this heuristic under different channel conditions and against the analytical upper and lower bounds.

1.3 Thesis Statement

The thesis statement can be summarized as follows:

> *Energy efficient transmission in delay constrained cooperative wireless networks is computationally hard in general, however, careful consideration of the combinatorial structure of the problem can yield (near-)optimal algorithms in typical settings.*

This dissertation makes several contributions that significantly enhance our understanding of complexity and algorithm design for cooperative transmission in wireless networks. The summary of the algorithmic results developed in this thesis for the single-flow problem are presented in Tables 3.1 and 4.1 and the results for the multiflow problem are highlighted in chapter 5. It is worth noticing that although we maintained a memory-less assumption throughout, all the negative results presented in Table 3.1 extend to the case where there is no memory.

1.4 Organization of the Thesis

The rest of this thesis is organized as follows: Chapter 2 provides a brief history of cooperative communication and places this dissertation in the context of prior related work. It also highlights some of the key concepts in computational complexity theory that are later used throughout the thesis. References are provided for the interested reader to sources with comprehensive discussions of each topic. Chapter 3 provides a generalized formulation of energy-efficient transmission problem in cooperative multihop wireless networks, encompassing both EA and MIA. We further establish hardness results for a variety of settings in that chapter. In chapter 4, we propose approximation algorithms and positive results for the hard problems described in chapter 3, and evaluate the performance of these algorithms using simulations. The multiflow problem is discussed in chapter 5, where we investigate the delay constrained minimum energy problem in presence of interfering flows. Concluding comments and directions for future work are discussed in chapter 6.

Chapter 2

Background and Related Work

In this chapter we provide a brief tutorial on key concepts in computational complexity theory that will be used later on in this thesis. We also provide a background on the concept of cooperation and briefly discuss the history of cooperation in wireless communication and highlight the current state of art as it relate to the topic of this dissertation.

2.1 Computational Complexity

In this section we briefly review some of the most basic concepts of computational complexity, including NP-hardness and NP-completeness that will be used later on in the thesis. The discussions in this chapter are largely from [3, 4], and the interested reader is referred to these sources and the references therein for a more thorough discussion.

Computational complexity theory[1] is a branch of the theory of computation in theoretical computer science and mathematics that focuses on classifying computational

[1]Definition adopted from Wikipedia, the free encyclopedia.

problems according to their inherent difficulty [3, 74]. In this context, a computational problem is understood to be a task that is in principle amenable to being solved by a computer (which basically means that the problem can be stated by a set of mathematical instructions). Informally, a computational problem can be viewed as an infinite collection of *instances* together with a solution for every instance. The input string for a computational problem is referred to as a problem instance, and should not be confused with the problem itself. In computational complexity theory, a problem refers to the abstract question to be solved. In contrast, an instance of this problem is a rather concrete utterance, which can serve as the input for a *decision problem*[2]. For example, *primality testing* is the problem of determining whether a given number is prime or not. The instances of this problem are natural numbers, and the solution to an instance is yes or no based on whether the number is prime or not.

The running time of many of the algorithms we encounter are bounded by some polynomial in the size of the input. These algorithms are *efficient* algorithms, and the corresponding problems are *traceable*. In other words, we say an algorithm is efficient if its running time is $O(P(n))$, where $P(n)$ is a polynomial in the size of the input n. The class of all problems that can be solved by efficient algorithms is denoted by P (for polynomial time).

[2]A decision problem is a question in some formal system with only a yes-or-no answer, depending on the values of some input parameters.

There are also many problems for which no polynomial time algorithm is known. Some of these problems may be solved by efficient algorithms that are yet to be discovered. For many such problems however, there is a strong belief that they cannot be solved efficiently. It is desirable to be able to identify such problems, so one does not have to spend time search for non-exissenet algorithms. One special class of such problems that we are interested in is a class of decision problems called *NP-complete* problems [13]. We can group these problems in one class because they are all equivalent in a strong sense, *there exists an efficient algorithm for any on NP-complete problem if and only if there exist efficient algorithms for all NP-complete problems.* NP-complete is a subset of NP, the set of all decision problems whose solutions can be verified in polynomial time. In computational complexity theory, NP is one of the most fundamental complexity classes. The abbreviation NP refers to "nondeterministic polynomial time". The complexity class P is also contained in NP, but NP contains many important problems, the hardest of which are called NP-complete problems, for which no polynomial-time algorithms are known. The most important open question in complexity theory, the P = NP problem, asks whether such algorithms actually exist for NP-complete, and by corollary, all NP problems. It is widely believed that this is not the case [29].

NP-hard, non-deterministic polynomial-time hard, is a class of problems [3, 4] that are, informally, *at least as hard as the hardest problems in NP*. NP-hard problems may be of any type: decision problems, search problems, or optimization problems.

The above mentioned notions are the basis for an elegant theory that allows us to identify the problems for which no polynomial algorithm is likely to exist. But proving that a given problem is *hard* does not make it go away, we still need to solve the problem! However, given that a polynomial algorithm is unlikely to exist, we need to make compromises. The most common compromises concern the optimality, robustness, guaranteed efficiency or the completeness of the solution. An algorithm that may not lead to the optimal (precises) result is called an *approximation algorithm*. Of particular interest are approximation algorithms that can guarantee a bound on the degree of imprecision. We will see an example of such algorithms in chapter 4.

2.2 Cooperative Communication

In this section we provide a brief background on cooperation concept and a brief history of cooperative communication, based largely on the materials in [1, 2, 9, 10] and the references therein. The interested reader is referred to these sources for a more thorough background. We also highlight the state of the art related to the the premiss of this thesis, in particular cross-layer techniques for cooperative transmission in multihop networks.

The word cooperate derives from the Latin words *co-* and *operate* (to work), connoting the idea of *working together*. Cooperation is the strategy of a group of entities working together to achieve a common or individual goal [1]. Cooperation has been the subject of intensive study in mathematics, artificial intelligence, social and biological sciences. Examples of cooperation can be found in different areas ranging from animal behavior in nature, including population of ants, termites, bees, hunting lions, vampire bats to human interactions, to information systems and success of open source [5–8].

Wireless networks provide yet another realm in which cooperation among groups of entities can be attained, provided that the right framework can be designed and implemented. Cooperative communication has become one of the fastest growing areas of research in wireless communication in recent years. The key idea in user-cooperation is the resource-sharing among multiple nodes in the network., which would often lead to savings of overall network resources. An enormous application space for user cooperation strategies is Mesh networks [1, 2] .

Cooperation is possible whenever there are more than two communicating terminals. As such, a three-terminal network, introduced by [11, 12], can be thought of as a simplest form and a fundamental unit of user cooperation and as such has been the subject of intense studies [2] . Indeed, a vast portion of the literature, especially in the realm of information theory, has been devoted to a special three-terminal channel, labeled the *relay* channel. In his original work, van der Meulen discovered upper and lower bounds on the capacity of the relay channel, and made several observations that led to improvement of his results in later years. The capacity of the general relay channel is still unknown, but the most prominent work on relaying to date is [14], in which the authors developed lower and upper bounds on the channel capacity for specific non-faded relay channel models. Most of the results in this work have still not been superseded [1], however there has been a lot of work in the area which has improved our understanding of the problem [15–20]. In particular, several works have studied the capacity of relay channels and developed coding strategies to achieve the ergodic capacity of the channel under certain scenarios (see [20] and the references therein).

The terms *decode-and-forward* and *amplify-and-forward* are introduced in [21, 22], where the authors propose different cooperative diversity protocols and analyze the performance in terms of outage behavior. In the former protocol, relays receive and decode the signal transmitted by the source, before forwarding the decoded message to the destination. The destination combines the copies in a proper way. The latter protocol works by the relay amplifying the received signal and forwarding it to the destination. This protocol is clearly simpler, and although it amplifies noise, it can be shown to achieve spatial diversity gain if the message is transmitted over spatially independent channels. *Compress-and-forward* is discussed in [14, 20]. More information on related distributed source coding techniques and on alternative cooperative diversity techniques can be found in [23] and [24, 25] respectively. In this work, we focus on decode-and-forward protocol.

The majority of the work discussed so far considers cooperation with very few nodes and in a two-hop setting. In a different direction, the author in [26] have proposed a new approach towards finding network information carrying capacity, which has led to research on finding scaling laws for wireless networks in a variety of settings. This work shows that an aggregate throughput scaling of $\Theta(\sqrt{n})$ is an upper bound for what is achievable by multihop transmission. Their results however are limited, in that in heir model they assume no cooperation is allowed between networks. As such, a signal not intended to be transmitted to a node is treated as interference. However, cooperation benefits from the broadcast nature of the wireless channel and utilizes this innate property of the channel instead of treating the overhead signal as interference.

In [27], the authors improve the throughput compared to traditional schemes by proposing distributed collaborative schemes over multihop networks, achieving an aggregate throughput of $\Theta(n^{2/3})$. The authors in [28] propose a hierarchical cooperation scheme, achieving linear scaling in ad hoc networks. This means that as the number of nodes per unit area increases the throughput per node does not drop. This is an interesting result which shows that cooperation can overcome the interference limitation in wireless networks. A more comprehensive discussion of this topic can be found in [1].

Although the topic of cooperation has been discussed extensively in physical layer and information theory, the majority of the work has been focused on single or two-hop settings [1, 2]. Recently there has been an increase interest in a cross-layer design for cooperative networks. In [30–32], the authors propose a cooperative MAC protocol to introduce cooperation in 802.11 networks. The proposed protocol is shown to achieve substantial throughput and delay performance improvements by integrating cooperation at the physical layer with the MAC sublayer. The protocol recruits a *single* relay on the fly to support the communication of a particular source-destination pair. This work is extended in [35], where the authors propose utilizing *multiple* cooperative nodes by developing a randomized cooperative framework [33, 34]. However, the focus is still on physical and MAC layer only with a single-source single-destination setup.

In order to harvest the cooperative gains predicted by analytical models in multihop settings, one needs to take routing, scheduling and resource allocation into account as well. While the optimum networking performance strongly depends on the physical-layer technique used, often routing, scheduling and power allocation and physical layer design are treated separately. In this thesis, we focus on cooperative communication in a multihop setting and investigate the complexity of the problem from an algorithmic point of view and the algorithmic aspects of address the problem of designing energy efficient cross-layer cooperative algorithms.

In this thesis, we focus on two of the main physical layer techniques used in the literature concerning cooperative communication in multihop networks, namely: Energy accumulation (EA) [37, 39, 41, 42] and mutual information accumulation (MIA) [44, 45, 80, 82]. Another technique is maximum-ratio transmission (virtual beamforming) [86–89], not considered in this thesis. For cooperation in a non-multihop context see [1, 21, 24, 93, 94].

Figure 2.1 information shows arriving at the receiving node r from a set of senders $\mathcal{S}$. Let p_s denote the power used by sender $s \in \mathcal{S}$ to transmit the message and let h_{sr} denote the mean channel gain between nodes s and r.

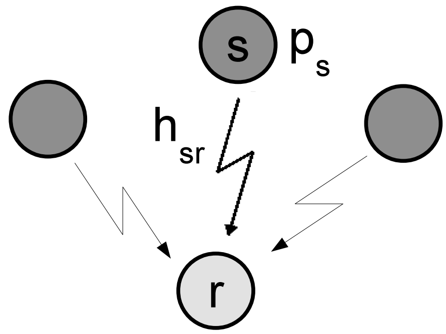

Figure 2.1: A node receiving information from multiple sources.

An ideal EA receiver can reliably decode the message so long as the accumulated energy can exceed some threshold τ. This can be shown as:

$$\sum_{s \in \mathcal{S}} \frac{h_{sr} p_s}{N} > \tau \tag{2.1}$$

where N represents the noise. Notice that τ can be re-adjusted to absorb noise in this formulation. A Rake receiver, in CDMA, is a good approximation for such an EA receiver the information from the different source nodes arrives with relative delays that are larger than the chip duration [60]. Alternatively, space-time codes [57] could be used for transmission. EA can also be achieved by providing orthogonal resources for each channel [67]. Recently a commercially developed cooperative mobile ad hoc network system has been developed which utilizes a pragmatic cooperation method requiring minimal information exchange, based on a combination of phase dithering and turbo codes [48, 49, 83]. It is shown in [48] that the performance of this pragmatic scheme is close to that of an ideal EA approach based on space-time coding.

In MIA [44, 45], the receiving node accumulates mutual information for a packet from multiple transmissions until it can be decoded successfully. The decoding criterion in this case can be expressed as:

$$\sum_{s \in \mathcal{S}} \log \left(1 + \frac{h_{sr} p_s}{N}\right) > \theta \tag{2.2}$$

where θ is the decoding threshold.

This can be achieved using fountain codes and decoders [44,45,47], so that information streams from different relay nodes can be distinguished, and the mutual information of signals transmitted by relay nodes can be accumulated. Note that in this case the CDMA system needs to used different spreading codes for different nodes in order for the destination to be able to resolve the different streams. Notice that although this technique is similar to that of coded cooperation (where different codes are transmitted from different nodes and the nodes can help each other, see e.g. [93]), it is different most notably in that the underlying source information in MIA is the same for all nodes, and the nodes can start transmission at different times, making it particularly appealing in a multihop setting. Notice also that in MIA, nodes are designed to use independently generated codes for relaying. If the same code were used by each transmitter, the receiver would get multiple looks at each codeword symbol. This would be EA; however by getting looks at different codes (generated from the same information bits) the receiver accumulates mutual information rather than energy [80].

It has been shown that one can achieve significant saving in energy and/or transmission time when using an EA, compared to traditional protocols [37, 42, 67]. If energy accumulation is achieved by transmitting the exact same packet either from different relays or through successive re-transmissions, the scheme is shown to achieve capacity in an asymptotically wideband regime [37].

As previously mentioned, in this dissertation, we are interested in energy-efficiency[3] in multihop settings (broadcast, multicast and unicast), in particular as it relates to delay constraint. We now briefly review the state of the art in the scope of this thesis.

Many network protocols in mobile ad hoc and sensor networks need to operate in broadcast mode to disseminate certain control messages to the entire network (for instance, to initiate route requests, or to propagate a query). The subject of broadcast transmission in multi-hop wireless networks has attracted a lot of attention from the research community in both non-cooperative [50, 51, 53] and cooperative settings [37, 39, 42, 43, 67]. For traditional non-cooperative wireless networks, Cagalj *et al.* [53] showed that the problem of minimum energy broadcast is NP-hard. In [39], Mergen *et al.* show through a continuum analysis the existence of a phase transition in the behavior of cooperative broadcast: if the decoding threshold is below a critical value then the broadcast is successful, else only a fraction of the network is reached. In [42], Mergen and Scaglione, show that the problem of scheduling and power control for minimum energy broadcast

[3]For a discussion of multihop throughput optimality, see [84, 90–92].

is tractably solvable for highly dense (continuum) networks and show the gains obtained with respect to noncooperative broadcast. In [67], we examined the delay performance of cooperative broadcast and show that cooperation can result in extremely fast message propagation, scaling logarithmically with respect to the network diameter, unlike the linear scaling for non-cooperative broadcast. We discuss the hardness of the broadcast problem with both MIA and EA in chapter 3 and provide a polynomial-time algorithm (given ordering) in chapter 4, as well as an $O(T \log^2(n))$ approximation algorithm for the case when EA is used.

Algorithmic aspects of cooperative communication and computational complexity are topics that have remained largely unaddressed in the literature. In [37], Maric and Yates address the computation in the context of cooperative broadcast with EA. Their work is similar to ours in that they also consider a minimum energy cooperative solution, however delay constraint is not addressed in their work. Furthermore, they consider a model with memory, where the nodes can save soft information from all previous transmissions throughout time and use it to decode data later on. They prove that the problem is NP-complete in this case. In their setting, because of the memory, it suffices to have each transmitter transmit only once; therefore there is no distinction between ordering and scheduling. This is no longer true in our memoryless setting where the energy from past transmissions cannot be accumulated. Moreover, unicast and multicast settings (which are discussed in this thesis) are not considered in [37]. We further consider MIA, which is not considered in [37].

MIA is most notably discussed in [44, 45, 82]. However, the discussions are focused on unicast routing. The algorithms presented are heuristics, the performance of which are verified via simulations. We are not aware of any hardness results on cooperative MIA or any discussions on the use of MIA in a general cooperative broadcast setting.

There is very limited work addressing the power-delay tradeoff in a cooperative setting which is a focus of this dissertation. One prior work addressing this issue is [60]; however, the focus of that work is on space-time codes used for unicast, and does not discuss broadcast or multicast. The authors in [82] also address the problem power-delay tradeoff in the context of unicast with MIA, by proposing a heuristic that runs a sequence of LP-based route optimizations under increasingly tight energy constraints, revealing a trade-off between energy consumption and delay. These work however do not address the problem of hardness in a delay constrained setting and do not consider the broadcast or multicast problem. In earlier work [68], we had considered this tradeoff in a broadcast cooperative setting using EA and conjectured that many of the results would extend to MIA but the investigation of that conjecture had remained an open problem. This open problem was addressed in [64]. The results of these works are discussed in chapter 3, and chapter 4.

The majority of the work in multihop cooperative communication considers a single flow. In [40], the authors consider the problem of broadcasting independent sources in a dense wireless network. They characterize the propagation of the source flows across the network and show that in the limit of an infinitely dense network, the relaying proceeds in levels. The problem of *jointly computing schedules, routing, and power allocation for multiple flows in cooperative networks* has recently been discussed in [61–63]. These papers propose heuristics and conjecture that the problem is in general NP-hard. We are not aware of any work considering energy-delay tradeoff in multiflow cooperative networks, or any proof of hardness for such problems in the literature. We will address this problem in chapter 5. Part of that chapter has previously appeared in [71].

2.3 Summary

In this chapter, we briefly discussed the history of cooperation in wireless communication. Mentioning in particular that the historically, the majority of the literature in this area has been on information theoretic and coding aspects of cooperation in small networks. We then highlighted the current state of art as it relates to the premiss of this thesis, in particular on cross-layer algorithms for energy-efficient cooperative transmission in multihop networks. We also provided a brief tutorial on key concepts in computational complexity that will be used later on in this thesis.

Chapter 3

Generalized Algorithmic Formulation & Computational Complexity

This chapter considers the problem of energy-efficient transmission in cooperative multihop wireless networks [1]. Although the performance gains of cooperative approaches are well known, the combinatorial nature of these schemes makes it difficult to design efficient polynomial-time algorithms for deciding which nodes should take part in cooperation, and when and with what power they should transmit. In this chapter, we tackle this problem in *memoryless* networks with or without delay constraints, i.e., quality of service guarantee. We analyze a wide class of setups, including unicast, multi-cast, and broadcast, and two main cooperative approaches, namely: energy accumulation (EA) and mutual information accumulation (MIA). We provide a generalized algorithmic formulation of the problem that encompasses all those cases. We investigate the similarities and differences of EA and MIA in our generalized formulation. We prove that the *broadcast*

[1]The work described in this chapter and the following chapter, was done in collaboration with B. Krishnamachari and A. F. Molisch and, has appeared part in [64, 68].

and multicast problems are, in general, not only NP hard but also $o(\log(n))$ inapproximable. We further prove that the problem is NP-hard for the *unicast* case with MIA.

3.1 Introduction

In this chapter we focus on formulating the problem of cooperative transmission in wireless networks, where a single node is sending a packet to either the entire network (broadcast), a single destination node (unicast) or more than one destination node (multicast), in a multihop wireless network. Other nodes in the network, that are neither the source nor the destination, may act as relays to help pass on the message through multiple hops. The transmission is completed when all the destination nodes have successfully received the message. We particularly focus on the case where there is a delay constraint, whereby the destination node(s) should receive the message within the delay constraint, however, we also discuss how our results apply to the unconstrained case.

As previously mentioned a key problem in such cooperative networks is routing and resource allocation, i.e., the question which nodes should participate in the transmission of data, and when, and with how much power, they should be transmitting. The situation is further complicated by the fact that the routing and resource allocation depends on the type of cooperation and other details of the transmission/reception strategies of the nodes. We consider in this work a time-slotted system in which the nodes that have received and decoded the packet are allowed to re-transmit it in future slots. During reception, nodes add up the signal power (energy accumulation, EA) or the mutual information (mutual information accumulation, MIA) received from multiple sources. EA, which has been discussed in prior work [37, 39, 41, 42], can be implemented by using maximal ratio combining (MRC) of orthogonal signals from source nodes that use orthogonal time/frequency channels, or spreading codes, or distributed space-time codes. MIA can be achieved using rateless codes [44, 45]. A brief background on these techniques is highlighted in Chapter 2. Although these techniques are often treated separately in the literature, we shall see how our formulation of the problem encompasses both approaches and allows many of the results to be extended to both.

We furthermore assume that the nodes are memoryless, i.e., accumulation at the receiver is restricted to transmissions from multiple nodes in the present time slot, while signals from previous time slots are discarded. This assumption is justified by the limited storage capability of nodes in ad-hoc networks, as well as the additional energy consumption nodes have to expand in order to stay in an active reception mode when they overhear weak signals in preceding timeslots. Note that much of the literature cited above has used the assumption of nodes with memory, so that their results are not directly comparable to ours.

A key tradeoff is between the total energy consumption[2] and the total delay measured in terms of the number of slots needed for all destination nodes in the network to receive the message. At one extreme, if we wish to minimize delay, each transmitting node should transmit at the highest power possible so that the maximum number of receivers

[2]As we consider fixed time slot durations, we use the words energy and power interchangeably throughout this thesis

can decode the message at each step (indeed, if there is no power constraint, then the source node could transmit at a sufficiently high power to reach all destination nodes in the first slot itself). On the other hand, reducing transmit power levels to save energy, may result in fewer nodes decoding the signal at each step, and therefore in a longer time to complete the transmission. We therefore formulate the problem of performing this transmission in such a way that the total transmission energy over all transmitting nodes is minimized, while meeting a desired delay constraint on the maximum number of slots that may be used to complete the transmission. The design variable in this problem is to decide which nodes should transmit, when, and with what power.

The rest of this chapter is organized as follows: The assumptions made on the system model is described in section 3.2. The generalized problem formulation is presented in section 3.3, encompassing both EA and MIA for unicast, multicast and broadcast scenarios. We discuss the computation complexity of different variations of the problem in section 3.4. The chapter is summarized in section 4.6.

3.2 System Model

We consider a wireless network with n nodes. Radio propagation is modeled by a given symmetric n by n static channel matrix, $H = \{h_{ij}\}$, representing the (power) gain on the channel between each pair of nodes i and j. Time is assumed to be discretized into fixed-duration slots; without loss of generality we assume unit slot durations. We assume cooperative communication in the receivers, encompassing two scenarios: EA and MIA. Only a single message is transmitted through the network.

In EA, the received power at a given receiver in a specific timeslot is sum of the powers received from the transmitters that are active during that slot. As described in [39, 41,42], this kind of additive received power can be achieved via *maximal ratio combining* under different scenarios including transmission using TDMA, FDMA channels, as well as with CDMA spreading codes and space-time codes. MIA can be implemented using rateless codes and decoders at receivers, as described in [44–46]. With proper design (e.g., different spreading codes), information streams from different relay nodes can be distinguished, and the mutual information of signals transmitted by different nodes can be accumulated. A brief background on this is highlighted in chapter 2. We consider a per-node bandwidth constraint and dynamic power allocation.

We assume appropriate coding is used so that each receiving node can decode the message so long as its accumulated received mutual information exceeds a given threshold θ that represents the bandwidth-normalized entropy of the information codeword in nats/Hz. Furthermore, all nodes are assumed to operate in half-duplex mode, i.e. they cannot transmit and receive simultaneously. If used in transmission, the nodes operate based on a decode and forward protocol. Therefore, they are not allowed to take part in transmission until they have fully decoded their message.

Assuming the noise power is the same at all receivers, we can assume without loss of generality the noise power to be normalized to unity so that the transmit power attenuated by the channel becomes equivalent to the signal to noise ratio (SNR). As previously mentioned, we assume a memory-less model in which nodes do not accumulate energy or information from transmissions occurred in previous time slots.

3.3 Generalized Formulation

In this section we provide a generalized formulation for the delay constrained minimum-energy cooperative transmission (DMECT) problem in the setting described in section 3.2.

We assume that the transmission begins from a single source node. The aim is to get the message to all the nodes in a destination set $\mathcal{D}$, with the minimum possible total energy, within a time T (which can take on any value from 1 to $n-1$). Every node in the network is allowed to cooperate in the transmission, so long as they have already decoded the message. The problem now becomes: which nodes should take part in the cooperation, when and with what power should they transmit to achieve this aim while meeting the constraints and incurring minimum total transmission power.

Recalling the memoryless assumption, the condition for successful decoding at some receiver node r at time t when a set of nodes $S(t)$ is transmitting packets, with transmit power $p_{st}, \forall s \in S(t)$ is:

$$y_{rt} \geq \theta \tag{3.1}$$

with y_{rt} being the mutual information accumulated by node r at time t. Let x_{it} be an indicator binary variable that indicates whether or not node i is allowed to transmit at time t. In other words, we define x_{it} to be 1, if node i is allowed to transmit at time t (i.e. has decoded the message by the beginning of time slot t as per equation (3.1)), and 0 otherwise. Let p_{it} be the transmit power for each node i at each time t. Without loss of generality, the source node is assigned node index 1.

The DMECT problem can then be formalized as a combinatorial optimization problem:

$$\begin{aligned} \min \quad & P_{total} = \textstyle\sum_{t=1}^{T} \sum_{i=1}^{n} p_{it} \\ \text{s.t.} \quad & 1. \quad p_{it} \geq 0, \ \ \forall i, \forall t \\ & 2. \quad x_{iT+1} \geq 1, \ \ \forall i \in \mathcal{D} \\ & 3. \quad x_{it+1} \leq \tfrac{1}{\theta} y_{it} + x_{it}, \ \ \forall i, \forall t \\ & 4. \quad x_{1t} = 1, \ \ \forall t \\ & 5. \quad x_{i1} = 0, \ \ \forall i \neq 1 \\ & 6. \quad x_{it} \in \{0, 1\} \end{aligned} \tag{3.2}$$

where, for the energy accumulation (EA) case[3]:

$$y_{it} = \log\left(1 + \sum_{s \in S(t)} p_{st} x_{st} h_{si}\right) \tag{3.3}$$

[3]Notice that because of the monotonicity of the log function, $y_{it} \geq \theta$ in this case is equivalent to $\sum_{s \in S(t)} p_{st} x_{st} h_{si} \geq e^{\theta} - 1$

and for mutual information accumulation (MIA) case:

$$y_{it} = \sum_{s \in S(t)} \log\left(1 + p_{st} x_{st} h_{si}\right) \tag{3.4}$$

Constraint 2 ensures that every nodes in the destination set successfully decodes the message within the time constraint T, constraint 3 ensures that a node cannot transmit unless it has already received the message while simultaneously making sure that a node that has decoded the message in previous time slots will not be prevented from transmitting in future time slots (if it wants to transmit), constraint 4 assigns the source node, and all other constraints are self-explanatory.

In general, there are three variations of this problem, based on the size of the destination set:

- Delay constrained minimum energy cooperative unicast (DMECU): where the set $\mathcal{D}$ includes a single destination node.
- Delay constrained minimum energy cooperative multicast (DMECM): where the set $\mathcal{D}$ includes more than one destination node.
- Delay constrained minimum energy cooperative broadcast (DMECB): where the set $\mathcal{D}$ includes all the nodes apart from the source node.

The decision version of these problem, can be defined correspondingly as follows: "Given some power bound C, does there exist an allocation of powers, p_{it}, satisfying the constraints in (3.2) such that $P_{total} \leq C$?" An instance of this decision problem is defined by giving the symmetric $n \times n$ matrix H, with a designated source node (vertex), a destination set $\mathcal{D}$, a delay bound T, and a power bound C.

Notice that assigning $T \geq n$, in the above formulation, results in the problem definition in the case where there is no delay constraint. Note also that a requirement for per-node maximum power can be trivially added to the above formulation as additional constraint; we have left that out for simplicity. Should the maximum power be added, it should be large enough to ensure a feasible solution exists for the given connectivity and delay constraint.

3.4 Computational Complexity

In this section, we prove that finding an optimal solution for DMECB and DMECM problems is not only NP-hard but also $o(\log(n))$ inapproximable i.e., finding any polynomial time algorithm that approximates the optimal solutions within a factor of $o(\log(n))$ is also NP-hard. We show this by demonstrating that any instance of the set cover problem can be reduced to an instance of DMECB (and by extension DMECM). We further prove NP-completeness for DMECU when MIA is used; note that DMECU with EA will be treated in section 4.2.

3.4.1 Interpreting the Set Cover Problem in Networking Context

The set cover problem is a classical problem in computer science [55]. It is stated as follows: Given a universe U of n elements and a collection of subsets of U, $S =$

$S_1, S_2, ...S_k$, find a minimum subcollection of S that covers all elements of U. This problem is NP-complete and was shown, in [56], to be $o(\log(n))$ inapproximable.

The set cover problem can be thought of as a bipartite graph $G(V, E)$, with $|V| = k + n$, representing the k sets and n elements and the edges are used to connect each set to its elements. This is shown in Figure 3.1 (a), where we assign a vertex for each set in the top part of the graph, and assign a vertex for each element in the bottom part of the graph. We connect each set to its elements using an edge. One can think of each vertex in this graph as a node in a network, in which edges exist between any pair of nodes for which $h_{ij} > 0$, and the edges are labeled with a weight w_{ij} that corresponds to the transmit power needed at node i to exceed a threshold of θ at the receiver j, in a single time slot if i was the only transmitter. Given an instance, G, of the set cover problem, the optimal solution to the set cover problem, OPT_{sc}, would find the minimum subset of vertices in the top part of the graph, so that their transmission of a message can broadcast the message to all the vertices in the bottom part of the graph.

3.4.2 Inapproximablity of DMECB

Given an instance, G, of the set cover problem, with k sets and n elements, let us construct a new graph G' as follows: Assign a root node r, which is the source with the message at the starting time, call this level 0. Include k nodes in level 1, representing the k sets in the set cover problem, all connected to the root node, as shown in Figure 3.1 (b). This is followed by the bipartite graph of G, which makes up level 2 and 3 of G'.

Connect each of the k nodes in level 2 to their representative in level 1 and to all the other nodes in level 2. Notice the nodes in level 2 are also connected to their elements in level 3 of the graph, as shown in the Figure. We make all the weight on the edges arbitrarily small (say 1), with the exception of the edges in between the nodes in level 1 and 2. We make those edges to be sufficiently large, say M, to be specified later.

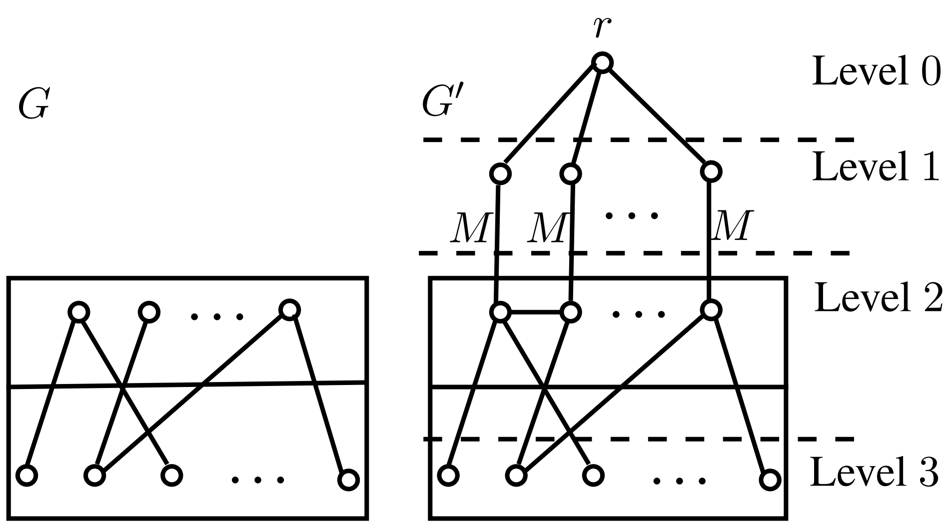

Figure 3.1: Construction of G' for a given G in DMECB.

Assume the the weight on the edges represent the power needed for the message to be transmitted across that edge. If we were to run the optimal DMECB algorithm on G' with $T = 3$ the algorithm would have to act as follows, to be able to cover all the nodes in the given time frame:

Step 1*:* Root transmits with power 1, turning on all its k neighbors on level 1.

Step 2*:* The algorithm picks a subset of the k nodes on level 1 to transmit the message. This subset must be chosen to be as small as possible, given the large weight they have to endure to pass on the message on to the bipartite graph, and the fact that DMECB is trying to minimize the total weight. Yet it has to be large enough so that when the nodes in level 2 transmit, all the nodes in level 3 would receive the message. The optimal algorithm must be able to find such a subset.

Step 3*:* The nodes that receive the message in level 2 transmit the message in this step, turning on all the nodes in level 3 of the graph, as well as all the nodes in level 2 of the graph that were not selected for transmission, thus covering the whole graph.

Let us call the solution[4] of this optimal algorithm OPT_{DMECB}. Then the following two lemmas with respect to the above construction of G and G' hold:

Lemma 1. $OPT_{DMECB} \leq M.OPT_{SC} + 1 + OPT_{SC}$

Proof. Consider an instance of SC (with graph G), whose optimal solution is OPT_{SC}. Construct a graph G', as explained and run the DMECB algorithm to get OPT_{DMECB}. The above inequality holds by construction of the graph. □

Lemma 2. $OPT_{SC} \leq \frac{OPT_{DMECB}}{M}$

Proof. Consider an instance of DMECB on G' and its optimal solution OPT_{DMECB} for delay $T = 3$. Notice that if $T > 3$, we add additional single nodes (as virtual roots) to reduce the problem to the case where $T = 3$. Looking at G', we observe that to meet the delay constraint, by end of step i, at least one node in level i must have heard the message - else it is impossible to get the message through to the rest of the levels in the time frame left. Let's say the root is on level 0. Consider the subset of level 1 that has come on at the end of time 1, $\mathbf{s_1}$, and from level 2 consider the set, $\mathbf{s_2}$, that came on at the end of time

[4]Minimum energy needed for transmission.

step 2. We now want to show that $\mathbf{s_2}$ is a feasible solution for set cover. To do so, we make the following two claims: *Claim* 1: Nodes responsible for turning on $\mathbf{s_2}$ must be a subset of $\mathbf{s_1}$. *Claim* 2: $\mathbf{s_2}$ is a feasible solution to set cover. Claim 1 holds because only nodes that have received the message by the end of time 1 can transmit the message at time 2. Not all of them might transmit though, so $\mathbf{s_2}$ is a subset of corresponding nodes in $\mathbf{s_1}$. Claim 2 is true because if there exists an element in level 3 that is not a corresponding node to any node in $\mathbf{s_2}$, it cannot decode by $T = 3$. Therefore, $\mathbf{s_2}$, is a feasible solution to set cover. OPT_{DMECB} must spend at least M for each element of $\mathbf{s_2}$ to come on, so $OPT_{SC} \leq \frac{OPT_{DMECB}}{M}$. □

Theorem 1. *The DMECB problem is* $o(\log(n))$ *inapproximable, for* $T \geq 3$*.*

Proof. For an instance of the set cover problem, with k being the total number of sets, lemma 1 can be re-written as $OPT_{DMECB} \leq M.OPT_{SC} + 1 + k$. We also know by lemma 2 that $OPT_{SC} \leq \frac{OPT_{DMECB}}{M}$. Therefore, for a sufficiently large M, we can write $OPT_{SC} = \frac{OPT_{DMECB}}{M} + o(1)$. Therefore, the reduction used in construction of the graph G' preserves the approximation factor. That is, if one can find an α-approximation for DMECB, by extension there must exist an α-approximation for set cover. We know, by [56], that the set cover problem is $o(\log(n))$ inapproximable, thus DMECB must be $o(\log(n))$ inapproximable. In other words, finding a polynomial time approximation algorithm that approximates OPT_{DMECB} with a factor of $o(\log(n))$ is NP-hard. □

The DMECB problem can be solved in polynomial time for cases when $T < 3$. The optimal algorithm for $T = 1$ is trivial and an optimal polynomial algorithm for

$T = 2$ is discussed in section 4.1[5]. It is also trivial to verify the feasibility of a given power allocation, and verify whether or not it satisfies the decision version of DMECB given in section 3.3. Therefore, the problem belongs to the class of NP. Notice that the inapproximability result, given by Theorem 1, is stronger than, and implies, the NP-completeness result. It is also worth noticing that without any delay constraint (i.e. when $T \geq n$), the problem is still NP-complete and the proof can be obtained, using directed Hamiltonean path, following the approach in [37].

3.4.3 Inapproximablity of DMECM

The proof of the following theorem, follows from Theorem 1 by noticing that broadcast can be thought of as a special case of multicast.

Theorem 2. *The DMECM problem is $o(\log(n))$ inapproximable, for $T \geq 3$.*

3.4.4 Hardness Results for DMECU

In the unicast case, the hardness of the problem depends on whether we are using EA or MIA. In the former case, DMECU can be shown to be polynomially solvable and the algorithm for that is provided in section 4.2. In the remainder of this section, we discuss DMECU with MIA.

Given an instance, G, of the set cover problem, with k sets and n elements, similar to that in section 3.4.2, let us construct a new graph G' as follows: Assign a root node

[5]DMECT_go algorithm, discussed in section 4.1, along with an ordering based on channel gains from the source, provides an optimal polynomial time algorithm for DMECB for the case when $T = 2$.

r, which is the source with the message at the starting time, call this level 0. Include k nodes in level 1, representing the k sets in the set cover problem, all connected to the root node with a small weight (say weight 1), as shown in Figure 3.2. This is followed by the bipartite graph of G, which makes up level 2 and 3 of G'. Connect each of the k nodes in level 2 to their representative in level 1 with edge weights, of say W. Notice the nodes in level 2 are also connected to their elements in level 3 of the graph, as shown in the Figure, with low-weight edges. Add a single destination node d, in level 4 and connect all the nodes in level 3 to d. Let the channel between all nodes on level 3 and destination d be equal and of gain h. Therefore, the edge weight on the edges connecting the level 3 nodes to d, can be assigned to be M, where M is defined so that the following equality holds: $\log(1 + Mh) = \theta$.

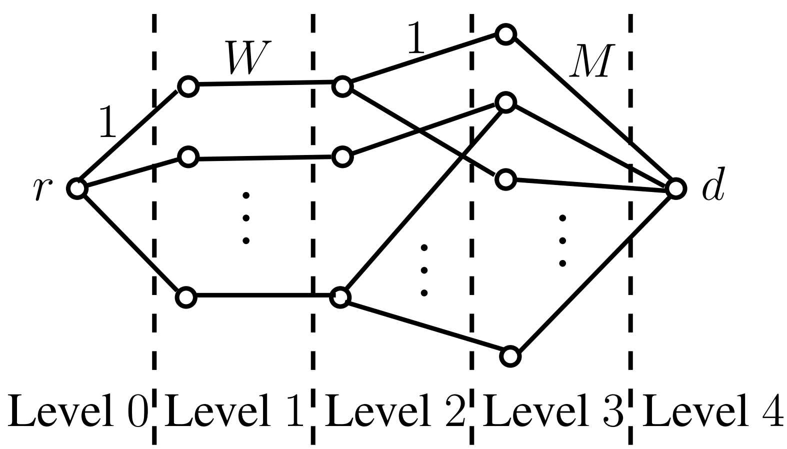

Figure 3.2: Construction of G' for a given G in DMECU.

Assume that the weight on the edges represent the power needed for the message to be transmitted across that edge. If we were to run the optimal DMECU algorithm on G' with $T = 4$ the algorithm would have to act as follows, to be able to turn node d on within the given time frame:

Step 1*:* Root transmits with power 1, turning on all its k neighbors on level 1.

Step 2*:* The algorithm picks a subset the k nodes on level 1 to transmit the message to the nodes in level 2.

Step 3*:* A subset of nodes that have received the message in level 2, transmit the message in this step, turning on a subset of nodes in level 3 of the graph.

Step 4*:* A subset of nodes that have received the message in level 3, transmit the message in this step with sufficient power to turn on d.

Let us call the solution of this optimal algorithm OPT_{DMECU}.

Theorem 3. *The DMECU problem, with MIA, is NP-complete for* $T \geq 4$*.*

Proof. Given an instance of G, we construct G' as above. Let us run DMECU on G' and call the optimal solution OPT_{DMECU} for delay $T = 4$. Notice that if $T > 4$, we add additional single nodes (as virtual roots) to reduce the problem to the case where $T = 4$. Define p to satisfy the following: $n \log(1 + ph) = \log(1 + Mh)$, meaning p is the power required for nodes on level 3 to turn on d, if all of them were transmitting at the same time. *Claim*: OPT_{DMECU} needs to use all the nodes in level 3 for transmission. This claim holds by contradiction, as follows: If all the nodes on level 3 are used for transmission, each node on that level must transmit with power p. Let's assume one of the nodes in that level is not used for transmission. Then the remaining nodes in level 3 need to transmit with power p', where $n \log(1 + ph) = (n - 1) \log(1 + p'h) = \theta$. Therefore, the ratio of the sum power needed with one fewer node transmitting to the case where all nodes in level 3 are transmitting can be written as $\frac{(n-1)p'}{np} = \frac{(n-1)(e^{\theta/n-1}-1)}{n(e^{\theta/n}-1)}$, for sufficiently large θ, this ratio can become arbitrarily large. Therefore, for sufficiently large θ, the claim holds. Given the claim holds, we know that by definition OPT_{SC} provides the optimal way (minimum energy) to turn on all the nodes in level 3 within the required time frame, therefore, for non-zero edge weights OPT_{DMECU} needs to optimally solve the set cover problem in step 2. □

It is worth noticing that all the hardness results presented in this section extend to the case where there is no memory.

3.5 Summary

In this chapter we formulated the novel problem of delay constrained minimum energy cooperative transmission in memoryless wireless networks, encompassing both EA and MIA. We analyzed a wide class of setups, including unicast, multi-cast, and broadcast, and two main cooperative approaches, EA and MIA. We provided a generalized algorithmic formulation of the problem that encompasses all those cases. We investigated the similarities and differences of EA and MIA in our generalized formulation. We proved that the *broadcast and multicast* problems are, in general, not only NP hard but also $o(\log(n))$ inapproximable. We further proved that the problem is NP-hard for the *unicast* case with MIA. Table 3.1 provides a summary of the algorithmic results proved in this chapter.

NEGATIVE RESULTS	Energy Accumulation	
	Delay Constraint (T)	*Unconstrained*
Broadcast	$o(\log(n))$ inapproximable for $T \geq 3$	NP-complete
Multicast	$o(\log(n))$ inapproximable for $T \geq 3$	NP-complete
Unicast	Polynomial time	Polynomial time

NEGATIVE RESULTS	Mutual Information Accumulation	
	Delay Constraint (T)	*Unconstrained*
Broadcast	$o(\log(n))$ inapproximable for $T \geq 3$	NP-complete
Multicast	$o(\log(n))$ inapproximable for $T \geq 3$	NP-complete
Unicast	NP-complete for $T \geq 4$	—

Table 3.1: Summary of the algorithmic negative results.

Chapter 4

Minimum Energy Delay Constrained Transmission

In chapter 3 we considered the problem of energy-efficient transmission in cooperative multihop wireless networks. We proved NP-hardness for several variations of the DMECT problem (namely, the DMECB, DMECM and DMECU (with MIA), with the former two being $o(\log(n))$ innaproximable). In this chapter we break these problems into three parts: ordering, scheduling and power control, and propose a novel algorithm that, given an ordering, can optimally solve the joint power allocation and scheduling problems simultaneously in polynomial time. We further show empirically that this algorithm used in conjunction with an ordering derived heuristically using the Dijkstra's shortest path algorithm yields near-optimal performance in typical settings. For the *unicast* case, we prove that although the problem remains NP complete with MIA, it can be solved optimally and in polynomial time when EA is used. We further use our algorithm to study numerically the trade-off between delay and power-efficiency in cooperative broadcast and compare the performance of EA vs MIA as well as the performance of our cooperative algorithm with a smart non-cooperative algorithm in a broadcast setting. We also briefly discuss how the algorithms discussed could be implemented in a decentralized fashion.

4.1 Optimal Transmission Given Ordering

In section 3.4, we proved NP-hardness for several variations of the DMECT problem. In this section, we break this NP-hard problem into three subproblems, namely ordering, scheduling and power allocation, and we propose an optimal polynomial time algorithm for joint scheduling and power allocation when the ordering is given. We evaluate a heuristic for the ordering in section 4.4.

Definition 1. *An ordering, for a vector of n nodes, is an array of indices from 1 to n; any node that has decoded the message will only be allowed to retransmit when all nodes with smaller index have also decoded the message (and are thus allowed to take part in transmission).*

Given an ordering, what remains to be determined is which nodes should take part in transmission, how much power they should transmit with and at what time slots, such that minimum energy is consumed while delay constraints are satisfied.

4.1.1 Instantaneous optimal power allocation

If we know which nodes are transmitting the message and which nodes are receiving it, at any single time-slot, we can use a convex program (CP) to determine the optimal power allocation for that time slot. Consider an *ordered* vector of n nodes $(1, ..., k, ..., i, ..., n)$. Let us assume that by time slot t, node 1 to i have decoded the message and nodes $i+1$ to n are to decode it during that time slot. At time instance t, the optimal instantaneous power allocation for a set of transmitting nodes (say $S(t) = (k, ..., i)$) to turn on a set of receiving nodes nodes (say $R(t) = (i+1, ..., n)$) can be calculated by the following CP:

$$\begin{aligned} \min \quad & \textstyle\sum_{s \in S(t)} p_{st} \\ s.t. \quad & p_{st} \geq 0, \;\; \forall s \\ & y_{rt} \geq \theta, \;\; \forall r \in R(t) \end{aligned} \tag{4.1}$$

We use the notation $CP([\{k...i\}, \{i+1...n\}], \theta, H)$ to refer to solution of the above CP. As a notation, $CP([\{x...y\}, \{z...\alpha\}], \theta, H) = 0$, if $z \geq \alpha$. Notice that in the case where EA is used, this CP simply reduces to a linear program, using the manipulation highlighted in footnote 2 in section 3.3.

4.1.2 Joint Scheduling and power allocation

Knowing the instantaneous optimal power allocation given the set of senders and receivers at each time slot, all that remains to be done is to determine these sets at each time slot, in order to minimize the overall power while meeting the delay constraint.

Let $C(j, t)$ be the minimum energy needed to cover up to node j in t steps or less. We can calculate this, using the following algorithm:

$$C(j,t) = \min_{k \in (1,..,j)} [C(k,t-1) + CP(\{1...k\}, \{k+1...j\}, \theta, H)] \tag{4.2}$$

where $C(k,1) = CP(1, \{2...k\}, \theta, H), C(1,t) = 0 \quad \forall t.$

Thus, in DMECB, the total minimum cost for covering n nodes by time T can be found by calculating $C(n,T)$. In DMECM, and DMECU (MIA), the same approach could be used, except for node n being replaced by the highest order destination in the former and by the destination order in the latter.

A pseudocode for the algorithm is presented in Algorithm 1. The complexity of the

Algorithm 1 Delay constrained minimum energy cooperative transmission, given an ordering (DMECT_go)

INPUT: an ordered array of nodes of size n (where node i is the ith node in the array), T (delay), d (destination), H (channel), θ (threshold).
OUTPUT: C (cost matrix)
Begin:
for $i := 2$ to n **do**
    $C(i,1) := CP([1, \{2...i\}], \theta, H);$
end for
for $t := 1$ to T **do**
    $C(1,t) := 0;$
end for
for $t := 2$ to T **do**
    for $i := 1$ to d **do**
        for $k := 1$ to i **do**
            $x(k) := C(k,t-1) + CP([\{1...k\}, \{k+1...i\}], \theta, H);$
        end for
        $C(i,t) := \min \mathbf{x}$
    end for
end for

optimal scheduling and power allocation can be obtained by inspection of the above algorithm: it invokes at most $O(n^2T)$ calls to the CP solver, each of which takes polynomial time. Hence the DMECT_go algorithm that does joint scheduling and power-control is a polynomial time algorithm.

Note that the delay constraint T can be made sufficiently large ($\geq n$), or removed entirely from the formulation, to cover the case of no delay constraints. In that case the two-dimensional dynamic program proposed in (4.2), reduces to a one-dimensional dynamic program:

$$C(n) = \min_{k} [C(k) + CP(\{1...k\}, \{k+1...n\}), \theta, H] \tag{4.3}$$

where $C(n)$ is the minimum cost of covering node n using our cooperative memoryless approach, starting from node 1 and $C(1) = 0$.

4.2 Optimal Unicast with Energy Accumulation

In this section we propose an optimal polynomial time algorithms for solving the unicast problem with EA.

Theorem 4. *In DMECU with EA, there exists a solution consisting of a simple path between source and destination, which is optimum.*

Proof. Let us prove by induction: For delay $T = 1$, the claim is trivially true, as the optimal solution is a direct transmission from the source, s, to the given destination, d. For $T > 1$, we prove the claim by induction. Assume that the claim is true for $T = k - 1$. Pick any node in the network as the desired destination d. If the message can be transmitted from source s to d with minimum energy in a time frame less than k, then an optimal simple path exists by the induction assumption. So consider the case when it takes exactly $T = k$ steps to turn on d. The system is memoryless, so d must decode by accumulating the energy transmitted from a set of nodes, **v**, at time k. This can be represented as $\log(1 + \sum_{v_i \in \mathbf{v}} p_{v_i k} h_{dv_i}) \geq \theta$. We observe that there must exist a node $v_o \in \mathbf{v}$ whose channel to d is equal or better than all the other nodes in **v**. Therefore, given $h_{dv_o} \geq h_{dv_i}, \forall v_i \in \mathbf{v} - \{v_o\}$ then $\log(1 + \sum_{v_i \in \mathbf{v}} p_{v_i k} h_{dv_o}) \geq \log(1 + \sum_{v_i \in \mathbf{v}} p_{v_i k} h_{dv_i}) \geq \theta$. In other words, if we add the power from all nodes in **v** and transmit instead from v_o, our solution cannot be worse. v_o must have received the message by time $k - 1$, to be able to transmit the message to d at time k. We know by the induction assumption that the optimal simple path solution exists from source to any node to deliver the message within $k - 1$ time frame. Thus, for $T = k$, there exists a simple path solution between s and d, which is optimum. □

Notice that the above theorem holds in the case where there is no delay constraint as well. The proof follows an straightforward modification of the above proof and is omitted for brevity.

Corollary 1. *The Dijkstra's shortest path algorithm provides the optimal ordering in the case of minimum energy memoryless cooperative unicast, when there is no delay constraint.*

Proof. We have already established that an optimal minimum energy solution exists between source and destination, which is a simple path. The well-known Dijkstra's shortest path algorithm can find the minimum cost simple path between source and destination. Therefore, Dijkstra's algorithm provides the optimal ordering. □

Using theorem 4 we know that the optimal unicast solution from source to any destination in DMECU (with EA) is given by a simple path. The cost paid by the optimal solution can be calculated using the following algorithm: Let $C(i,t)$ be the minimum cost it takes for source node s to turn on i, possibly using relays, within at most t time slots. Then we can write:

$$C(i,t) = \min_{k \in Nr(i)} [C(k,t-1) + w(k \to i)] \tag{4.4}$$

with $C(s,t) = 0$, for all t and $C(i,1) = w(s \to i)$, where $Nr(i)$ is the set that contains i and its neighboring nodes that have a non-zero channel to i, $w(k \to i)$ represents the power it takes for k to turn on i using direct transmission. Given that, the solution to OPT_{DMECU} (with EA) is given by $C(d,T)$. Computing this lower-bound incurs a running time of $O(n^3)$.

The unicast case (with EA), with no delay constraint, is still polynomially solvable. Given Theorem 4 and Corollary 1, the optimal solution is simply the weight of the shortest path given by the Dijkstra's algorithm.

It is worth noticing that the crux of the difference between DMECU with EA and with MIA, that allows the former to be polynomially solvable, while the latter is NP-complete, lies in the optimality of single-node transmission. Namely, in the EA case, the multi-transmitters single-receiver case (multi-single) makes no sense as explained above and instead it is optimal to put the combined power into the best channel. This allows for the overall solution to be a simple path. However, in the MIA case, the many-to-one transmission case does in fact make sense. That is due to the property of the $\log$ function, creating an effect similar to what we observe in *water-filling*, where it is best to transmit from the best channel up until some point, then from the second best channel and so forth.

4.3 Approximation Algorithm for Broadcast with Energy Accumulation

In section 3.4, we proved that DMECB is NP-complete and $o(\log(n))$ inapproximable, therefore it is *hard* to approximate DMECB to a factor strictly better than $\log(n)$. It is of theoretical interest to know how close we can get to the optimal solution, using a polynomial-time algorithm. In this section we show that existing approximation algorithms for the *bounded-diameter directed Steiner tree problem* can be used to provide an $O(n^{\epsilon})$ approximation for DMECB in the case where EA is used. We do so by proposing an approximation-preserving reduction to the directed Steiner tree problem.

The Steiner tree problem is a classic problem in combinatorial optimization [55]. We focus on a variation of this problem, namely bounded diameter directed Steiner tree, defined as follows. Given a directed weighted graph $G(V, E)$, a specified root $r \in V$, and a set of *terminal nodes* $X \subseteq V$ ($|X| = n$), the objective is to find the minimum cost arborescence rooted in r and spanning all vertices in X, subject to a maximum diameter T. Diameter refers to the maximum number of edges on any path in the tree. Notice that the tree may include vertices not in X as well, these are known as *Steiner nodes*. Directed Steiner tree problem is known to be NP-complete and $O(\log(n))$ inapproximable [55]. In [54], the authors give the first non-trivial approximation algorithms for Steiner tree problems and propose approximation algorithms that can achieve an approximation factor of $O(n^{\epsilon})$ for any fixed $\epsilon > 0$ in polynomial time. To the best of our knowledge this is currently the tightest approximation algorithm known for this problem.

In order to reduce a given instance of the DMECB to an instance of the Steiner tree problem, we first restrict DMECB by not allowing many-transmitter-to-one-receiver (many-to-many) transmissions. Notice that in the proof of theorem 4, we had established that many-to-one transmissions can be replaced with one-to-one transmissions without loss of optimality. Therefore, by not allowing many-to-many transmissions, we are left with one-to-one and one-to-many transmissions. We call this an integral version of DMECB, DMECB-int. The integrality gap of the weighted set cover problem is shown to be $\log(n)$ [55]; it is straightforward to extend that result to show that DMECB-int also loses a factor of $\log(n)$, compared to optimal DMECB.

Consider an instance of DMECB-int, $G(V, E)$, with ($|V| = n$) and $s \in V$ being the source node. To reduce this problem to an instance of directed Steiner tree problem, let us construct a new graph G', consisting of n clusters, x', each corresponding to each node in G. Let each cluster be a bipartite graph, with n nodes on the left (marked as " $-$ ") and n nodes on the right (marked as " $+$ "), as shown in Figure 4.1. The " $-$ " nodes are intra-connected within a cluster with edges of weight 0. In each cluster, $x' \in G'$ corresponding to node $v \in G$, the " $+$ " and " $-$ " nodes on each level, i, of the bipartite graph are connected to each other with an edge of weight w_i, representing the power needed by the corresponding node $v \in G$ to turn on its i closest neighbors. The i_+ node is then connected, with edges of weight 0, to all the " $-$ " nodes in the corresponding neighbor clusters. We further add a single root node, $r \in G'$, and connect it via a zero-weight edge to all the " $-$ " nodes in the cluster corresponding to s, x'_s. We assign the root r and one desired " $-$ " node from each cluster as terminal nodes and all other nodes in G' as Steiner nodes.

Let us look at an example of this construction, say node $v_1 \in G$, whose closest 3 neighbors are (v_2, v_4, v_6). We have an equivalent cluster $x'_1 \in G'$ corresponding to node v_1. x'_1 has $2n$ nodes, arranged in n levels. The weight between the two nodes in say level 3 is equivalent to the power it takes for v_1 to turn on (v_2, v_4, v_6). Furthermore, the node 3_+ in cluster x'_1 is connected to the " $-$ " nodes in clusters (x'_2, x'_4, x'_6) with edges of weight 0. This construction allows us to find a way to allow v_1 to transmit with different power levels, without knowing what those powers might be in advance. We first add a single root node, r, and connect it via a zero-weight edge to all the " $-$ " nodes in the cluster corresponding to s.

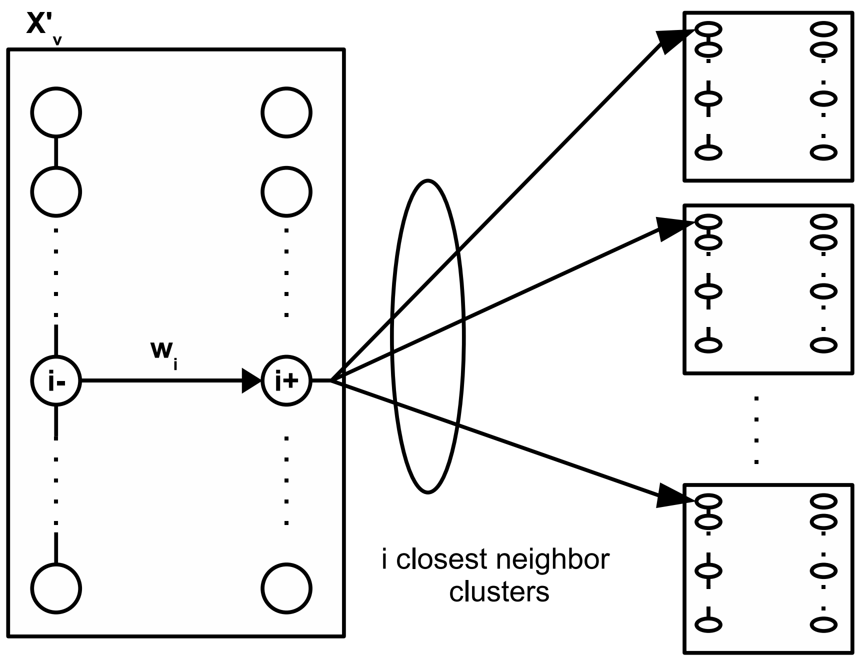

Figure 4.1: A simplified example of how clusters are constructed in G'.

Run the directed Steiner tree algorithm on G' to obtain a solution. The solution must choose at least one node from each cluster, to meet the mandatory terminal nodes requirement. Recall that each cluster in G' corresponds to a node in G and that multi-multi was not allowed. To convert the solution of the Steiner tree algorithm on G' to a solution of DMECB-int on G, we look at the parent of each cluster, which is a "$+$" node in another cluster. Let's say we want to see which node turns on v_6 by looking at G'. We look at the parent of x'_6 and see that it's $3_+ \in x'_1$. So in G, we figure out that v_1 must transmit with enough power to turn on 3 of its closest neighbor (w_3), and it is as a result of this transmission that v_6 comes on. Going through all the clusters and their parents, we can establish an ordering and transmission power for all the nodes that should take part in cooperation in G, and thus we have a solution for DMECB-int.

Theorem 5. *For DMECB problem with EA, an $O(n^\epsilon)$ approximation ratio can be achieved in polynomial time, for any fixed $\epsilon > 0$.*

Proof. As mentioned, the directed Steiner tree is $o(\log(n))$ inapproximable, and the best approximation algorithm currently available [54] gives an $O(n^\epsilon)$ approximation on the optimal solution. We had already lost $O(\log(n))$ to convert DMECB to its integral form. The approximation algorithm proposed in [54] can approximate the optimal integral solution within $O(n^\epsilon)$. Therefore, using the above reduction, and applying the directed Steiner tree approximation algorithm, we can approximate the optimal solution to DMECB within $O(n^\epsilon \times \log(n))$, which is equivalent to $O(n^\epsilon)$. □

The running time of the Steiner tree approximation algorithm is a function of ϵ, and the tighter the approximation, the worse the running time. Similarly, using the above mentioned reduction, the following result holds by directly applying the approximation algorithms in [54]. Detailed discussions of the algorithms in [54] are beyond the scope of this dissertation.

Theorem 6. *For any fixed $T > 0$, there is an algorithm which runs in time $n^{O(T)}$ and gives an $O(T \log^2(n))$ approximation of the DMECB with EA.*

4.4 Performance Evaluaion

For the simulations, we focus on the broadcast case. We consider a network of n nodes uniformly distributed on a 15 by 15 square surface. The transmission starts from a node, arbitrarily located at the left center corner of the network $(0, 7)$. The channels between all nodes are static, with independent and exponentially distributed channel gains (corresponding to Rayleigh fading), where $h_i j$ denotes the channel gain between node i and j. The mean value of the channel between two nodes, $\overline{h_{ij}}$, is chosen to decay with the distance between the nodes, so that $\overline{h_{ij}} = d_{ij}^{-\eta}$, with d_{ij} being the distance between nodes i and j and η being the path loss exponent. The corresponding distribution for the channel gains is then given by

$$f_{h_{ij}}(h_{ij}) = \frac{1}{\overline{h_{ij}}} \exp\left(\frac{h_{ij}(k)}{\overline{h_{ij}}}\right)$$

Based on the intuition developed in section 4.2, we use the Dijkstra's shortest path algorithm as our ordering heuristic. Simulations are repeated multiple times with the same node locations but different fading realizations and average values are shown in the graphs. Notice that the minimum power calculated by different algorithms, shown on the y-axes of the graphs in this section, are normalized by unit power (rendering it unit-less). The value of θ is, arbitrarily, chosen to be $\log(2)$ throughout this section.

In Figure 4.2, we calculate the optimal ordering by brute-force for a small number of nodes and compare the performance of our algorithm, which uses Dijkstra's shortest path-based ordering, with the optimal performance. The results for the broadcast case, with EA, is shown in this figure. As can be seen, Dijkstra's algorithm provides a good heuristic for ordering in this example and will be used throughout this section. Note that, as can be seen in the figure, although the problem was proved to be $o(\log(n))$ in-approximable, it is possible to achieve near-optimal results in polynomial time in certain practical settings where the network does not have any pathological properties. The in-approximablity results contain all possible (including pathological networks) scenarios.

We next compare the performance of our cooperative algorithm with a smartly designed non-cooperative algorithm (both using EA). Notice that in our cooperative algorithm we make use of the *wireless broadcast advantage (WBA)*, where transmission by one node can be received by multiple nodes and *cooperative advantage*, where a node can accumulate power from multiple transmitters.

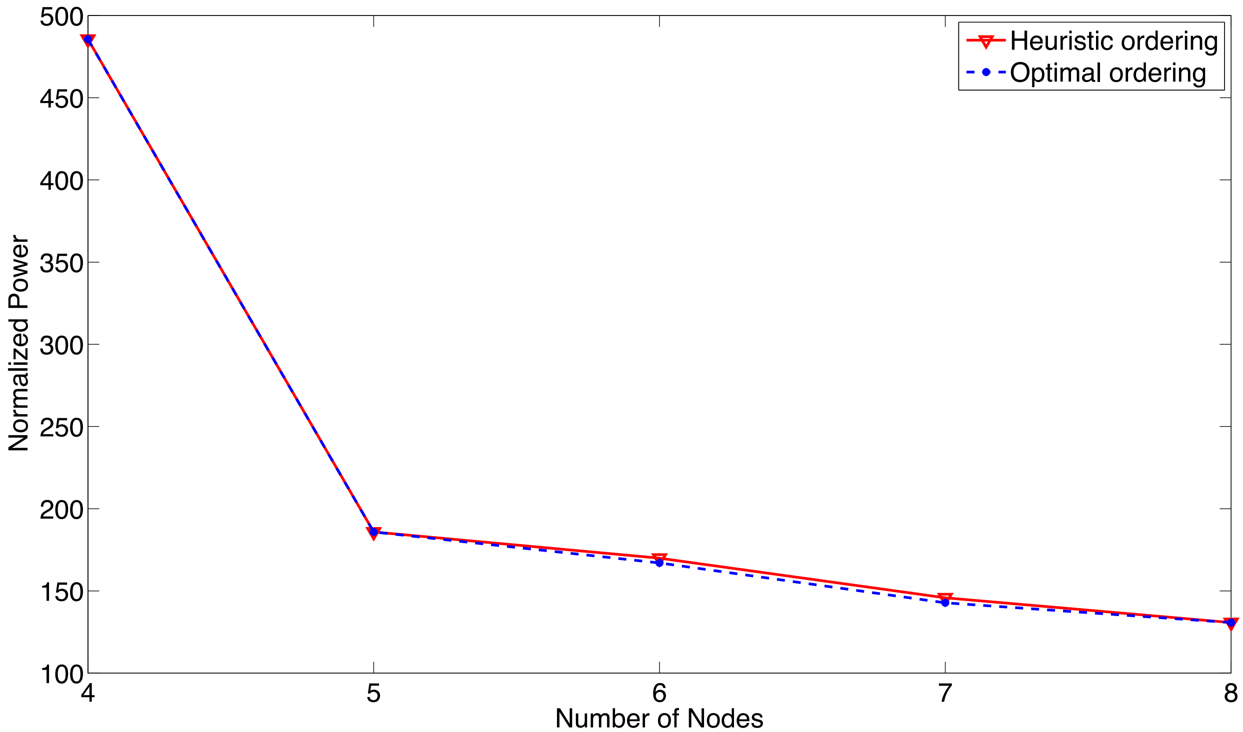

Figure 4.2: Performance with optimal ordering vs Dijkstra's algorithm-based heuristic ordering.

If an algorithm is using the WBA, but not the cooperative advantage, it can be thought of as an integral version of DMECB. This means that each node can receive the message from one transmitter only (and cannot accumulate from multiple transmitters), however one transmitter can transmit to multiple receivers.

We had established in section 4.3 that DMECB-int is also NP complete. It is however interesting to note that DMECB-int needs to solve a weighted set cover problem when allocating powers as well; we know that set cover problem is $o(\log(n))$ inapproximable [55], so the non-cooperative case is $o(\log(n))$ inapproximable, even when ordering is provided. Greedy algorithms exist [55] that give $O(\log(n))$ approximations for the weighted set cover problem, and thus provide a tight polynomial time approximation.

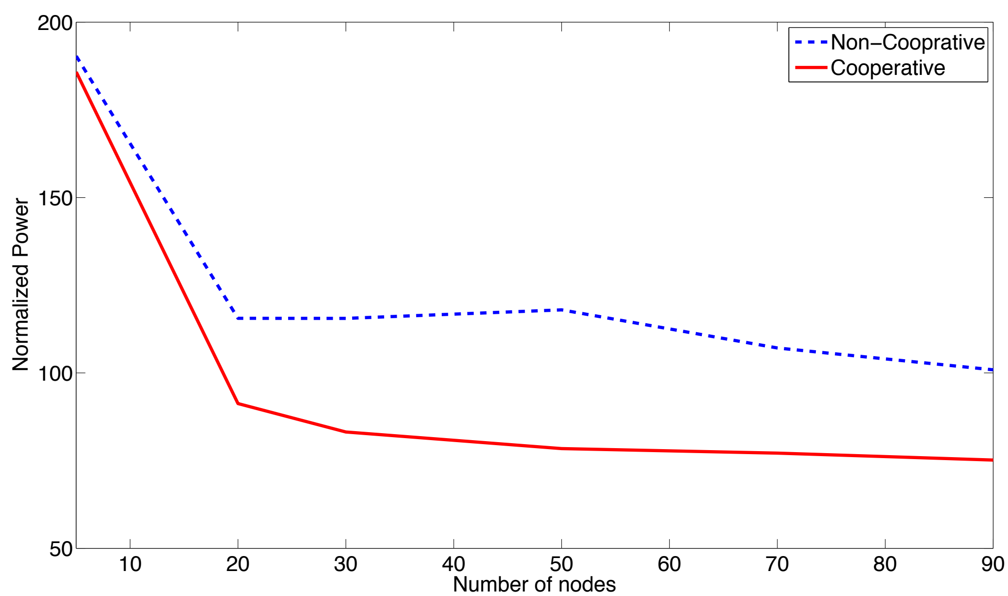

Figure 4.3: Effect of cooperation for varying network size.

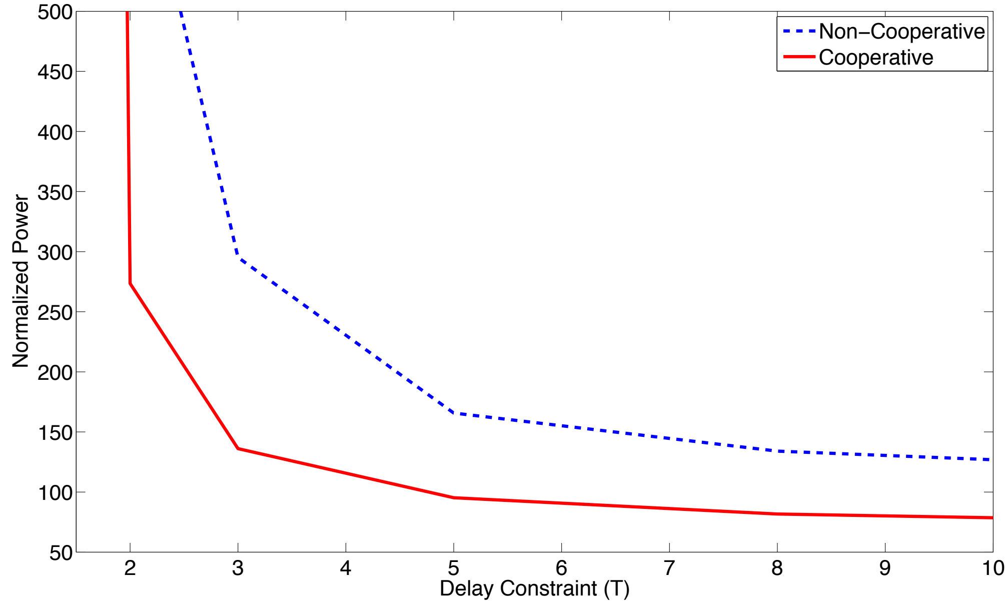

Figure 4.4: Power-delay tradeoff in cooperative vs non-cooperative case.

Therefore, to simulate a smart non-cooperative algorithm, we use Dijkstra's algorithm-based ordering and the DMECT algorithm of section 4.1, with the exception that instead of using an LP we use the greedy algorithm for power allocation.

The performance comparison between our proposed cooperative algorithm and the smart non-cooperative algorithm, for different values of n is shown in Figure 4.3 and the power-delay tradeoff for cooperative and non-cooperative algorithms are presented in Figure 4.4. As can be seen, the cooperative algorithm outperforms the non-cooperative algorithm, and the advantage is more pronounced when a delay constraint is imposed.

The performance gains obtained by using MIA is shown in Figure 4.5, for a sample network of 30 nodes.

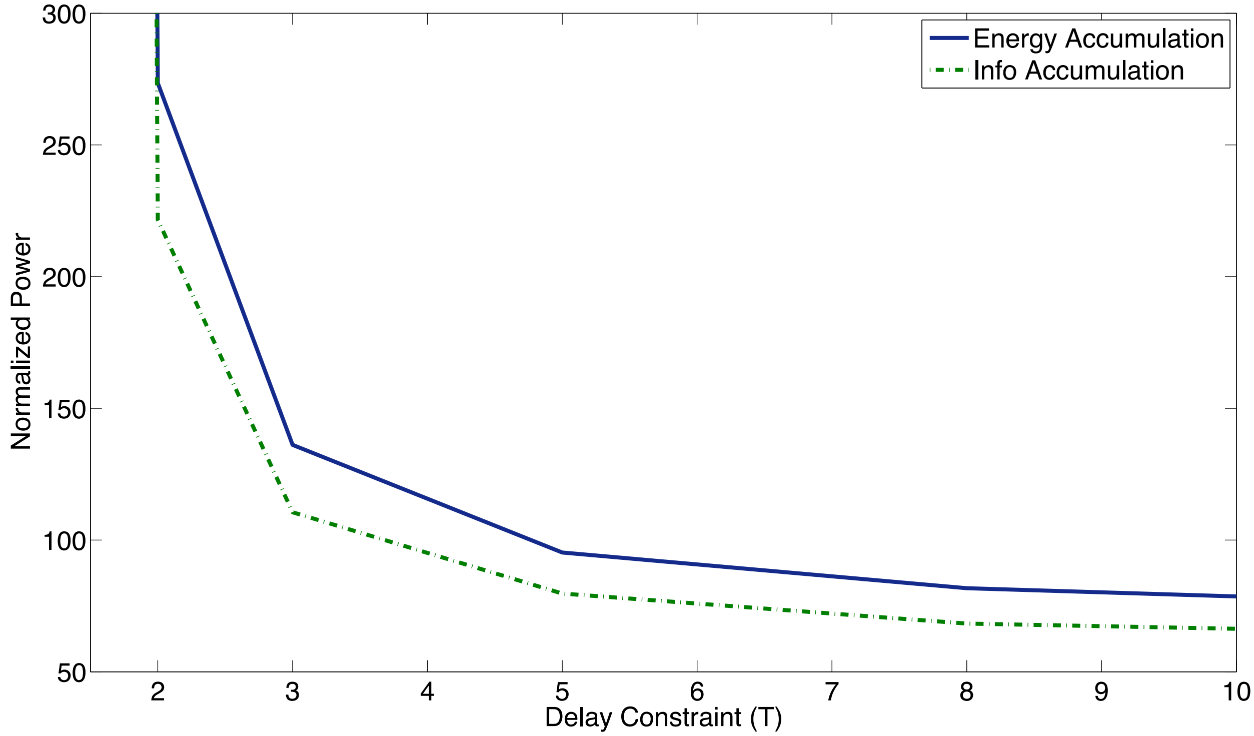

Figure 4.5: Energy accumulation vs mutual information accumulation.

We next study the power-delay tradeoff of the cooperative algorithm for different channel conditions and different values of network density ρ (in nodes/area). Figure 4.6 and Figure 4.7, show results for EA and MIA, respectively. These figures highlight the sensitivity of the dense networks and those with poor channel conditions to delay constraints and the importance of having smart algorithms to minimize the energy consumption.

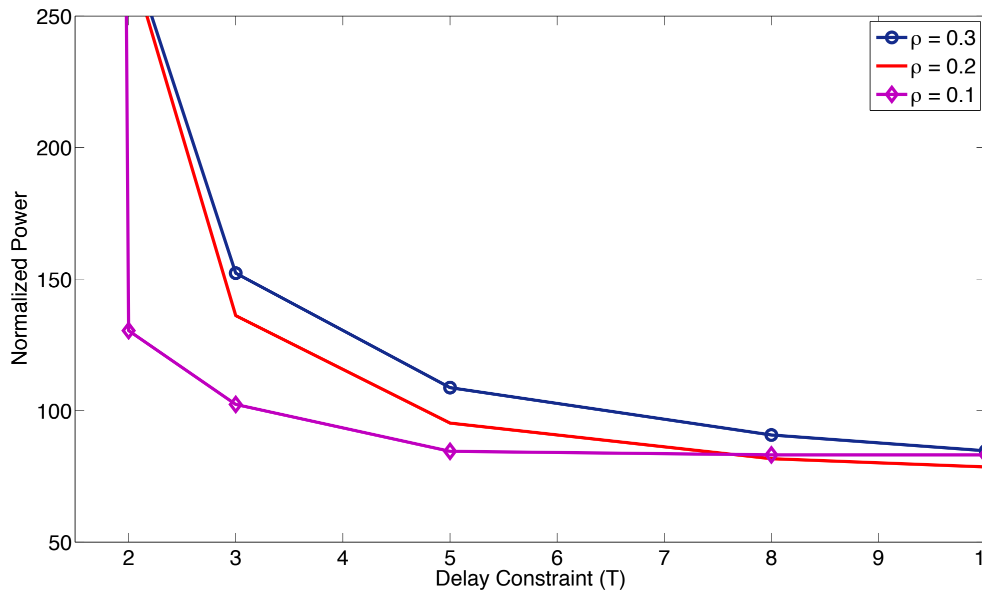

Figure 4.6: Effect of network density on power-delay tradeoff.

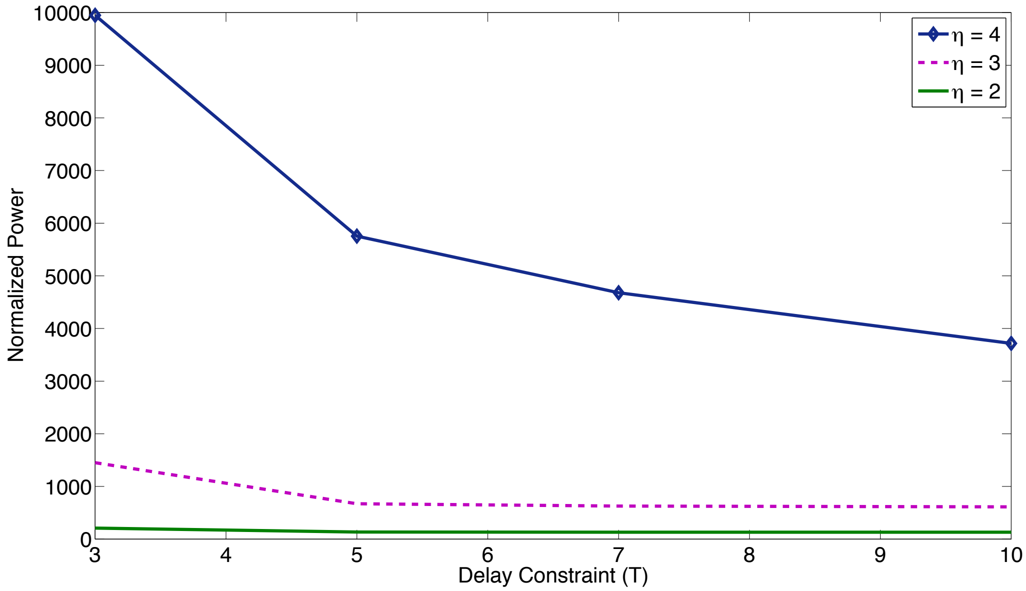

Figure 4.7: Different η values for information accumulation.

4.5 Decentralized Implementation

The algorithms discussed in this chapter are centralized algorithms. In this section, we discuss a possible approach for a decentralized implementation of these algorithms in the case where EA is used.

For simplicity, we start off by considering the single source with energy accumulation without a delay constraint. As previously discussed, the unicast problem in this case is optimally solvable and consists of a single path. We observed that the Dijkstra algorithm could provide us with that optimal single path. An alternative algorithm is Bellman's shortest path algorithm developed in 1957. The distributed algorithm based on this work is often referred to as Bellman-Ford algorithm, named after its inventors. Although, not regarded to be as fast as Dijkstra's algorithm for graphs with non-negative edges, the distributed Bellman-Ford algorithm seems to provide a viable alternative to Dijkstra's that would allow us to have a distributed solution for the unicast case, in energy accumulation.

Next, let us consider the algorithm discussed in section 4.1, with EA. For this algorithm, we already established that, given an ordering, an optimal polynomial time solution exists and can be found using dynamic programming, for scheduling, and linear programming, for power control, jointly. We proposed using Dijkstra's algorithm for the ordering and had verified, using simulations, that this ordering performs near-optimally in typical settings. So, if ordering is given, using this insight we could try to find a distributed approach.

Inspecting the dynamic program closely, for the case with delay constraint, we can see that it works by filling out the elements of an $n \times T$ table, with n being the number of nodes in the network and T being the delay. Assuming all nodes have the ordering vector, we can have each node calculate their own corresponding row of the table using the information that has been passed on to them by the node immediately preceding them and then amend this to what they have received from the previous node and pass it on to the node that proceeds them in the ordering. Notice that since the next node in the neighboring is not necessarily their closest neighbor (in terms of channel strength) they might need to pass on this information using some sort of distributed shortest path algorithm. The final node in the ordering will then be in the possession of the entire table and can pass it on to all the nodes in the network. Once the nodes get a copy of the table, they can locally calculate when they need to transmit and with what power.

Assuming channel conditions do not vary too frequently, this initial step need not happen too frequently. This way, given ordering, we can find a distributed solution with each node only having access to the ordering vector and the channel gains of its neighbors.

If ordering is not given, the problem becomes more challenging. We have already established that, without ordering, the problem is $o(\log(n))$ inapproximable and have proposed an $O(n^{\epsilon})$ approximation algorithm based on the best existing approximation algorithms for directed Steiner tree. A possible decentralized approximation algorithm for the broadcast version of the problem with EA is using the observation that if we are to broadcast the message to all the nodes in the network, we cannot possibly do better than what it takes to transmit the message in an optimal unicast fashion to each of the nodes in the network. Thus, for a given source, the minimum unicast optimal solution over all nodes provides a lower bound on the optimal solution for the broadcast. We can further observe that the optimal solution cannot be worse than n times the maximum unicast optimal solution over all nodes. Using these observations, we can think of the following distributed algorithm (in absence of ordering): We first run the distributed Bellman-Ford algorithm, to establish a shortest path tree in the network - in the unconstrained case and then use the distributed unicast solution developed above with the leaf nodes being the destinations. Similarly, in the delay constrained case, we use the distributed solution for the delay constrained unicast case. This provides a distributed approximate for the optimal broadcast problem. The approximation is $O(n)$ in this case.

4.6 Summary

We considered the DMECT problem in memoryless wireless networks. We had proved in the previous chapter that this problem is $o(\log n)$ inapproximable in broadcast and multicast cases and is NP-complete in the unicast case when mutual information accumulation is used. In this section we developed a polynomial-time algorithm that can solve this NP-hard problem optimally for a fixed transmission ordering. Our empirical results suggest that for practical settings, a near-optimal ordering can be obtained by using Dijkstra's shortest path algorithm. We have further showed that the unicast case can be solved optimally and in polynomial time when EA is used. We have studied the energy-delay tradeoffs and the performance gain of MIA using simulations, and evaluated the performance of our algorithm under varying conditions. For the broadcast case with EA, we presented an $O(T \log^2(n))$ approximation algorithm. We discussed how some of the algorithms discussed could be implemented in a decentralized fashion and proposed a decentralized $O(n)$ approximation algorithm for the broadcast problem with EA. The summary of the algorithmic results developed in this chapter are presented in Tables 4.1.

POSITIVE RESULTS	Energy Accumulation		Mutual Information Accumulation	
	Delay Constraint (T)	*Unconstrained*	*Delay Constraint (T)*	*Unconstrained*
Broadcast	•$O(n^\epsilon)$ for $\epsilon > 0$ •$O(T \log^2(n))$, for fixed T • Polynomial time given ordering (DMCT_go) • Polynomial time algorithm for $T = 1, 2$	Polynomial time given ordering 1D dynamic program	• Polynomial time given ordering (DMCT_go) • Polynomial time algorithm for $T = 1, 2$	Polynomial time given ordering 1D dynamic program
Multicast	Polynomial time given ordering (DMCT_go)	Polynomial time given ordering (DMCT_go)	Polynomial time given ordering (DMCT_go)	Polynomial time given ordering (DMCT_go)
Unicast	Polynomial time	Polynomial time	• Polynomial time given ordering (DMCT_go) $T \geq 4$ • Polynomial time algorithm for $T = 1, 2$	—

Table 4.1: Summary of the algorithmic positive results.

Chapter 5

Transmission in Presence of Interfering Flows

In this thesis so far, we have focused on the case where there is no interference present. In this chapter, we consider the problem of energy-efficient transmission in multi-flow multihop cooperative wireless networks [1]. As we discussed in previous chapters, the combinatorial nature of these schemes makes it difficult to design efficient polynomial-time algorithms for joint routing, scheduling and power control. This becomes more so when there is more than one flow in the network. It has been conjectured by many authors, in the literature, that the multiflow problem in cooperative networks is an NP-hard problem. In this chapter, we formulate the problem, as a combinatorial optimization problem, for a general setting of k-flows, and formally prove that the problem is not only NP-hard but it is $o(n^{1/7-\epsilon})$ inapproxmiable. To our knowledge, the results in this chapter provide the first such inapproxmiablity proof in the context of multiflow cooperative wireless networks. For the special case of $k = 1$, we prove that the solution is a simple

[1]The work described in this chapter, was done in collaboration with B. Krishnamachari and D. S. Hochbaum and, is presented in part in [71]

path, and offer a polynomial time algorithm for jointly optimizing routing, scheduling and power control. We then use this algorithm to establish analytical upper and lower bounds for the optimal performance for the general case of k flows. Furthermore, we propose a polynomial time heuristic for calculating the solution for the general case and evaluate the performance of this heuristic under different channel conditions and against the analytical upper and lower bounds.

5.1 Introduction

In a wireless network, a transmit signal intended for one node is received not only by that node but also by other nodes. In a traditional point-to-point system, where there is only one intended recipient, this innate property of the wireless propagation channel can be a drawback, as the signal constitutes undesired interference in all nodes but the intended recipient. However, this effect also implies that a packet *can* be transmitted to multiple nodes simultaneously without additional energy expenditure. Exploiting this broadcast advantage, broadcast, multicast and multihop unicast systems can be designed to work cooperatively and thereby achieve potential performance gains. As such, cooperative transmission in wireless networks has attracted a lot of interest not only from the research community in recent years [36, 37, 39, 42, 43, 67, 68] but also from industry in the form of first practical cooperative mobile ad-hoc network systems [49]. The majority of the work in the cooperative literature has so far focused on the single flow problem, though recently there has been an increased interest in considering multiflow settings in cooperative networks [58, 59, 61–63].

As previously discussed, networks is routing and resource allocation are key problems in cooperative networks. The situation is further complicated by the fact that the routing and resource allocation depends on the type of cooperation and other details of the transmission/reception strategies of the nodes. We consider a time-slotted system in which the nodes that have received and decoded the packet are allowed to re-transmit it in future slots. During reception, nodes add up the signal power (EA) received from multiple sources. Details of EA, and possible implementations have been extensively discussed in prior work [37, 39, 42, 59] and have been briefly highlighted in Chapter 2.

We focus on the problem of minimum-energy multiflow cooperative transmission in this chapter, where there are k source-destination pairs, with the source node wanting to send a packet to its respective destination nodes, in a multihop wireless network. Other nodes in the network, that are neither the source nor the destination, may act as relays to help pass on the message through multiple hops. The transmission is completed when all the destination nodes have successfully received their corresponding messages. It has been noted in the literature ([64,68]) that a key tradeoff in cooperative settings is between the total energy consumption and the total delay measured in terms of the number of slots needed for all destination nodes in the network to receive the message. Therefore, we take delay into consideration and focus on the case where there is a delay constraint, whereby the destination node(s) should receive the message within some pre-specified delay constraint. We therefore formulate the problem of performing this transmission in such a way that the total transmission energy over all transmitting nodes is minimized, while meeting a desired delay constraint on the maximum number of slots that may be used to complete the transmission. The design variables in this problem determine which nodes should transmit, when, and with what power.

We furthermore assume that the nodes are memoryless, i.e., accumulation at the receiver is restricted to transmissions from multiple nodes in the present time slot, while signals from previous time slots are discarded. This assumption is justified ([64, 68]) by the limited storage capability of nodes in ad-hoc networks, as well as the additional energy consumption nodes have to expand in order to stay in an active reception mode when they overhear weak signals in preceding time-slots.

The main contribution of the work presented in this chapter is as follows: It has been *conjectured* in the literature that *the problem of jointly computing schedules, routing, and power allocation for multiple flows in cooperative networks* is NP-hard [61–63]. In this chapter we formulate the joint problem of scheduling, routing and power allocation in a multiflow cooperative network setting and formally prove that not only it is NP-hard, but it is also $o(n^{1/7-\epsilon})$ inapproximable. (i.e., unless $P = NP$, it is not possible to develop a polynomial time algorithm for this problem that can obtain a solution that is strictly better than a logarithmic-factor of the optimum in all cases). We are not aware of prior work on multiflow cooperative networks that shows such inapproximability results. We further prove that for a special case of $k = 1$, the solution is a simple path and offer an optimal polynomial time algorithm for joint routing, scheduling and power control. We establish analytical upper and lower bounds based on this algorithm and propose a polynomial-time heuristic, the performance of which is evaluated against those bounds.

The rest of this chapter is organized as follows: In section 5.2 we provide a mathematical formulation of the problem. In section 5.3 we consider the special case of $k = 1$ and prove the solution is a simple path and can be found optimally in polynomial time. The inapproximablity results are presented in section 5.4 using reduction from minimum graph coloring problem. We establish analytical upper and lower bounds for optimal performance in section 5.5. A polynomial-time heuristic is proposed in section 5.6 and its performance is evaluated under different channel conditions and against the performance bounds. Concluding remarks are summarized in section 5.8.

5.2 Problem Formulation

Consider a network, G, with a total of n nodes, $I = \{1, .., n\}$. Assume we have r source nodes, labeled $\mathcal{S} = \{s_1, s_2, ..., s_r\}$, and r corresponding destination nodes, $\mathcal{D} = \{d_1, d_2, ..., d_r\}$. The source-destination nodes can be thought of as pairs, $\{(s_k, d_k)\}_{k=1}^{r}$, all with the same delay constraint T. The goal is to deliver a unicast message from each source to its corresponding destination, possibly using other nodes in the network as relays. The objective is to do so using the minimum amount of sum transmit power and within the delay constraint.

We consider a cooperative wireless setting with EA and consider signal-to-intereference-plus-noise (SINR) threshold model, [37,59,69,70]. That is, in order for node i to be able to decode message k at time t, the following inequality needs to be satisfied:

$$\frac{\sum\limits_{j \in s_k(t)} p_{jt} h_{ji}}{\sum\limits_{u \notin s_k(t)} p_{ut} h_{ui} + N} \geq \theta. \tag{5.1}$$

Here $s_k(t)$ is the set of nodes transmitting the message k at time t, h_{ij} is a constant between 0 and 1 representing the channel gain between node i and j, and N and θ are constants representing the noise and the decoding threshold respectively.

Equation (5.1) can be re-written as

$$\sum_{j=1}^{n} h_{ji} p_{jt}^{k} - \theta \sum_{\substack{q=1 \\ q \neq k}}^{r} \sum_{u=1}^{n} h_{ui} p_{ut}^{q} - \theta N \geq 0, \tag{5.2}$$

where p_{it}^{k} is the power used by node i at time t to transmit message k.

The system is memoryless, meaning although we are allowed to accumulate the same message from multiple sources during each time slot, we cannot accumulate over time. The relays are half-duplex, meaning they cannot transmit and receive simultaneously. The relays cannot transmit more than one message at the same time either.

In order to apply ideas driven by the rich literature on multicommodity flows [3] to our problem, we need to somehow introduce the notion of delay constraint into the multicommodity setting. What follows is a transformation of our network graph that would allow for the multicommodity flow technique to be applied, while observing the delay constraint: For a delay constraint T, map the given network to a layered graph with T layers as shown in Figure 5.1. Place a copy of all the nodes in the network on each of the layers. Connect each node, on each layer, to its corresponding copy on its neighboring layers with an edge weight of 0. Also create directed edges between each node, on each layer k, and the nodes on the next layer $k + 1$, with edge weights representing the amount of power required to transmit the message from the node on the top level to the node on the bottom level, as a whole. Notice that there is no edge between the nodes on the same level. Call the new graph G'. Assign the nodes corresponding to the source nodes of G on level 1 of G' as source nodes in G' and the destination nodes on level T of G', corresponding to destination nodes in G, as destinations in G', as shown in the figure. Similar transformations have been used in the literature in the context of multiflow transmission [62].

Without loss of generality, we assume unit length time slots. The nodes who want to transmit are to do so at the beginning of each time slot, and the decoding (by nodes who receive enough information during that time slot) will happen by the end of that time slot. Let z_{it}^k be an indicator binary variable that indicates whether or not node i decodes the message k during time slot t, as per inequality in equation (5.1). In other words, we define z_{it}^k to be 1, if node i decodes message k during time slot t, and 0 otherwise. Let p_{it}^k be the transmit power used by node i at each time t to transmit message k. We define another binary variable x_{it}^k, that is 1 if node i is *allowed* to transmit message k at time t, and 0 otherwise. A node is *allowed* to transmit during a particular time slot, if it has already decoded that message in previous time slots, and it's not receiving or transmitting any other messages during that time slot. Notice that being allowed to transmit does not necessarily mean that a transmission actually occurs. To take care of actual transmissions, let us define v_{it}^k to be a binary variable that is 1 if node i transmits message k at time t, and 0 otherwise.

The problem can then be formalized as a combinatorial optimization problem:

$$\min \quad P_{total} = \sum_{t=1}^{T} \sum_{i=1}^{n} \sum_{k=1}^{r} p_{it}^k \tag{5.3}$$

$$\begin{aligned}
\text{s.t.} \quad & 1. \quad p_{it}^k \geq 0, \;\; \forall i, t, k \\
& 2. \quad x_{d_k T+1}^k = 1, \;\; \forall k \\
& 3. \quad x_{it+1}^k \leq z_{it}^k + x_{it}^k, \;\; \forall i, t \\
& 4. \quad (-M)(1 - z_{it}^k) \leq y_{it}^k, \;\; \forall i, t \\
& 5. \quad p_{it}^k \leq M v_{it}^k, \;\; \forall i, t \\
& 6. \quad \textstyle\sum_{k=1}^{r} \left(v_{it}^k + z_{it}^k \right) \leq 1, \;\; \forall i, t \\
& 7. \quad v_{it}^k \leq x_{it}^k, \;\; \forall i, t, k \\
& 8. \quad x_{s_k 1}^k = z_{s_k 1}^k = 1, \forall k \\
& 9. \quad x_{i1}^k = z_{i1}^k = 0, \forall i \in I \backslash \{s_k\} \\
& 10. \quad x_{it}^k \in \{0, 1\} \\
& 11. \quad z_{it}^k \in \{0, 1\} \\
& 12. \quad v_{it}^k \in \{0, 1\}.
\end{aligned}$$

Here $y_{it}^{k} = \sum_{j=1}^{n} h_{ji} p_{jt}^{k} - \theta \sum_{\substack{q=1 \\ q \neq k}}^{r} \sum_{u=1}^{n} h_{ui} p_{ut}^{q} - \theta N$, M is a large positive constant, and the constraints have the following interpretations:

1. No negative power is allowed.

2. Every node in the destination set is required to have decoded the data by the end of time slot T.

3. If a node has not decoded a message by the end of time slot t, that node is not allowed to transmit that message at time $t+1$.

4. z_{ti}^{k} is forced to be 0 if message k is not decoded in time slot t.

5. p_{it}^{k} is forced to be 0, if node i is not transmitting message k at time t (i.e. if $v_{it}^{k} = 0$).

6. A node cannot transmit and receive at the same time and can only transmit or receive a single message at each time slot.

7. v_{it}^{k} is forced to be 0, node i is not allowed to transmit message k at time t (i.e. if $x_{it}^{k} = 0$).

8. Only sources have the message at the beginning.

9. No one else has the message at the beginning.

10. x, z and v are binary variables.

We call this optimization problem MCUE, for multiflow cooperative unicast with Energy Accumulation.

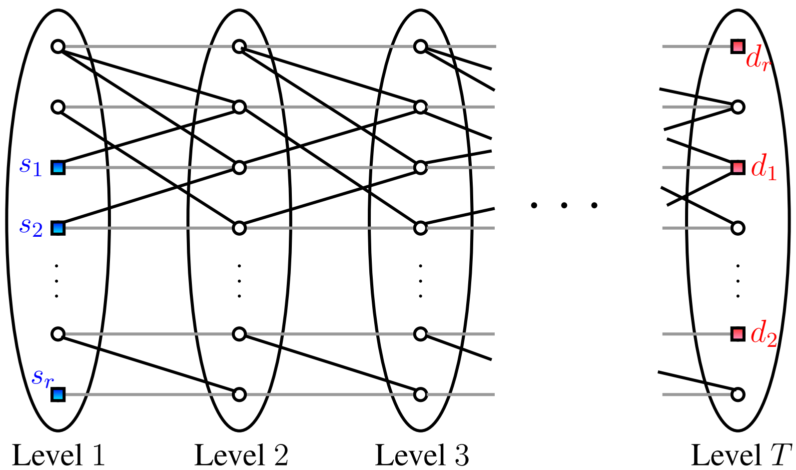

Figure 5.1: Applying the multicommodity flow technique for unicast cast

5.3 Special Case of $k = 1$

In this section we consider MCUE for the special case of $k = 1$ and prove the problem can be solved optimally and in polynomial time for this special case. We also provide a polynomial-time algorithm to achieve the optimum solution.

Theorem 7. *The optimal solution for MCUE is a simple path for $k = 1$, but not necessarily so for $k > 2$.*

Proof. The claim can be proved by induction on T: For delay $T = 1$, the claim is trivially true, as the optimal solution is direct transmission from the source, s, to the given destination, d. Let us assume the claim is true for $T = t - 1$. To complete the proof, we need to show the claim holds for $T = t$. Pick any node in the network as the desired destination d. If the message can be transmitted from source s to d with minimum energy in a time frame less than t, then an optimal simple path exists by the induction assumption. So consider the case when it takes exactly $T = t$ steps to turn on d. The system is memoryless, so d must decode by accumulating the energy transmitted from a set of nodes, $\mathbf{v}$, at time t. This can be represented as $\sum_{v_i \in \mathbf{v}} p_{v_i t} h_{dv_i} \geq \theta$. We observe that there must exist a node $v_o \in \mathbf{v}$ whose channel to d is equal or better than all the other nodes in $\mathbf{v}$. Therefore, given $h_{dv_o} \geq h_{dv_i}, \forall v_i \in \mathbf{v} \backslash \{v_o\}$ then $\sum_{v_i \in \mathbf{v}} p_{v_i t} h_{dv_o} \geq \sum_{v_i \in \mathbf{v}} p_{v_i t} h_{dv_i} \geq \theta$. In other words, if we add the power from all nodes in $\mathbf{v}$ and transmit instead from v_o, our solution cannot be worse. v_o must have received the message by time $t - 1$, to be able to transmit the message to d at time t. We know by the induction assumption that the optimal simple path solution exists from source to any node to deliver the message within $t-1$ time frame. Thus, for $T = t$, there exists a simple path solution between s and d, which is optimum. □

Considering the above theorem, the MCUE problem formulation (for the special case of $k = 1$) reduces to:

$$\min \quad P_{total} = \textstyle\sum_{t=1}^{T} \sum_{i=1}^{n} p_{it} \tag{5.4}$$

s.t.

1. $p_{it} \geq 0, \ \ \forall i, t$
2. $x_{dT+1} = 1$
3. $-M(1 - x_{it+1}) \leq \sum\limits_{j=1}^{n} h_{ji} p_{jt} - \theta N, \ \ \forall i, t$
4. $p_{it} \leq M x_{it}, \ \ \forall i, t$
5. $x_{s1} = 1$
6. $x_{i1} = 0, \forall i \neq s$
7. $x_{it} \in \{0, 1\}$

This can be solved optimally in polynomial time using dynamic programming. Let $C(i,t)$ be the minimum cost it takes for source node s to turn on i, possibly using relays, within at most t time slots. Then we can write:

$$C(i,t) = \min_{j \in Nr(i)} [C(j, t-1) + w_{ji}] \tag{5.5}$$

with $C(s,t) = 0$, for all t and $C(i,1) = w_{si}$, where $Nr(i)$ is the set that contains i and its neighboring nodes that have a non-zero channel to i, w_{ji} represents the power it takes for j to turn on i using direct transmission. Thus the solution to (5.4) is given by $C(d,T)$ and its computation incurs a running time of $O(n^3)$.

5.4 Inapproximability Results

For $k = 1$, we proved in Theorem 5.3, that the optimal solution is a simple path. For $k > 2$, we can consider the following counter-example to argue that the solution is not necessarily a single-path. Consider the scenario shown in Figure 5.2, where $T = 3$, where the edge weights are equal and the edges shown in gray show strong interference. The red nodes cannot by themselves transmit the message to d_2, as it causes interference for d_1 and d_3 preventing them from being able to decode the data. However, they can cooperate with each other, by each sending with half power to get the message to d_2 without causing too much interference for the other destinations.

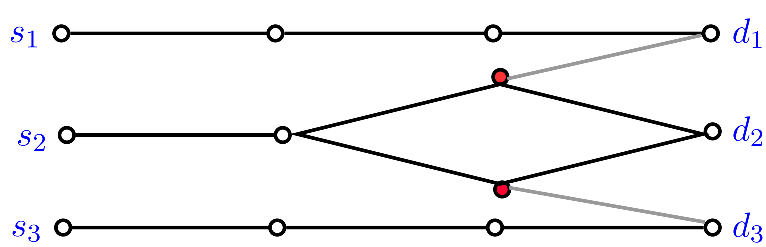

Figure 5.2: An example of $k > 2$, with $T = 3$, where the optimal solution is not a single path.

To investigate the complexity of MCUE, let us start by looking at a sub-problem. Imagine a one hop setting of k source nodes and their corresponding k destination nodes, with no relay nodes. Due to interference, not all sources can transmit simultaneously. The task is to schedule the sources appropriately, so that everyone can get their message delivered to their corresponding destination within a time delay T. The problem is to find the minimum such T. Let us call this problem MOSP, for multi-source one-hop scheduling problem[2]. It is important to note that MCUE is at least as hard as MOSP. Thus, any hardness results obtained for MOSP imply hardness of MCUE.

In this section, we derive inapproximablity results for MOSP by showing that any instance of *minimum graph coloring problem* [3] can be reduced to an instance of MOSP.

Lemma 3. *MOSP is* $o(n^{1/7-\epsilon})$ *inapproximable, for any* $\epsilon > 0$.

Proof. Given an instance $G(V, E)$, $|V| = n$, of the minimum graph coloring, we construct a bipartite graph G', with the bi-partition X and Y with $|X| = |Y| = n$. For each node $v_i \in G$, we place two nodes $u_i \in X$ and $u'_i \in Y$ and connect them with an edge (u_i, u'_i). Also for every edge in G, $e_{ij} = \{v_i, v_j\}$, place two edges (u_i, u'_j) and (u_j, u'_i) in G'. We assign u_i and u'_i to be a source and destination pair respectively for all i. We set equal edge weights for all the edges in G' and set $\theta > 1$ to get an instance of MOSP.

A simple example is shown in Figure 5.3. Notice that the gray edges in the figure represent interference, and by setting $\theta > 1$, a message can be successfully decoded if and only if there is no interference at that node.

[2]This is essentially the problem considered in [66], though no proof of complexity is given in that paper.

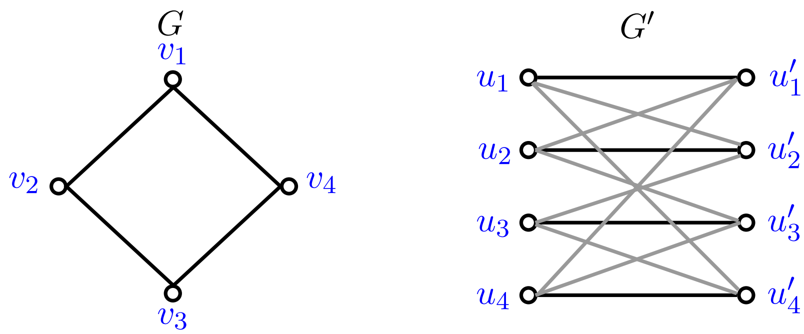

Figure 5.3: Example construction of G', for a given G.

□

This in turn means two sources in G' can simultaneously transmit if and only if there is no edge in between them in G. Thus, the set of nodes that are transmitting simultaneously in G' correspond to an independent set in G. Consequently, the optimal solution to MOSP is equal to the minimum graph coloring of G, which is known to be $o(n^{1/7-\epsilon})$ inapproximable [65]. The following theorem follows by noticing that MOSP is a special case of MCUE.

Theorem 8. *MCUE is $o(n^{1/7-\epsilon})$ inapproximable,for any $\epsilon > 0$.*

Notice that the inapproximability result, given by Theorem 8, is stronger than, and implies, the NP-hardness result. In other words, it implies that not only finding the optimal solution is NP-hard but finding a polynomial time approximation algorithm that approximates the optimal solution to MCUE with a factor of $o(n^{1/7-\epsilon})$ is also NP-hard.

5.5 Performance Bounds

In section 5.4, we proved that MCUE problem is in general inapproximable. However, it was shown in section 5.3 that the problem can be solved optimally and in polynomial time for the special case of $k = 1$. In this section, we used the results of section 5.3 to obtain performance bounds for MCUE.

5.5.1 An Analytical Lower Bound

In this section we establish a lower bound on the optimum solution to MCUE.

To get a better intuition for this lower bound, let us start off by considering the optimal solution to MCUE for the case when there is only one flow present in the network. As before, we have n nodes and a channel H, but this time the source s wants to transmit the message to a particular destination d, using the minimum energy within a given delay constraint T. The system is cooperative in that other nodes in the network, may be utilized as memoryless energy accumulating relays to help achieve the minimum energy goal. Based on section 5.3, the solution can be found by calculating $C(d, T)$ where $C(d, T)$ is defined as per equation (5.3).

To find a lower bound for MCUE for a general case of r flows, with source-destination pairs $\{(s_k, d_k)\}_{k=1}^{r}$, all with the same delay constraint T, we notice that the cost paid by optimal MCUE to cover each node cannot be lower than the optimal minimum cost paid by each source s_k to cover its corresponding destination d_k in the absence of other interfering flows. Based on that observation we derive the following lower-bound, $LB(T)$, for the OPT_{MCUE} for r flows when the delay constraint is T:

$$LB(T) = \sum_{k=1}^{r} C(d_k, T) \tag{5.6}$$

where $C(d_k, T)$ is defined as per equation (5.3). In other words, $C(d_k, T)$ calculates the minimum cost of optimal single flow transmission to cover a destination d_k, starting from its corresponding source under a delay constraint T. $LB(T)$ takes the sum of those costs and use it as lower-bound - since we know OPT_{MCUE} has to cover all these flows and cannot do so any better than the optimal solution for a single flow. Computing this lower bound incurs a running time of $O(n^3)$.

5.5.2 An Analytical Upper Bound

In this section we establish an upper bound on the optimum solution to MCUE, for the general case of r flows, with $T \geq r$.

The upper bound is established by considering the multiplexing solution. At the extreme end of $T = r$, we would allow one time slot for each of the r flow to transmit its message, while the other flows are silent. For a general time $T(> r)$ we break the time into r blocks $\mathcal{T} = (\tau_1, \tau_k, ...\tau_r)$, such that $\sum_{k=1}^{r} \tau_k = T$. We assign each block to one of the flows, while the other flows are silent. We calculate $C(d_k, \tau_k)$, defined as per equation (5.3). For a given tuple $\mathcal{T}$, the summation of the total energy required by all flows to complete their transmission can be achieved by calculating:

$$UB(\mathcal{T}) = \sum_{k=1}^{r} C(d_k, \tau_k) \tag{5.7}$$

This sum would provide an upper bound for OPT_{MCUE}. For a general $T \leq r$, we will have $\binom{T-1}{k-1}$ possibilities for assigning the time slots to different flows. The upper bound is calculated as follows:

$$UB(T) = \min_{\mathcal{T}} UB(\mathcal{T}) \tag{5.8}$$

To compute this upper bound we need to carry on the computation for calculating a single flow MCUE, discussed in section 5.3, $\binom{T-1}{k-1} \times r$ times. Thus the upper bound incurs a running time of $O(n^3)$.

5.6 A Polynomial-time Heuristic

In this section we propose a polynomial time heuristic for MCUE, the performance of which is later evaluated against that of the bounds established earlier in this chapter.

To recap, consider a network, G, with a total of n nodes, $I = \{1, .., n\}$. Assume we have r source nodes, labeled $\mathcal{S} = \{s_1, s_2, ..., s_r\}$, and r corresponding destination nodes, $\mathcal{D} = \{d_1, d_2, ..., d_r\}$. The source-destination nodes can be thought of as pairs,

$\{(s_k, d_k)\}_{k=1}^{r}$, all with the same delay constraint T. The goal is to deliver a unicast message from each source to its corresponding destination, possibly using other nodes in the network as relays. The objective is to do so using the minimum amount of sum transmit power and within the delay constraint.

The algorithm works greedily by scheduling flows one by one. Each flow is given more slots than its previous flows, to ensure a feasible solution always exist. That means the algorithm works for $T \geq r$. Each flow, with the exception of the final flow, uses more power than required to deliver its message. This is achieved by assigning a higher threshold to that flow when scheduling the flow. After scheduling, the nodes that will be transmitting at each time slot and the power they use for transmission is passed on to the next flow. Each flow, when scheduling itself, will ensure that its transmission will not disturb the transmission of previously scheduled flows. A lower threshold is used to check for disturbance, than the one used for scheduling the flow itself. Let us now look at the details of the algorithm.

We schedule the r flows greedily, starting from the one that causes the least disturbance. Without loss of generality, let us assume that we are scheduling the flows in the order 1 to r. We have a total of T time slot, for flow 1, we assign T_1 time slots for transmission, for flow 2, we assign T_2 and so forth, such that:

$$1 \leq T_1 < T_2 < ... < T_k < ..T_r = T \tag{5.9}$$

Recall that time-slots are in unit durations, thus can increment in integer units, thus each flow has at least one more time slot at its disposal than its immediate predecessor, ensuring that a feasible solution always exists.

In section 5.2, we defined θ to be the decoding threshold as per equation (5.1). For this multi-flow setting, each flow is assigned its own θ value, such that:

$$\theta_1 > \theta_2 > ... > \theta_k > ...\theta_r = \theta \tag{5.10}$$

Flow 1 is scheduled with T_1 and θ_1, as per algorithm in section 5.3. We store the nodes that are scheduled to transmit in each time slot, and their transmit power and their corresponding receivers in a black list $\mathcal{B}$, as such $\mathcal{B}(t)$ gives us the set of already scheduled nodes that are transmitting at time t and their corresponding powers, and their corresponding receiving nodes.

For the kth flow, we pick a subset of nodes (as potential relays) and use a modified version of the algorithm discussed in section 4.1, and assign an ordering to those nodes. Let us call this ordered subset $\mathcal{I}_k = (1_k, 2_k, ..., j_k, ..., n_k)$, where 1_k corresponds to s_k and n_k corresponds to d_k. This set could for instance be obtained by picking the nodes that would have been picked if we were to run the single-flow algorithm of section 5.3 for flow k. Recall that given ordering the algorithm in section 4.1, would find the optimal scheduling and power allocation in polynomial-time. In order to use that algorithm, we need to modify the power allocation part to ensure that when assigning powers to the current flow we are not disturbing the previously scheduled flows. Let us call the modified version of power allocation algorithm $LPMF$ (for linear program multi-flow). $LPMF$, to be specified shortly, would calculate the instantaneous optimal power allocation for flow k at time t, given the set of instantaneous senders and receivers for flow k and the set $\mathcal{B}(t)$ (of senders and receivers of flows 1 to $k-1$ and their corresponding powers at that time slot).

Given this modified power allocation algorithm, all that remains to be done is to determine these sets at each time slot, in order to minimize the overall power while meeting the delay constraint. Let $C_k(j_k, t)$ be the minimum energy needed for flow k to cover up to node j_k in t steps or less. We can calculate this, using the following deterministic dynamic program:

$$C_k(j_k, t) = \min_{i_k \in (1,..,j_k)} [C_k(i_k, t-1) + LPMF(\{1...i_k\}, \{i_k+1...j_k\}, \theta_k, H, \mathcal{B}(t))] \tag{5.11}$$

where $C_k(i_k, 1) = LPMF(1_k, \{2_k...i_k\}, \theta_k, H, \mathcal{B}(t)) \quad \forall i_k \in \mathcal{I}_k \backslash 1_k$, and $C_k(1_k, t) = 0 \quad \forall t$.

Thus, for flow k, the total minimum cost for covering n_k nodes by time T_k can be found by calculating $C_k(n_k, T_k)$.

Let us now look at the design of $LMPF$. $LMPF$ is defined similar to that of power allocation algorithm in (4.2), except we have to take a few new points into account. Consider $LMPF$ for flow k, the algorithm takes as an input a set of transmitters ($TX_k(t) = \{1...i_k\}$) and a set of receivers ($RX_k(t) = \{i_k+1...j_k\}$), and the set of already scheduled nodes for that time-slot and their corresponding powers $\mathcal{B}(t)$, the channel between the nodes and the receiving threshold θ_k. For this flow, the new set of rules to abide by would be:

1. A node cannot transmit a message for flow k at time t, if it has already been scheduled to participate in another flow in that time slot. In another words,
   $\forall q \in TX_k(t),$ **if** $q \in \mathcal{B}(t)$ **then** $p^k_{qt} = 0$.

2. A node cannot receive a message for flow k at time t, if it has already been scheduled to participate in another flow in that time slot. This renders the power allocation task infeasible with the given set of transmitting and receiving nodes. In another word,
   $\forall q \in RX_k(t),$ **if** $q \in \mathcal{B}(t)$ **then** $LPMF(\{1...i_k\}, \{i_k + 1...j_k\}, \theta_k, H, \mathcal{B}(t)) = \infty$.

After having taken the above two conditions into account, we can proceed with the linear program as follows:

$$\min \quad \sum_{q \in TX_k(t)} p^k_{qt} \tag{5.12}$$

$$\text{s.t.} \quad 1.\ \ p^k_{qt} \geq 0, \ \ \forall q \in TX_k(t)$$

$$2.\ \ \sum_{q \in TX_k(t)} h_{qj} p^K_{qt} - \theta_k \sum_{u \in TX_f(t)} h_{uj} p^f_{ut} - \theta_k N \geq 0, \quad {\scriptstyle \forall j \in RX_k(t) \atop \forall f \in \{1,\dots,k-1\}}$$

$$3.\ \ \sum_{v \in TX_f(t)} h_{vz} p^f_{vt} - \theta_f \sum_{\substack{u \in TX_g(t) \\ g \neq f}} h_{uz} p^g_{ut} - \theta_f N \geq 0, \quad {\scriptstyle \forall z \in RX_f(t) \atop {\forall g \in \{1,\dots,k\} \atop \forall f \in \{1,\dots,k-1\}}}$$

Where constraint 1 ensures that there are no negative powers. Constraint 2 ensures that the nodes assigned to receive flow k at time t will in fact accumulate enough energy to decode the message, despite the existing interference. Constraint 3 ensures that the power being assigned to nodes in flow k, is not disturbing the previously scheduled flows. This algorithm is referred to as $LPMF(\{1...i_k\}, \{i_k + 1...j_k\}, \theta_k, H, \mathcal{B}(t))$. As a notation, $LPMF([\{x...y\}, \{z...\alpha\}], \theta, H, \mathcal{B}(t)) = 0$, if $z \geq \alpha$

5.7 Performance Evaluation

In this section we compare the performance of the proposed heuristic against the analytical bounds for an example network with an arbitrarily chosen three flows. We also look at the effect of channel degradation in the overall performance.

We consider a network of 100 nodes uniformly distributed on a 20 by 20 square surface. The channels between all nodes are static, with independent and exponentially distributed channel gains (corresponding to Rayleigh fading), where h_{ij} denotes the channel gain between node i and j. The mean value of the channel between two nodes, $\overline{h_{ij}}$, is chosen to decay with the distance between the nodes, so that $\overline{h_{ij}} = d_{ij}^{-\eta}$, with d_{ij} being the distance between nodes i and j and η being the path loss exponent. The corresponding distribution for the channel gains is then given by

$$f_{h_{ij}}(h_{ij}) = \frac{1}{\overline{h_{ij}}} \exp\left(\frac{h_{ij}(k)}{\overline{h_{ij}}}\right)$$

Notice that the minimum power calculated by different algorithms, shown on the y-axes of the graphs in this section, are normalized by value of θ (rendering it unit-less).

Figure 5.4 shows the performance of the heuristic against that of the analytical bounds. As can be seen the heuristic is performing close to the lower bound. Notice that the lower bound is an unachievable lower bound, in that it assumes no interference is present. This means that its performance is not achievable by any algorithm. This is more emphasized when we have fewer time slots available, and thus we need to use more power to transmit the message creating a lot of interference that is ignored by the lower bound. As we get more time-slots available to us, the performance of the heuristic and the bounds seem to converge, which is what we expect as the solution goes to a multiplexing solution in all cases.

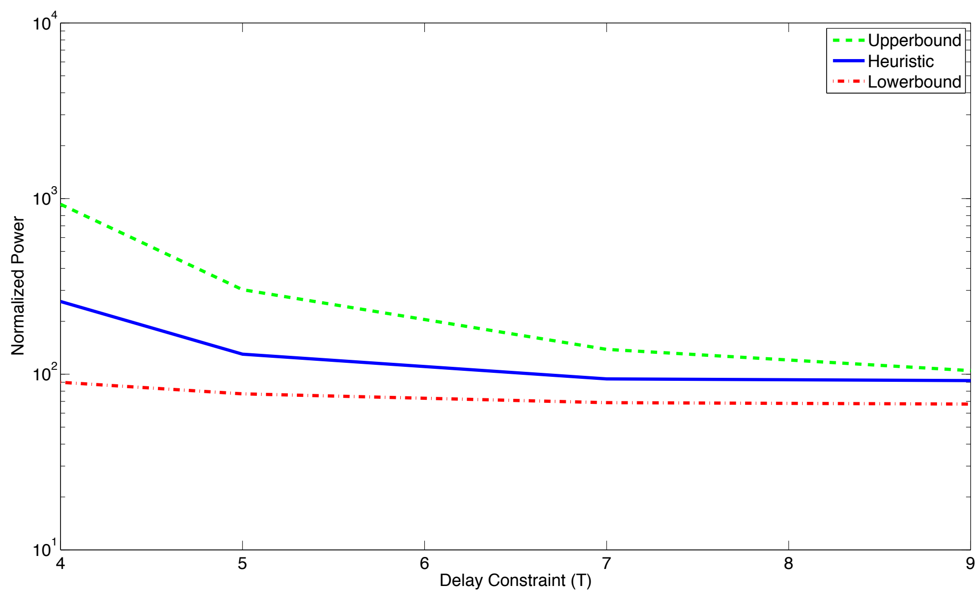

Figure 5.4: Performance of the heuristic against the analytical upper and lower bound.

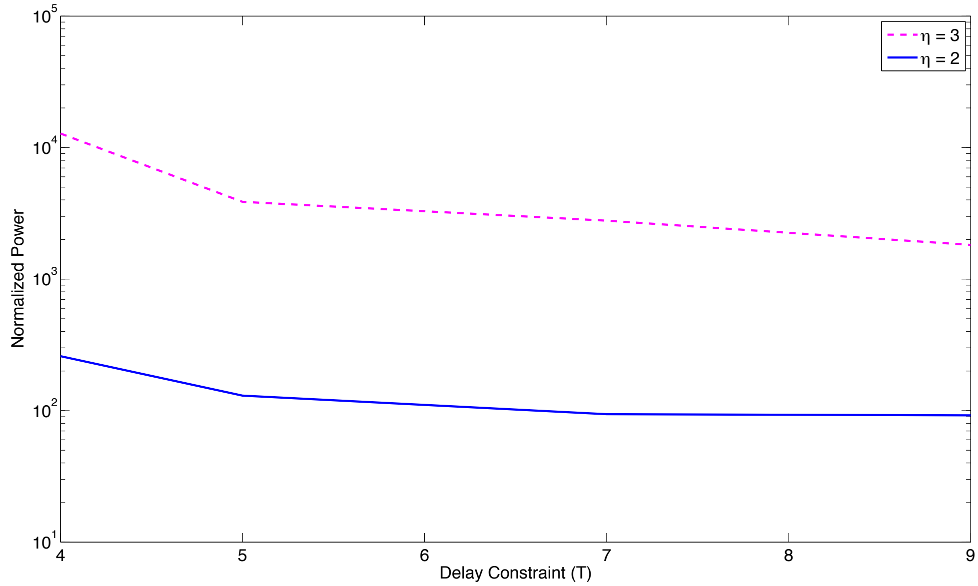

Figure 5.5: Effect of channel degradation on the total energy consumed.

We see the effect of poor channel conditions in Figure 5.5. As expected the performance is degraded as the channel conditions become poor, this highlights the importance of having smart algorithms to minimize the energy consumption in such scenarios.

5.8 Summary

In this chapter we formulated the problem of minimum energy cooperative transmission in a delay constrained multiflow multihop wireless network, as a combinatorial optimization problem, for a general setting of k-flows and formally proved that the problem is not only NP-hard but it is $o(n^{1/7-\epsilon})$ inapproxmiable. To our knowledge, the results in this chapter provide the first such inapproxmiablity proof [64] in the context of multiflow cooperative wireless networks. It is interesting to note that although the minimum graph coloring problem is NP-hard, the fractional graph coloring can be solved in polynomial time. That presents an interesting venue for future work and for designing possible approximation algorithms for this problem.

We further proved that for a special case of $k = 1$, the solution is a simple path and offered an optimal polynomial time algorithm for joint routing, scheduling and power control. We then used this algorithm to establish analytical upper and lower bounds for the optimal performance for the general case of k flows. Furthermore, we proposed a polynomial time heuristic for calculating the solution for the general case and evaluated the performance of this heuristic under different channel conditions and against the analytical upper and lower bounds.

Chapter 6

Conclusions and Future Directions

In this thesis, we formulated the novel problem of delay constrained minimum energy cooperative transmission (DMECT) in wireless networks, encompassing both EA and MIA. We have shown that this problem is $o(\log n)$ inapproximable in broadcast and multicast cases and is NP-complete in the unicast case when mutual information accumulation is used . For the broadcast case with EA, we have presented an $O(T \log^2(n))$ approximation algorithm.

Another key algorithmic contribution has been to show a polynomial algorithm that can solve this NP-hard problem optimally for a fixed transmission ordering. Our empirical results suggest that for practical settings, a near-optimal ordering can be obtained by using Dijkstra's shortest path algorithm. We have further showed that the unicast case can be solved optimally and in polynomial time when EA is used. We have studied the energy-delay tradeoffs and the performance gain of MIA using simulations, and evaluated the performance of our algorithm under varying conditions. The summary of the algorithmic results developed are presented in Tables 3.1 and 4.1.The empty slots are still open problems.

We further formulated the problem of minimum energy cooperative transmission in a delay constrained multiflow multihop wireless network, as a combinatorial optimization problem, for a general setting of k-flows and formally proved that the problem is not only NP-hard but it is $o(n^{1/7-\epsilon})$ inapproxmiable. To our knowledge, the results in this chapter provide the first such inapproxmiablity proof in the context of multiflow cooperative wireless networks.

We observed that for a special case of $k = 1$, the solution is a simple path and offered an optimal polynomial time algorithm for joint routing, scheduling and power control. We then used this algorithm to establish analytical upper and lower bounds for the optimal performance for the general case of k flows. Furthermore, we proposed a polynomial time heuristic for calculating the solution for the general case and evaluated the performance of this heuristic under different channel conditions and against the analytical upper and lower bounds.

There are many interesting open problems and research directions yet to be investigated in this field. In the following, we highlight a number of these directions, which are of particular interest to the author and the results of which we consider to be of significant theoretical and practical impacts.

- One area, that is of particular interest, is investigating the approximability gap in the broadcast case with EA. We have already proved that this problem is $o(\log(n))$

inapproximable, however the current best positive approximation result we have for this problem gives an approximation factor of $O(T \log^2(n))$. It would be interesting to see whether or not we can tighten up this gap.

Notice that we obtained our approximation algorithm by proposing a mapping between our problem and the directed Steiner tree problem that preserved the approximation factor. We, thus, argued that the best known approximation results for directed Steiner tree applies directly to our work. Hence, we are so far using the existing approximation algorithms of directed Steiner tree as a block box. The conjecture is that a closer examination of how those approximation algorithms for directed Steiner tree were developed, and applying those techniques [72, 73] directly to our problem, might lead to a tighter approximability gap.

- Throughout the thesis, we did not make any assumptions on the structure of the network when deriving inapproximability results. We did however note, through simulations, that our proposed algorithms were performing close to optimal for uniform distribution of nodes for example. It would be interesting to investigate achieving better inapproximablity results by making assumptions on the structure of the network.

- We proved that the multiflow problem is $o(n^{1/7-\epsilon})$ inapproxmiable and proved a heuristic that performed good in simulations. It would be interesting to establish approximation algorithms for this problem and see how close we can get to the theoretical limit. We drove the inapproximablity results using minimum graph coloring problem. Good approximation algorithms exists for this problem [95–97], and it is interesting to note that the fractional graph coloring can be solved in polynomial time. That presents an interesting venue for future work and for designing efficient heuristics for this problem. Investigating multicommodity flow problem and its approximation algorithms [74, 76] in the context of our problem might also lead to interesting results.

- Applying game theory to cooperative wireless settings is currently a vibrant field of research [98–100]. Another interesting venue that might be promising to investigate is applying game theory to the above setting by either trying to develop a new distributed approach based on the assumption that the nodes are selfish and might not be willing to, left on their own, cooperate; or by applying game theory analysis to the distributed version of our existing algorithms and analyze the performance under the assumption that the nodes are selfish.

- More recently there has been a growing interest in the research community in investigating the practical aspects of cooperative schemes setting geared toward developing practical schemes capable of harvesting the gains predicted by analytical models [49, 81, 83]. Evaluating the proposed algorithms under more realistic settings (perhaps through direct implementation of distributed protocols on software radio platforms) remains an interesting venue for future research and would certainly help in moving this work towards practice.

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