Algorithmic Aspects of Throughput-Delay Performance for Fast Data Collection in Sensor Networks

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Thesis Statement:

Multi-channel scheduling, optimal routing topologies, and transmission power control are necessary in order to improve the throughput-delay trade-off for aggregated convergecast in large-scale, multi-hop, wireless sensor networks.
Outline

1. **Maximizing Convergecast Throughput**
   - Motivation and Preliminaries
   - Scheduling with Unlimited Frequencies
   - Approximations on Unit Disk Graphs
   - Approximations on General Disk Graphs

2. **Multi-Channel Scheduling in SINR**
   - Motivation and Interference Models
   - Problem Formulation and Approach
   - Approximations in SINR

3. **Throughput-Delay Trade-off**
   - Motivation
   - Bi-criteria Formulation
   - Optimal Spanning Trees (parts)

4. **Distributed Topology Control in 3-D**
   - Motivation
   - Two Approaches: Orthographic Projections, SDT
Outline

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   - Motivation
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   - Optimal Spanning Trees

4. **Conclusions**
Model and Assumptions

e.g., Applications with fast & efficient data collection requirements.
Model and Assumptions

- **TDMA periodic** scheduling
  
  - every node generates a single packet at the beginning of each frame
  
  - every node transmits once per slot

- **Graph-based** network & interference model

- **Half-duplex, single transceiver**

- **Receiver-based Frequency Assignment**

- **Non-interfering** orthogonal channels

- **In-network** aggregation
  
  (distributive, algebraic
  
  - aggregation happens at each hop)

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**Secondary conflict**

- e.g., Applications with fast & efficient data collection requirements.
Convergecast

A many-to-one communication pattern (i.e., flow of data): from all the nodes to a sink (opposite of broadcast).

Figure: Schedule length is 5.
Convergecast and Benefits of Multiple Frequencies

**Convergecast**

A many-to-one communication pattern (i.e., flow of data): from all the nodes to a sink (opposite of broadcast).

**Figure**: Schedule length is 5.

**Figure**: Schedule length is 3.
Convergecast

A many-to-one communication pattern (i.e., flow of data): from all the nodes to a sink (opposite of broadcast).

<table>
<thead>
<tr>
<th>Frame 1</th>
<th>Frame 2</th>
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<tbody>
<tr>
<td>Receiver</td>
<td>Slot 1</td>
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<td>s</td>
<td>c</td>
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<tr>
<td>a</td>
<td></td>
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<tr>
<td>b</td>
<td>e</td>
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<tr>
<td>c</td>
<td>g</td>
</tr>
</tbody>
</table>

Pipelining

Figure: Schedule length is 3.
Joint Frequency and Time Slot Assignment

Given a connected network $G = (V, E)$ of arbitrarily deployed nodes, $K$ orthogonal frequencies, and a routing tree $T \subseteq G$ rooted at the sink $s \in V$

- Assign a frequency to each receiver in $T$
- Assign a time slot to each edge in $T$

such that the aggregated throughput at the sink is maximized, where

\[
\text{Aggregated Throughput} = \frac{1}{\text{Schedule Length}}
\]

(equivalent to minimizing the Schedule Length)
Theorem

Joint Frequency and Time Slot Assignment is NP-hard on general graphs.
Proof: Reduction from Vertex Color.

Questions

- Does the problem still remain NP-hard with unlimited frequencies?
- If not, how many frequencies (at a min, or at most) are needed?
- Can we design algorithms that have provable, worst-case guarantee?
Maximizing Convergecast Throughput

Scheduling Complexity

**Theorem**

Joint Frequency and Time Slot Assignment is NP-hard on general graphs.

Proof: Reduction from Vertex Color.

**Questions**

- Does the problem still remain NP-hard with unlimited frequencies?
- If not, how many frequencies (at a min, or at most) are needed?
- Can we design algorithms that have provable, worst-case guarantee?

**Theorem**

Given a spanning tree $T$ on a graph $G = (V, E)$, finding the minimum number of frequencies required to remove all secondary conflicts is NP-hard.
A Frequency Upper Bound

**Constraint Graph**

- For each receiver in $G$ create a node in $G_C$.
- Create an edge between two nodes in $G_C$ if the corresponding receivers in $G$ form a secondary conflict.

**Lemma**

$$K_{max} \leq \Delta(G_C) + 1,$$
where $\Delta(G_C)$: max degree in $G_C$. (Vertex Color)
**Algorithm:** BFS-\textsc{TimeSlotAssignment}

1. while $E_T \neq \emptyset$ do
2. \hspace{1em} $e \leftarrow$ next edge from $E_T$ in BFS order
3. \hspace{1em} Assign minimum time slot to $e$ respecting adjacency constraint
4. \hspace{1em} $E_T \leftarrow E_T \setminus \{e\}$
5. end while

**Theorem**

\textsc{BFS-TimeSlotAssignment} gives a schedule of minimum length $\Delta(T)$, where $\Delta(T)$: maximum node degree in $T$. 

[Diagram of tree with time slots assigned]
Evaluation

Parameters:

- Number of nodes: \( n : 200 \), square region of area \( A : 200 \times 200 \)
- Shortest Path Tree, Unit Disk Graph

Figure: Number of frequencies required to remove all secondary conflicts as a function of network density \( n/A \) on SPT.
So far...

Joint Frequency and Time Slot Assignment is

- **NP-hard** with **limited** (constant number) frequencies on arbitrary graphs.
- **Polynomial** with **unlimited** (at most $\Delta(G_C) + 1$) frequencies for arbitrary graphs.

Next...

Approximation algorithms for **Random Geometric Graphs** with **limited** (constant number) frequencies.

- Uniform transmission range (**Unit Disk Graphs**)
- Non-uniform transmission range (**General Disk Graphs**)
Algorithm $A_{UDG}$ Overview

- **Phase 1**: Assign frequencies to the receivers in each cell such that the maximum number of nodes transmitting on the same frequency (load) is minimized. (Load-Balanced Frequency Assignment)
- **Phase 2**: Assign time slot greedily to each edge.
Algorithm $\mathcal{A}_{UDG}$ Overview

- **Phase 1**: Assign frequencies to the receivers in each cell such that the maximum number of nodes transmitting on the same frequency (load) is minimized. (Load-Balanced Frequency Assignment)
- **Phase 2**: Assign time slot greedily to each edge.

**Theorem**

Algorithm $\mathcal{A}_{UDG}$ achieves a constant factor $8\mu_\alpha \cdot \left(\frac{4}{3} - \frac{1}{3K}\right)$ approximation on the optimal schedule length, where $\mu_\alpha > 0$ is a constant for any given cell size $\alpha \geq 2R$, and given node deployment, where $\mu_\alpha$: maximum number of edges on the same frequency in any cell that can be scheduled simultaneously by any algorithm.
**Evaluation**

### Parameters

UDG on a region of size $200 \times 200$ with transmission radius $R = 25$

### Shortest Path Tree

<table>
<thead>
<tr>
<th>Number of Nodes</th>
<th>Schedule Length</th>
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<tbody>
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<td>5</td>
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<tr>
<td>200</td>
<td>10</td>
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<td>300</td>
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<td>600</td>
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<tr>
<td>700</td>
<td>35</td>
</tr>
<tr>
<td>800</td>
<td>40</td>
</tr>
</tbody>
</table>

**Schedule length decreases with increasing number of frequencies**

**However, with diminishing returns**

**Structure of the routing tree (high node degree) may lead to bottlenecks**
Approximation on Disk Graphs
Approximation on Disk Graphs

Notations

- **\( r(u) \):** Transmission range of node \( u \)
- **\( \ell(e) \):** Length of edge \( e \)
- **\( I(e) \):** Set of edges that are either adjacent to \( e \) or form a secondary conflict with \( e \)
- **\( I_{\geq}(e) \):** Subset of \( I(e) \) that have larger disks than \( e \)

\[
I_{\geq}(e) = \{ e' = (u', v') : e' \in I(e), \ell(e') \geq \ell(e) \}\]
Algorithm $A_{DG}$ Overview

- **Phase 1**: Assign frequencies such that the maximum number of edges interfering with any given edge is minimized.
  - **0–1 Integer Linear Program**: Define indicator variables $X_{vk}$ for edge $e = (u, v)$ as:
    
    $$X_{vk} = \begin{cases} 
    1, & \text{if receiver } v \text{ is assigned frequency } f_k \\
    0, & \text{otherwise} 
    \end{cases}$$

    A frequency assignment is a 0–1 assignment to $X_{vk}$, $\forall e$, $\forall f_k$. 
Algorithm $A_{DG}$ Overview

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    \end{cases}
    \]
    A frequency assignment is a 0 − 1 assignment to $X_{vk}, \forall e, \forall f_k$.

- **Phase 2**: Sort edges in non-increasing order of lengths, and assign smallest available slots starting from the largest.
Approximation on Disk Graphs

\textbf{0-1 Integer Linear Program of Frequency Assignment}

\begin{align*}
\text{Minimize} \quad & \lambda \\
\text{subject to} \quad & : \sum_{v'} n(e, v') X_{vk} \leq \lambda \quad (1) \\
& : \sum_{f_k} X_{vk} = 1, \quad (2) \\
& : X_{vk} \in \{0, 1\} \quad (3)
\end{align*}

- Solve the Linear Relaxation (LP) by modifying constraint (3)
- **Randomized Rounding**: Assign $Y_{vk} = 1$ with probability $X^*_{vk}$, where $X^*_{vk}$ are the optimal fractional LP solutions, and $Y_{vk}$ are the new integral random variables.
**Lemma**

Let $Y_{vk}$ be the rounded solution, as described above. Then,

$$\max_{e,f_k} \left\{ \sum_{e'=(u',v') \in I_{\geq}(e)} Y_{v'k} \right\} = O(\Delta(T) \log n \cdot \lambda^*),$$

with probability at least $(1 - 1/n)$.

Proof: Apply Chernoff bound.
Lemma

Let $Y_{vk}$ be the rounded solution, as described above. Then,

$$\max_{e,f_k} \left\{ \sum_{e'=(u',v') \in I \geq (e)} Y_{v'k} \right\} = O(\Delta(T) \log n \cdot \lambda^*),$$

with probability at least $(1 - 1/n)$.

Proof: Apply Chernoff bound.

Theorem

The schedule constructed by time slot assignmentt strategy along with the fre- quency assignment using the above randomized rounding procedure results in a schedule of length $O(\Delta(T) \cdot \log n)$ times the optimum.
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4. Conclusions
Realistic Interference Model

Limitations of a Graph-Based Model (Protocol Model)

- Interference is *binary* and stops abruptly beyond a distance - idealizes physical laws
- Fails to capture *cumulative interference* from (many) far-away transmitters
- Sometimes *pessimistic* decisions and *conflicting* schedules
SINR Model (Physical Model)

For a given frequency $f$, the received signal power from sender $s_i$ to its intended receiver $r_i$ is:

$$P_{r_i}(s_i) = \frac{P}{d(s_i, r_i)^\alpha},$$

where $\alpha \in [2, 6]$ is called the path-loss exponent; value depends on external conditions.
Realistic Interference Model

**SINR Model (Physical Model)**

For a given frequency $f$, the received signal power from sender $s_i$ to its intended receiver $r_i$ is:

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where $\alpha \in [2, 6]$ is called the path-loss exponent; value depends on external conditions.

A transmission from sender $s_i$ to its intended receiver $r_i$ is successful if the ratio, $SINR_{r_i}$, of the received signal power to the cumulative interference plus noise $\mathcal{N}$ at $r_i$ is more than a hardware-dependent threshold $\beta$:

$$SINR_{r_i} = \frac{P}{I_{r_i} + \mathcal{N}} = \frac{P}{\sum_{j \neq i} \frac{P}{d(s_j, r_i)^\alpha} + \mathcal{N}} \geq \beta$$
Realistic Interference Model

Parameters
- CC2420 radio parameters with receiver sensitivity $-95$ dB
- Path-loss exponent $\alpha = 3.5$, transmit power $-6$ dB

Observations
- For a given frequency, the amount of conflict increases as the network gets denser, reaching almost 40% for densest deployment with a single frequency.
- With multiple frequencies, the amount of conflict is much less.

Percentage of Conflicting Schedules: SINR vs. Graph-based Model
Problem Formulation in SINR

**Joint Frequency and Time Slot Assignment**

Given a connected network of arbitrarily deployed nodes, $K$ orthogonal frequencies, and a routing tree $T = (V, E_T)$ rooted at the sink $s \in V$

- Assign a frequency to each receiver in $T$
- Assign a time slot to each edge in $T$

such that the schedule length is minimized, subject to

$$SINR_{r_i} \geq \beta$$

at every receiver $r_i$
Approach

**Link Diversity**

Classify the edges in $E_T$ based on their lengths $\ell_i = d(s_i, r_i)$. Link diversity is the number of magnitude of different lengths

$$g(E_T) = |\{m | \exists e_i, e_j \in E_T : \lfloor \log (\ell_i / \ell_j) \rfloor = m\}|$$

Usually a small constant in practice, but theoretically can be equal to the number of nodes
Link Diversity

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Usually a small constant in practice, but theoretically can be equal to the number of nodes

Overall Idea

- **Normalize** (w.l.o.g) the minimum edge length to one
- Let

$$E = \{ E_0, \ldots, E_{\log(e_{max})} \}$$

denote the set of non-empty length classes, where $E_k$ is the set of edges of lengths in $[2^k, 2^{k+1})$, and $e_{max}$ is the longest edge
- **Partition** the problem into disjoint length classes, and process edges in each class separately
For Length Class $E_k$

- Divide the 2-D region into a set $C_k$ of square grids of length $\eta_k = \delta \cdot 2^k$, for some constant $\delta$ (will choose $\delta$ later)
- **Frequency Assignment:**
  - For each cell in $C_k$, run *FrequencyGreedy* on the receivers
- **Time Slot Assignment:**
  - Four color the cells in $C_k$
  - Can we still reuse time slots across non-adjacent cells of the same color? If so, how many?
Approximation Algorithm in SINR

**Theorem**

For cells of the same color in $C_k$, we can simultaneously schedule at most one edge in $E_k$ from every non-adjacent cell that lies on the same frequency, so long as the cell size satisfies:

$$\eta_k = \delta \cdot 2^k, \quad \text{for } \delta = 4 \left[ 8\beta \cdot \frac{\alpha - 1}{\alpha - 2} \right]^{\frac{1}{\alpha}},$$

where $\beta \geq 1$ is the SINR threshold, and $\alpha > 2$ is the path-loss exponent.
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where $\beta \geq 1$ is the SINR threshold, and $\alpha > 2$ is the path-loss exponent.

**Proof Sketch**

- Consider one specific edge $e_i = (s_i, r_i) \in E_k$ in any cell $c \in C_k$
- Received power at $r_i$

$$P_{r_i}(s_i) = \frac{P}{\ell_i^\alpha} \geq \frac{P}{2^{\alpha(k+1)}}, \quad \therefore 2^k \leq \ell_i < 2^{k+1}$$

- Show that the cumulative interference $I_{r_i}$ caused by concurrent transmissions from all non-adjacent cells is still within $\beta$
Consider edge $e_i = (s_i, r_i) \in E_k$. 

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<th>$\gamma_1$</th>
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</tbody>
</table>
Consider edge $e_i = (s_i, r_i) \in E_k$.

At most 8 from layer 1.
Consider edge $e_i = (s_i, r_i) \in E_k$

- At most 8 from layer 1
- At most 16 from layer 2, ..., at most $8q$ from layer $q$
- Add up all the interferences up to layer $\infty$
Computing Cumulative Interference $I_{r_i}$

- From transmitter $s_j$ in a layer 1 non-adjacent cell of the same color

$$I_{r_i}(s_j) = \frac{P}{\ell_{ij}^\alpha} \leq \frac{P}{[2^k(\delta - 2)]^\alpha}, \quad \therefore \ell_{ij} \geq \delta \cdot 2^k - 2^{k+1}$$
Computing Cumulative Interference $I_{r_i}$

- From transmitter $s_j$ in a layer 1 non-adjacent cell of the same color

$$I_{r_i}(s_j) = \frac{P}{\ell_{ij}^\alpha} \leq \frac{P}{[2^k(\delta - 2)]^\alpha}, \quad \therefore \ell_{ij} \geq \delta \cdot 2^k - 2^{k+1}$$

- In general, $\ell_{ij} \geq 2^k \{(2q - 1)\delta - 2\}$ in layer $q$
Computing Cumulative Interference $I_{r_i}$

- From transmitter $s_j$ in a layer 1 non-adjacent cell of the same color

$$I_{r_i}(s_j) = \frac{P}{\ell_{ij}^\alpha} \leq \frac{P}{[2^k(\delta - 2)]^\alpha}, \quad \therefore \ell_{ij} \geq \delta \cdot 2^k - 2^{k+1}$$

- In general, $\ell_{ij} \geq 2^k \{(2q - 1)\delta - 2\}$ in layer $q$

- Adding up

$$I_{r_i} \leq \sum_{q=1}^{\infty} \frac{8qP}{[2^k \{(2q - 1)\delta - 2\}]^\alpha} = \frac{8P}{2^{(k-1)\alpha} \delta^\alpha} \cdot \frac{\alpha - 1}{\alpha - 2} \leq \frac{P_{r_i}(s_i)}{\beta}$$
Theorem: $O(g(E_T))$ Approximation

Given a routing tree $T$ on an arbitrarily deployed connected network in 2-D, and $K$ orthogonal frequencies, the algorithm gives a $O(g(E_T))$ approximation on the optimal schedule length.
Approximation Algorithm in SINR

**Theorem**: $O(g(E_T))$ Approximation

Given a routing tree $T$ on an arbitrarily deployed connected network in 2-D, and $K$ orthogonal frequencies, the algorithm gives a $O(g(E_T))$ approximation on the optimal schedule length.

**Proof Sketch**

- Choice of the critical cell $\hat{c}$ in which the number of edges on the same frequency $L_{\hat{c}}^m \ast$ is maximum across all length classes.
- Show that $OPT$ will take at least $L_{\hat{c}}^m \ast / \mu$ slots, for

  $$\mu = \frac{\left(2(\sqrt{2} \cdot \delta + 1)\right)^\alpha}{\beta}$$

- Show by contradiction that $OPT$ cannot schedule $\mu + 1$ edges on the same frequency in any cell.
Algorithmic Aspects of Throughput-Delay Performance for Fast Data Collection in Sensor Networks

Multi-Channel Scheduling in SINR

Approximations in SINR

Approximation Algorithm in SINR

Proof Sketch (cobming together)

\[
\Gamma \leq 4 \cdot \max_{k,c} \{|\gamma_{c \in C_k}|\} \cdot g(E_T)
\]

\[
\leq 8 \cdot \max_{k,c} \left\{ L_{c \in C_k}^{\phi} \right\} \cdot g(E_T)
\]

\[
\leq 8 \cdot \max_{k,c} \left\{ \left( \frac{4}{3} - \frac{1}{3K} \right) \cdot L_{c \in C_k}^{m^*} \right\} \cdot g(E_T)
\]

\[
\leq 8\mu \cdot \left( \frac{4}{3} - \frac{1}{3K} \right) \cdot g(E_T) \cdot OPT
\]

\[
= O \left( g(E_T) \right)
\]
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   - Bicriteria Formulation
   - Optimal Spanning Trees

4. Conclusions
Properties of Spanning Trees

Shortest Path Tree (shallow and fat)

- High node degree $\Rightarrow$ low throughput
- Few hops $\Rightarrow$ low delay
Algorithmic Aspects of Throughput-Delay Performance for Fast Data Collection in Sensor Networks

### Throughput-Delay Trade-off

### Motivation

#### Properties of Spanning Trees

**Shortest Path Tree** *(shallow and fat)*
- High node degree $\Rightarrow$ low throughput
- Few hops $\Rightarrow$ low delay

**Minimum Interference Tree** *(weighted MST)*
- [Mobihoc '04] *(deep and skinny)*
- $w(u, v)$: no. of nodes covered by the union of two disks centered at $u$ and $v$, each of radius $|uv|$
- Low node degree $\Rightarrow$ high throughput
- More hops $\Rightarrow$ high delay
Evaluation of $A_{UDG}$ on SPT & MIT

Parameters

Random geometric graph on a region of size $200 \times 200$ with $R = 25$

Shortest Path Tree

Minimum Interference Tree
Tree Property for Bounded Throughput

**Theorem**

If the maximum node degree of the routing tree is bounded by a constant \( \Delta_C > 0 \), then there exists an algorithm that gives a constant factor \( 8\mu_\alpha \cdot \Delta_C \) approximation on the optimal schedule length, where \( \mu_\alpha > 0 \) is a constant for a given cell size \( \alpha \geq 2R \) and given node deployment.
Tree Property for Bounded Throughput

**Theorem**

If the maximum node degree of the routing tree is bounded by a constant $\Delta_C > 0$, then there exists an algorithm that gives a constant factor $8\mu_\alpha \cdot \Delta_C$ approximation on the optimal schedule length, where $\mu_\alpha > 0$ is a constant for a given cell size $\alpha \geq 2R$ and given node deployment.

**Properties**

- Degree-bounded spanning trees? Minimum Degree Spanning Tree is NP-hard [SODA’92]
- Is constant factor approximation on schedule length and delay achievable?
Bicriteria Formulation

**Bounded-Degree Minimum-Radius Spanning Tree**

\[ \mathcal{P} = (\text{Degree}, \text{Radius}, \text{Spanning Tree}) \]

- **Radius**: \( R(T) \): Maximum hop distance from any node to sink in \( T \)
- **\( \Delta^* \)**: Budget on max degree

**Goal**: Find a spanning tree of **min radius** subject to **max degree** \( \Delta^* \)
Bounded-Degree Minimum-Radius Spanning Tree

\( \mathcal{P} = (\text{Degree, Radius, Spanning Tree}) \)

Radius: \( R(T) \): Maximum hop distance from any node to sink in \( T \)

\( \Delta^* \): Budget on max degree

Goal: Find a spanning tree of min radius subject to max degree \( \Delta^* \)

Approximation Definition

Algorithm \( \mathcal{A} \) is an \((\alpha, \beta)\)-approximation to a bicriteria optimization problem \( \mathcal{P} = (\text{Degree, Radius, Spanning Tree}) \) if:

- \( \Delta(T) \leq \alpha + \Delta^* \)
- \( R(T) \leq \beta \cdot R^*(T) \)

where \( R^*(T) \) is the min radius of any spanning tree \( T \) whose max degree is \( \Delta^* \).
Algorithm Overview

**Algorithm** $A_{BDMRST}$

Three phases:

1. Construct a **Global Backbone Tree of low radius**.
2. Construct **degree-bounded Local Spanning Trees**.
3. Merge the local spanning trees and the global backbone tree.
Algorithm Overview

Global Backbone Tree
Algorithmic Aspects of Throughput-Delay Performance for Fast Data Collection in Sensor Networks

Throughput-Delay Trade-off

Optimal Spanning Trees

Algorithm Overview

**Global Backbone Tree**

**Local Spanning Tree**
Properties of Spanning Trees

Bounded-Degree Minimum-Radius Spanning Tree, $\Delta^* = 4$
Theorem

Algorithm $A_{BDMRST}$ gives a constant factor $(\alpha, \beta)$ bicriteria approximation for the Bounded-Degree Minimum-Radius Spanning Tree, where $\alpha = 10$ and $\beta = 5$. 
Algorithm $A_{BDMRST}$ gives a constant factor $(\alpha, \beta)$ bicriteria approximation for the Bounded-Degree Minimum-Radius Spanning Tree, where $\alpha = 10$ and $\beta = 5$. 

**Schedule Length**

**Max Delay**
Outline

1. Maximizing Convergecast Throughput
   - Motivation and Preliminaries
   - Scheduling with Unlimited Frequencies
   - Approximations on Unit Disk Graphs
   - Approximations on General Disk Graphs

2. Multi-Channel Scheduling in SINR
   - Motivation and Interference Models
   - Problem Formulation and Approach
   - Approximations in SINR

3. Throughput-Delay Trade-off
   - Motivation
   - Bicriteria Formulation
   - Optimal Spanning Trees

4. Conclusions
Contributions

- Maximizing aggregated convergecast throughput using **multi-channel scheduling**
  - Constant factor approximation on UDG
  - $\Delta(T) \cdot \log(n)$ approximation on DG
  - $O(g(\mathcal{E}_T))$ approximation in SINR
- **Throughput-delay** trade-off in the same optimization framework
  - Bicriteria formulation
  - Constant factor approximation on constructing bounded-degree minimum-radius spanning trees
- Efficient, distributed **topology control in 3-D**
  - Simple orthographic projection based approach extending 2-D results
  - Robust spherical Delaunay triangulation based algorithm
Publications

Book Chapters


Journal Papers


Conclusions

Publications

Conference Papers (reverse chronologically)


Manuscripts


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