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ANRG

Bounded-Degree Minimum-Radius Spanning Trees for Fast Data Collection in Sensor Networks

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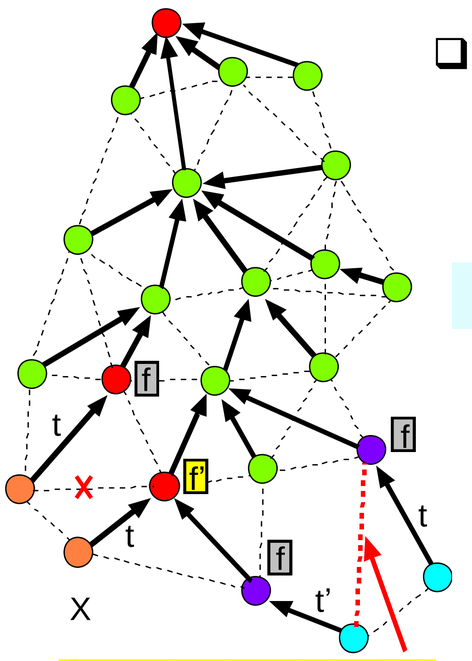
Motivation

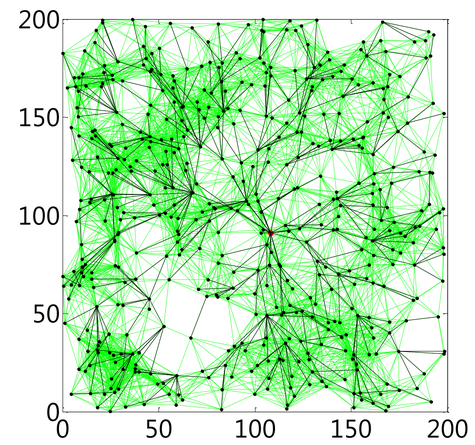
Maximize Aggregated Sink Throughput + Minimize Delay



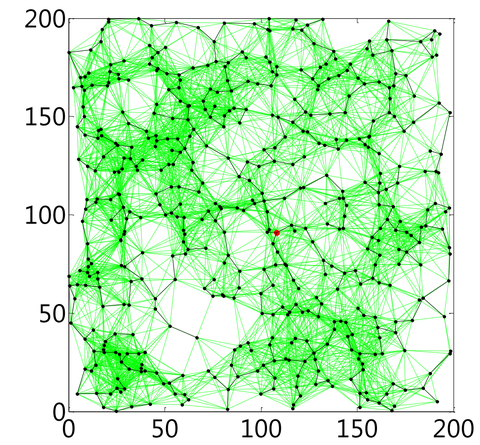
- **Applications**
  - High data-rate mission critical (e.g., SHM)
  - Real-time continuous monitoring (e.g., wildfire, security-surveillance, environmental)

Challenges: Scheduling and Routing

- **TDMA Scheduling**
  - Half-duplex transceivers (primary conflict)
  - 2-hop interfering links (secondary conflict)
- **Routing Topology**
  - High node degree + small radius (SPT) => fewer concurrent transmissions but low delay
  - Low node degree + large radius (MIT) => more concurrent transmissions but high delay



Shortest Path Tree (SPT): high node degree, small radius

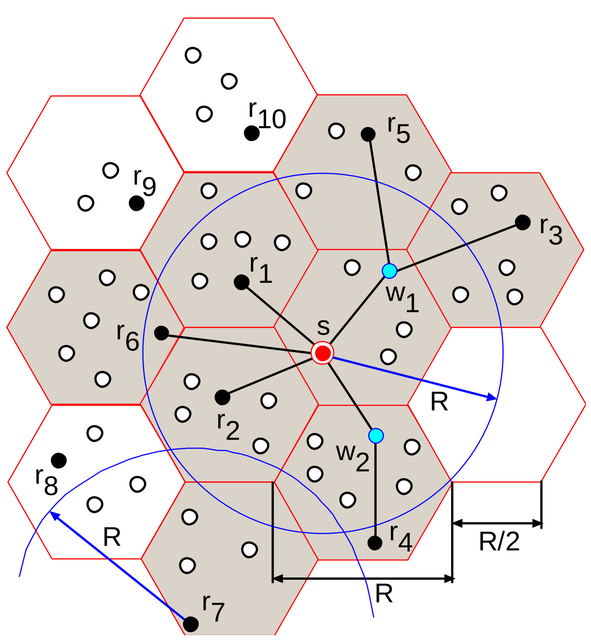


Minimum Interference Tree (MIT): low node degree, large radius

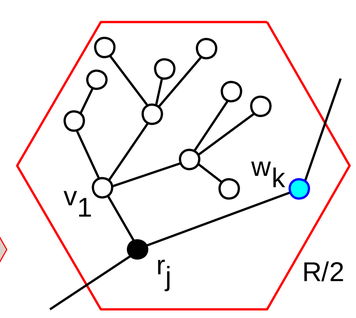
Problem Formulation and Solution

Bi-Criteria Optimization Formulation

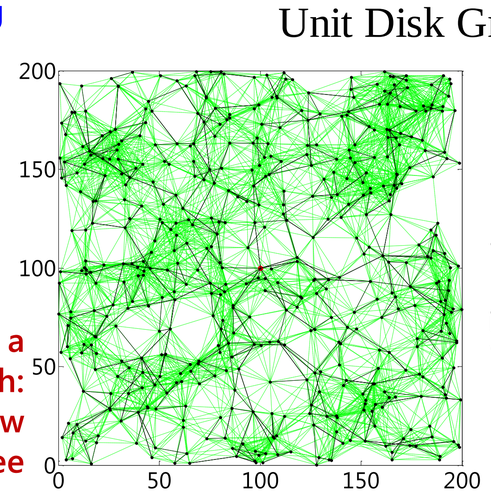
- **Bounded-Degree Minimum-Radius Spanning Tree (NP-Hard)**
  - Given a graph $G = (V,E)$, communication range R, and a constant $\Delta^* \geq 2$, construct a spanning tree T of G rooted at sink $s \in V$, such that:
    - Radius (max hop count to sink) of T is minimized
    - Degree of any node in T is at most Δ^*
- **Approximation**
  - Design an (α,β) bi-criteria approximation to BDMRST on Unit Disk Graphs such that:
    - Maximum node degree in T is at most $\Delta^*+\alpha$
    - Radius of T is at most β times the optimal radius



Backbone Tree : Local roots r_1, r_2, r_3, r_4, r_5, r_6 are connected from non-empty adjacent cells (shaded) of sink s. Nodes r_3 and r_5 are connected using helper node w_1 (blue), and r_4 using helper w_2.



Local Spanning Tree : Nodes within each cell form a complete graph. $\Delta^* = 4$.



BDMRST on a 500 node graph: small radius, low degree

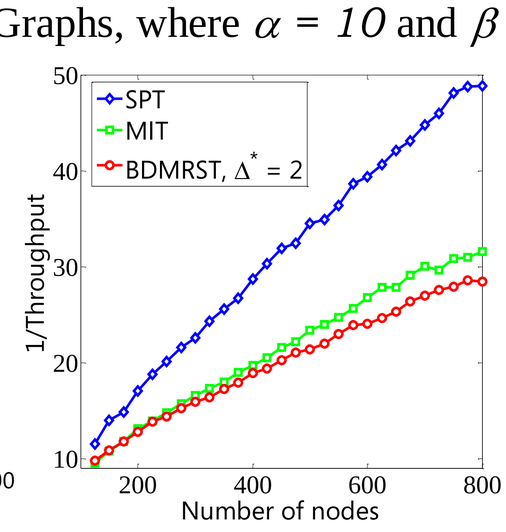
Approximation Algorithm ($\mathcal{A}$) for BDMRST

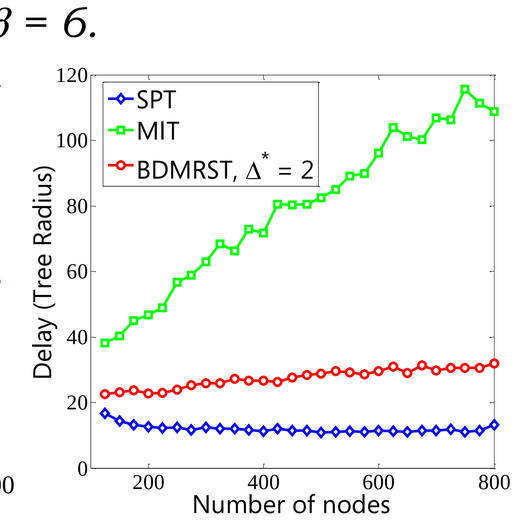
- **Phase 1: Backbone Tree (T_B)**
  - Tessellate the region into hexagonal grid cells of length $R/2$
  - Choose one local root arbitrarily from each non-empty cell
  - Let $\mathcal{R} = \{r_1, \ldots, r_n\}$ be the set of local roots
  - Connect the local roots in a **Breadth-First-Search** order starting from the sink s (using **helper nodes** when direct links do not exist), while ensuring the radius is not too long compared to an SPT on $\mathcal{R}$
- **Phase 2: Local Spanning Trees (T_j)**
  - Nodes within each hexagonal cell form a **complete graph**
  - Construct local spanning trees T_j with the remaining nodes in each cell such that no node exceeds degree Δ^*
- **BDMRST = $T_B \cup \{T_j\}$**

Main Result and Evaluation

- **Theorem**

  Algorithm $\mathcal{A}$ gives an (α,β) **constant factor approximation** to the Bounded-Degree Minimum-Radius Spanning Tree problem on Unit Disk Graphs, where $\alpha = 10$ and $\beta = 6$.





Evaluation of the interference-aware link scheduling algorithm in [1] on different trees. BDMRST achieves the best trade-off in terms of maximizing the aggregated sink throughput and minimizing the maximum delay.

[1] A. Ghosh, O.D. Incel, V.S. Anil Kumar, and B. Krishnamachari, "Multi-Channel Scheduling Algorithms for Fast Convergecast in Sensor Networks", MASS '09, pp. 363-372.

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