Topography Control in 3-Dimensional Networks & Algorithms for Multi-Channel Aggregated Convergecast

Amitabha Ghosh†  Yi Wang†  Ozlem D. Incel‡  V.S. Anil Kumar◊
Bhaskar Krishnamachari†
†Dept. of Electrical Engineering, University of Southern California
‡Dept. of Computer Science, University of Twente, Netherlands
◊Bio-Informatics Institute, Dept. of Computer Science, Virginia Tech
{amitabhg, wangyi, bkrishna}@usc.edu, o.durmaz@cs.utwente.nl,
akumar@vbi.vt.edu
Why Topology Control?

Goal: Reduce Power Consumption

Given a network connectivity graph, compute a subgraph with certain properties: connectivity, spanner, low interference, etc.

Leads to: High energy consumption, High interference, Low throughput, Partitioned network
Why Topology Control?

Benefits

1. Global connectivity
2. Low energy consumption
3. Low interference
4. High throughput

Problem

To find optimal transmission power levels using local information such that network connectivity is maintained.
3-Dimensional Wireless Networks

Challenges

1. Node density if prohibitively high to guarantee connectivity
   - Critical avg. node degree $(d)$: 15 in 2D, 34 in 3D for $n = 1000$ [Poduri, EmNets ‘06]
2. Many 2D algorithms not readily extensible to 3D (e.g., geographic routing)
3. No ordering of nodes based on angular information
4. High complexity

Applications

Structural health monitoring, Marine-life monitoring (Underwater networks)
Cone Based Topology Control

2D CBTC

1. Global connectivity from local geometric constraints
   [Wattenhofer, Infocom ‘01], [Li Li, PODC ‘01]
2. Maximum Power Graph $G = (V, E)$ is connected
3. Receivers can determine direction of senders, needs node ordering
4. Complexity $O(d \log d)$, $d =$ average node degree

Main Result: The $\theta$ constraint

If every node adjusts its power level so that there exists at least one neighbor at every $\theta = 2\pi/3$ sector around itself, then the network is connected.
Cone Based Topology Control

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Part I: Topology Control in 3D Wireless Networks
Part II: Algorithms for Multi-Channel Aggregated Convergecast

Introduction
Related Works
Solution
Simulation Results

Cone Based Topology Control

2D CBTC

1. Global connectivity from local geometric constraints
   [Wattenhofer, Infocom ‘01], [Li Li, PODC ‘01]
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4. Complexity $O(d \log d)$, $d = \text{average node degree}$

Main Result: The $\theta$ constraint

If every node adjusts its power level so that there exists at least one neighbor at every $\theta = \frac{2\pi}{3}$ sector around itself, then the network is connected.
Cone Based Topology Control

3D CBTC

1. Each node increases its power level until there is at least one neighbor at every 3D cone of apex angle $\theta = 2\pi/3$ around it

   [Bahramgiri, ICCCN ‘05, Wireless Networks ‘06]

2. Assumes directional information

3. High time complexity $O(d^3 \log d)$
Approach

2 Phases

1. **Phase 1**
   - Use Multi-Dimensional Scaling (MDS) to find relative location maps for each node's neighbors when they use $P_{max}$
     - **Input:** Pairwise distances
     - **Output:** Relative locations

2. **Phase 2**
   - Using orthographic projections (convert the 3D problem into similar problems in 2D)
     - OR
   - Using Spherical Delaunay Triangulation (SDT)

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Amitabha Ghosh† Yi Wang† Ozlem D. Incel‡ V.S. Anil Kumar

Dept. of Electrical Engineering, University of Southern California

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Orthographic Projection

Algorithm

1. Each node starts with minimum tx power
2. For a given tx power, project the neighbors on xy, yz, and zx plane
   - If any of the 3 planes does not satisfy the $\theta = \frac{2\pi}{3}$ constraint, increase power to the next level
   - Else STOP, settle with current power
3. Run 2D CBTC on each plane
4. Go back to Step 2 unless $P_{\text{max}}$ is reached

Intuition

Satisfying 2D CBTC on 3 planes $\Rightarrow$ non-empty 3D cones with apex angle $\frac{2\pi}{3}$
**Lemma**

Consider the projections of a node \( u_i \)ˈs neighbor on the three planes \( xy, yz, \) and \( zx \).

*If there exists at least one empty sector of angle \( \theta \) in any one of the planes, then there exists an infinite number of empty 3D cones with apex angle \( \theta \) around \( u_i \).*

**Connectivity?**

Theoretically NO, practically YES in almost all cases
Spherical Delaunay Triangulation

Delaunay Triangulation

1. Dual of a Voronoi diagram
2. Given $N$ points in 2D, a Voronoi diagram tessellates the plane into $N$ convex polygons such that any point in a polygon is closest to the point that lies in that polygon
3. Empty circum-circle property

Spherical triangles, and spherical caps
Spherical Delaunay Triangulation

SDT using Quickhull for 100 points randomly distributed on the surface of a sphere of radius 50
Algorithm SDT

1. Each node starts with minimum tx power
2. For a given tx power, project the neighbors on the spherical surface
3. Construct Delaunay triangulation on the surface of the sphere
4. Calculate the area of the (empty) spherical caps
5. If any cap area is $> 2.7R^2$, increase the power to next level; go to Step 2
6. Else, stop and settle down with current power

Lemma

If none of the spherical caps have a surface area greater than $2.7R^2$, the network is at least one-connected
Probability of network connectivity as the $\theta$ constraint is satisfied on 1, 2, or all 3 planes for 100 nodes.
Node deg of max power graph vs. final topology: $n = 200$, $P_{\text{max}} = 40$
Part I: Topology Control in 3D Wireless Networks
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SDT

Avg node deg vs. network size and max power

Comparison of CPU execution times with $P_{\text{max}} = 40$

- Amitabha Ghosh†
- Yi Wang†
- Ozlem D. Incel‡
- V.S. Anil Kumar
- Bhaskar Krishnamachari††

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Part II: Algorithms for Multi-Channel Aggregated Convergecast

Joint work: Amitabha Ghosh, Ozlem Durmaz Incel, V. S. Anil Kumar, Bhaskar Krishnamachari
Part I: Topology Control in 3D Wireless Networks
Part II: Algorithms for Multi-Channel Aggregated Convergecast

Introduction
Assignment on General Graphs
Assignment on Unit Disk Graphs: Approximation Algorithms
Evaluation

An Illustration

Single Frequency

Setting

1. Half-duplex transceiver: cannot transmit and receive simultaneously
2. Single transceiver: cannot receive multiple packets simultaneously
3. Interference causes packet loss
4. Receiver-based frequency assignment

Half-duplex transceiver: cannot transmit and receive simultaneously
Single transceiver: cannot receive multiple packets simultaneously
Interference causes packet loss
Receiver-based frequency assignment
An Illustration

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An Illustration

Single Frequency

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An Illustration

Single Frequency

In the diagram, nodes are connected with different colors indicating different channels. Each node is labeled with a letter (a, b, c, d, e, f, g) and numbers (1, 2, 3, 4, 5, 6) to represent the connections and channel assignments.
Single Frequency

- \( f_1 \)
- \( f_1 \)
- \( f_1 \)

Nodes: a, b, c, d, e, f, g

Connections:
- a to b, weight 1
- a to d, weight 1
- b to c, weight 5
- b to f, weight 4
- c to g, weight 2
- e to f, weight 3

This diagram illustrates a network topology with single frequency assignment.
An Illustration

Single Frequency

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An Illustration

Single Frequency

Multiple Frequencies

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3Bio-Informatics Institute, Dept. of Computer Science, Virginia Tech

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Frame 1: s sends data to b, which aggregates and forwards to c.
Frame 2: a sends data to d, which aggregates and forwards to e.

Amitabha Ghosh† Yi Wang† Ozlem D. Incel‡ V.S. Bhaskar Krishnamachari††
### An Illustration

**Multiple Frequencies**

```
  s  \\
 /   \\
|    \\
a----b----c
|     |     |
|     |     |
|     |     |
d----e----f
|     |     |
|     |     |
|     |     |
g
```

**Frame 1 (Aggregated data)**

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<tbody>
<tr>
<td>s</td>
<td>c</td>
<td>a, d</td>
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<td>1</td>
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An Illustration

Question
What is the fastest rate at which aggregated data can be collected from the network? (minimizing the schedule length)
**Optimal Frequency Assignment**

**MFAP: Minimum Frequency Assignment Problem**

Given a spanning tree $T$ on an arbitrary graph $G = (V, E)$, find the minimum number of frequencies that could be assigned to the receivers of $T$ such that all the interfering link constraints are removed.

![Figure: Interfering edge structure](image)

**Theorem**

*MFAP is NP-complete.*

*Proof: Reduction from Vertex Color.*
Lemma

Construct a constraint graph $G_C = (V_C, E_C)$ from $G = (V, E)$ as follows: For each receiver $v_i \in G$, create a vertex $u_i \in G_C$. Create an edge between two such vertices $u_i$ and $u_j$ if their corresponding receivers are part of an interfering edge structure. Then, the number of frequencies that will remove all the interfering link constraints is: $K_{\text{max}} \leq \Delta(G_C) + 1$, where $\Delta(G_C)$: maximum degree in $G_C$. 
BFS Time Slot Assignment

Theorem

Algorithm BFS-TIME-SLOT-_ASSIGNMENT on a tree gives the minimum schedule length equal to $\Delta(T)$. 

Diagram:

```
  s
 /  \
 f2  f1
 /    /
 a    b
 /  \
 f2  f1
 /    /
 c    d
 /  \
 f2  f2
 //   /  \
 i    f1  e
 /     /  \
 f3   f3  f1
 /  \
 f1  g
 /  \
 2  f3
 /  \
 1  h
 /  \
 3  p
 /  \
 1  m
 /  \
 2  n
 /  \
 3  l
```
MFMTSP: Multiple-Frequency Minimum Time Scheduling Problem

Given a spanning tree $T$ on an arbitrary graph $G = (V, E)$ and a constant number $q$ of frequencies, find an assignment of frequencies and time slots such that the schedule length is minimized.

Theorem

**MFMTSP is NP-complete.**

Proof: Reduction from Vertex Color.
Lemma

Suppose $\gamma_{c_i}$ denote the set of time slots to schedule the edges in cell $c_i$. Then, the minimum schedule length $\Gamma$ for the whole network is:

$$\Gamma \leq 4 \cdot \max_{c_i} \{|\gamma_{c_i}|\}, \forall \alpha \geq 2.$$
Load-Balanced Frequency Assignment

\[ R_{c_i} = \{ v_1, \ldots, v_n \} \] set of receivers on \( T \)

\[ m : R_{c_i} \rightarrow \{ f_1, \ldots, f_K \} \] mapping of frequencies to the receivers

Define **load** on \( f_k \) under \( m \) in \( c_i \) as:

\[
l^m_{c_i}(f_k) = \sum_{v_j \in R_{c_i}, m(v_j) = f_k} \deg^{in}(v_j)
\]

Then, a **load-balanced frequency assignment** \( m^* \) in \( c_i \) is

\[
m^* = \arg \min_m \max_{f_k} \{ l^m_{c_i}(f_k) \}
\]

**Lemma**

*Load-balanced frequency assignment is NP-complete.* [Garey, Johnson]
Algorithm **FrequencyGreedy**

(1) Sort the nodes in $R_{c_i}$ in non-increasing order of their in-degrees. Let this order be: $\text{deg}^i(v_1) \geq \text{deg}^i(v_2) \geq \ldots \geq \text{deg}^i(v_n)$.

(2) Starting from $v_1$, assign each successive node a frequency from $\{f_1, \ldots, f_K\}$ that has the least load on it so far, breaking ties arbitrarily.

**Lemma**

Algorithm **FrequencyGreedy** in each cell $c_i$ gives a $(4/3 - 1/3K)$ approximation on the optimal load $L^m_{c_i}$.

*Proof: Follows from Graham’s algorithm for job scheduling on identical machines according to longest processing time first (LPT).*
Lemma

If $L_{c_i}^\phi$ denote the load on the maximally loaded frequency in $c_i$ under mapping $\phi : R_{c_i} \rightarrow \{f_1, \ldots, f_K\}$ achieved by $\text{FrequencyGreedy}$, then any greedy time slot assignment can schedule all the edges in $c_i$ within $2L_{c_i}^\phi$ time slots, i.e.,

$$|\gamma_{c_i}| \leq 2L_{c_i}^\phi.$$
A Constant Factor Approximation

Theorem

Given a tree $T$ on a UDG $G$, and $K$ frequencies, there exists an algorithm $G$ that achieves a constant factor $8\mu_\alpha \left(\frac{4}{3} - \frac{1}{3}K\right)$ approximation on the optimal schedule length, where $\mu_\alpha > 0$ is a constant for a given cell size $\alpha \geq 2$.

Proof Sketch: $G$ consists of 2 phases:

(1) Run $\text{FrequencyGreedy}$ in each cell $c_i$

(2) Run any greedy time slot assignment scheme, e.g., greedily schedule a maximal number of edges at each slot

Lower bound on OPT:

$$\Gamma_{OPT} \geq \max_{c_i} \left\{ L_{c_i}^m \right\} / \mu_\alpha$$

$$\Gamma_G \leq 4 \cdot \max_{c_i} \left\{ |\gamma_{c_i}| \right\} \leq 8 \cdot \max_{c_i} \left\{ L_{c_i}^\phi \right\}$$

$$\leq 8 \cdot \max_{c_i} \left\{ (\frac{4}{3} - \frac{1}{3}K) \cdot L_{c_i}^m \right\}$$

$$\leq 8\mu_\alpha \left(\frac{4}{3} - \frac{1}{3}K\right) \cdot \Gamma_{OPT}$$
Given a UDG $G$ and $K$ frequencies, there exists an algorithm $\mathcal{H}$ that achieves a constant factor $8\mu_\alpha \Delta_C$ approximation on the optimal schedule length, where $\mu_\alpha > 0$ is a constant for a given cell size $\alpha \geq 2$, and $\Delta_C > 0$ is a constant.

Proof Sketch:

$V_{c_i}$: set of nodes in $c_i$;
$R_{c_i}$: set of receivers in $c_i$ for an arbitrary $T$

Lower bound on OPT:

$\Gamma_{OPT} \geq \max_{c_i} \left\{ \left\lceil \frac{|V_{c_i}|}{K} \right\rceil \right\} / \mu_\alpha$

Run a **CyclicFrequencyAssignment**: $\psi(v_i) = i \mod K$ if $i \neq qK$, and $K$ otherwise, $q \in \mathbb{N}^+$
Frequency Bounds

Largest Degree First (LDF)

1. Input: Constraint Graph $G_C = (V_C, E_C)$
2. while $V_C \neq \emptyset$ do
3. $u \leftarrow$ vertex with maximum degree in $V_C$
4. Assign the first available frequency to $u$ different from its neighbors
5. $V_C \leftarrow V_C \setminus \{u\}$

Comparison of LDF and $\Delta(G_C) + 1$
Schedule Length

Algorithm $G$ on Shortest Path Tree for Different Network Sizes

Square region of size $200 \times 200$
Maximum transmission range 25
Schedule Length

Algorithm $G$ on Minimum Interference Tree for Different Network Sizes

Square region of size 200 $\times$ 200
Maximum transmission range 25