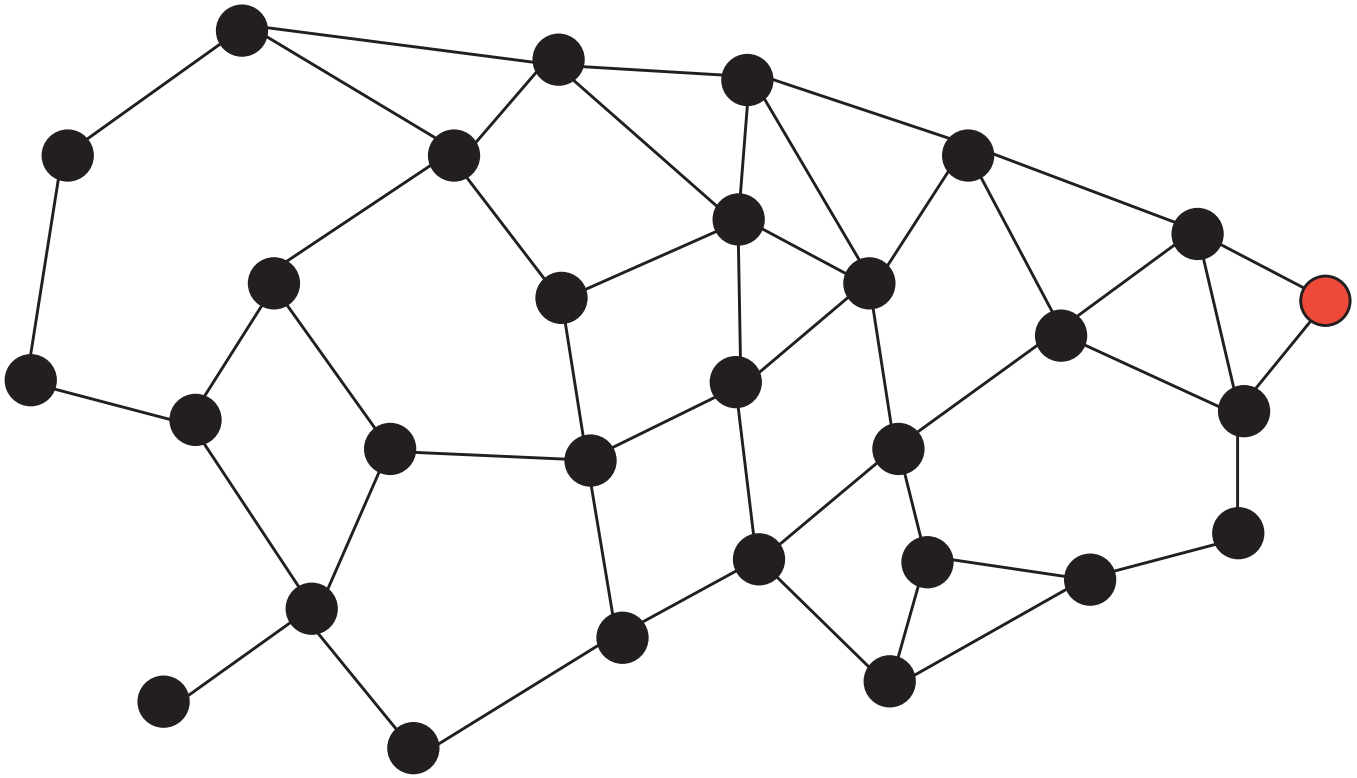
Algorithms for Multi-Channel Aggregated Convergecast in Sensor Networks

Amitabha Ghosh† Ozlem D. Incel‡ V.S. Anil Kumar◇
Bhaskar Krishnamachari†
†Dept. of Electrical Engineering, University of Southern California
‡Dept. of Computer Science, University of Twente, Netherlands
◇Bio-Informatics Institute, Dept. of Computer Science, Virginia Tech
{amitabhg, bkrishna}@usc.edu, o.durmaz@cs.utwente.nl,
akumar@vbi.vt.edu

A Data Aggregation Network



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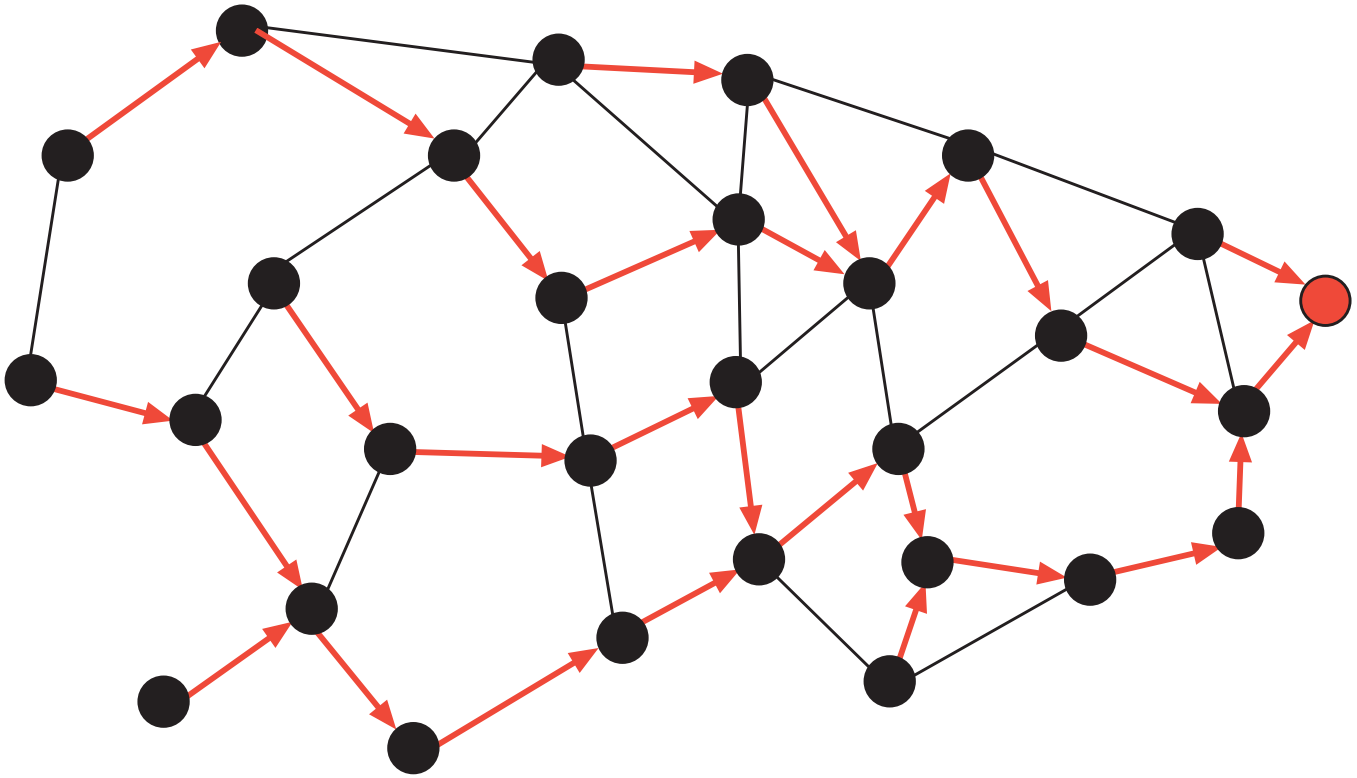
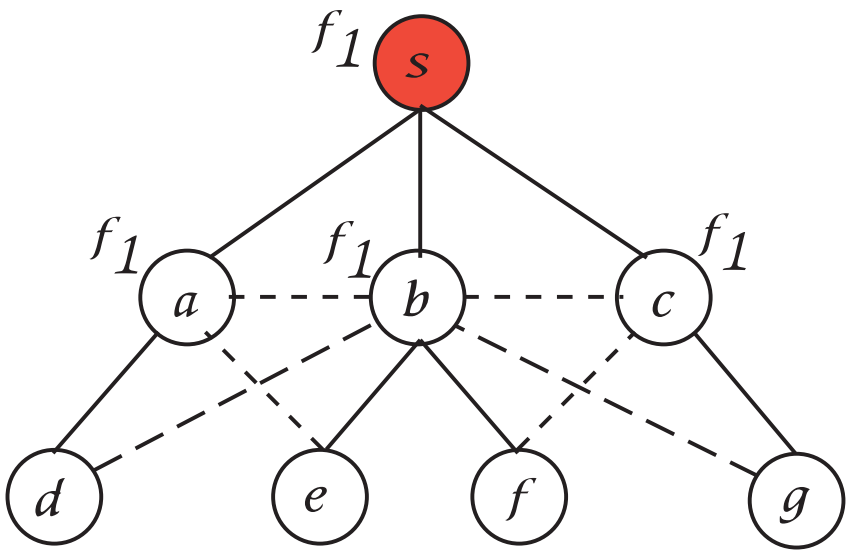


An Illustration



Setting

(1) Half-duplex single transceiver

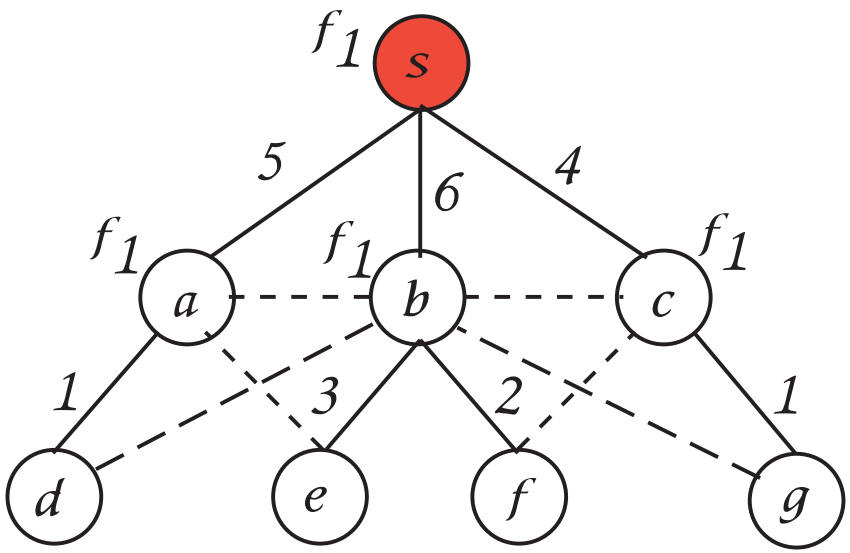
(2) Nodes aggregate pkts from children and transmit only one pkt

(3) TDMA

(4) Interference causes pkt loss

(5) Receiver-based frequency assignment

An Illustration



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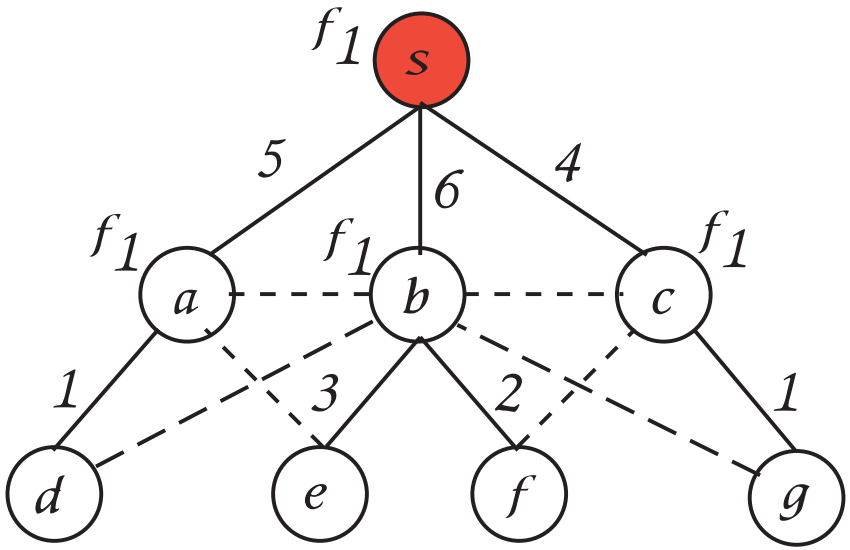

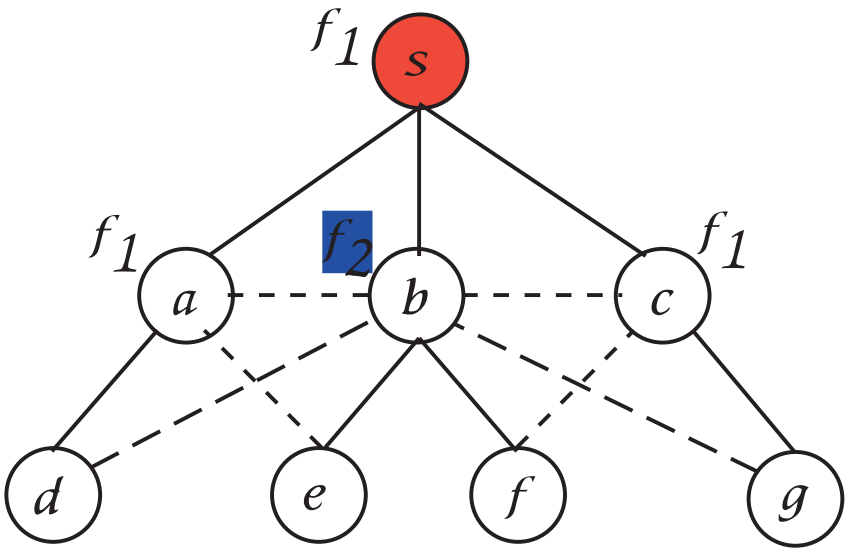

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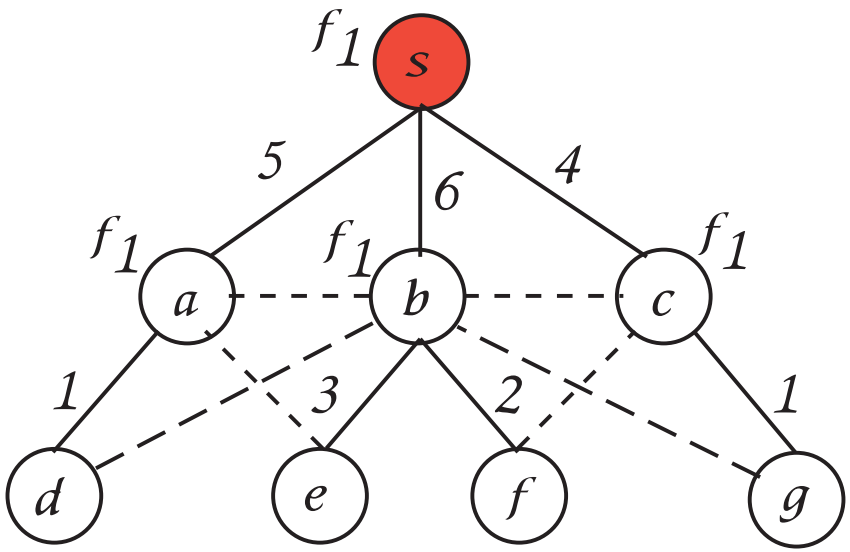
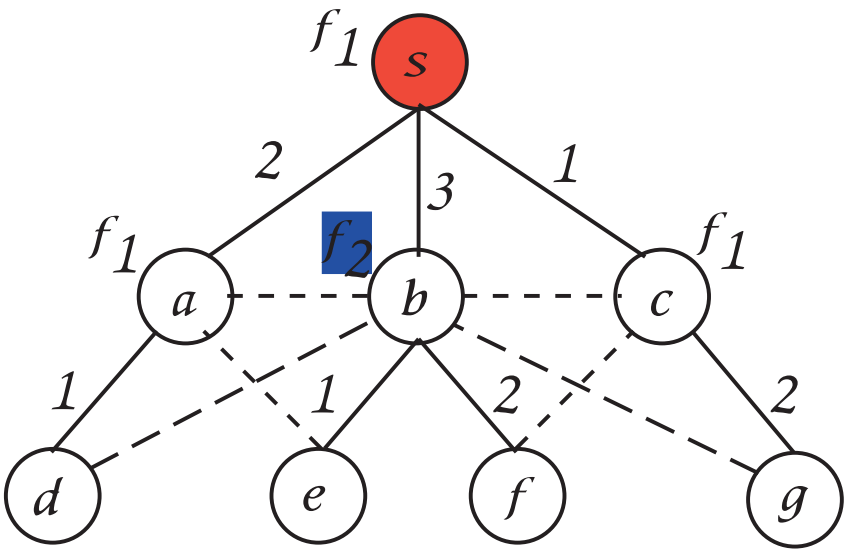




Question

What is the fastest rate at which aggregated data can be collected from the network? (minimizing the schedule length)

	Frame 1			Frame 2		
Receiver	Slot 1	Slot 2	Slot 3	Slot 1	Slot 2	Slot 3
s	c	a,d	b,e,f	c,g	a,d	b,e,f
a	d			d		
b	e	f		e	f	
c		g			g	

Minimum Frequency Assignment Problem

MFAP

Given a spanning tree T on an arbitrary graph $G = (V, E)$, find the minimum number of frequencies that can be assigned to the receivers of T such that all the interfering link constraints are removed.

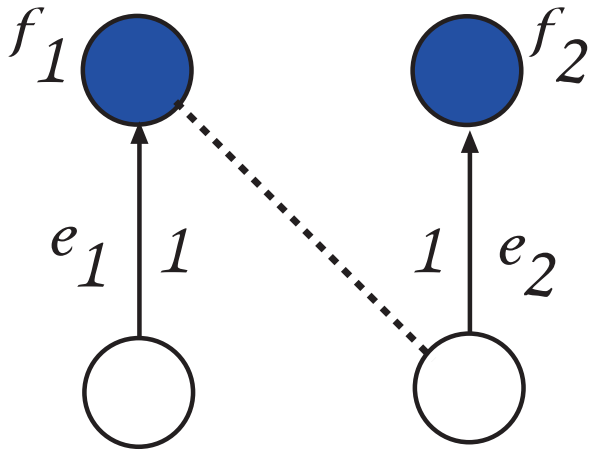


Figure: Interfering edge structure

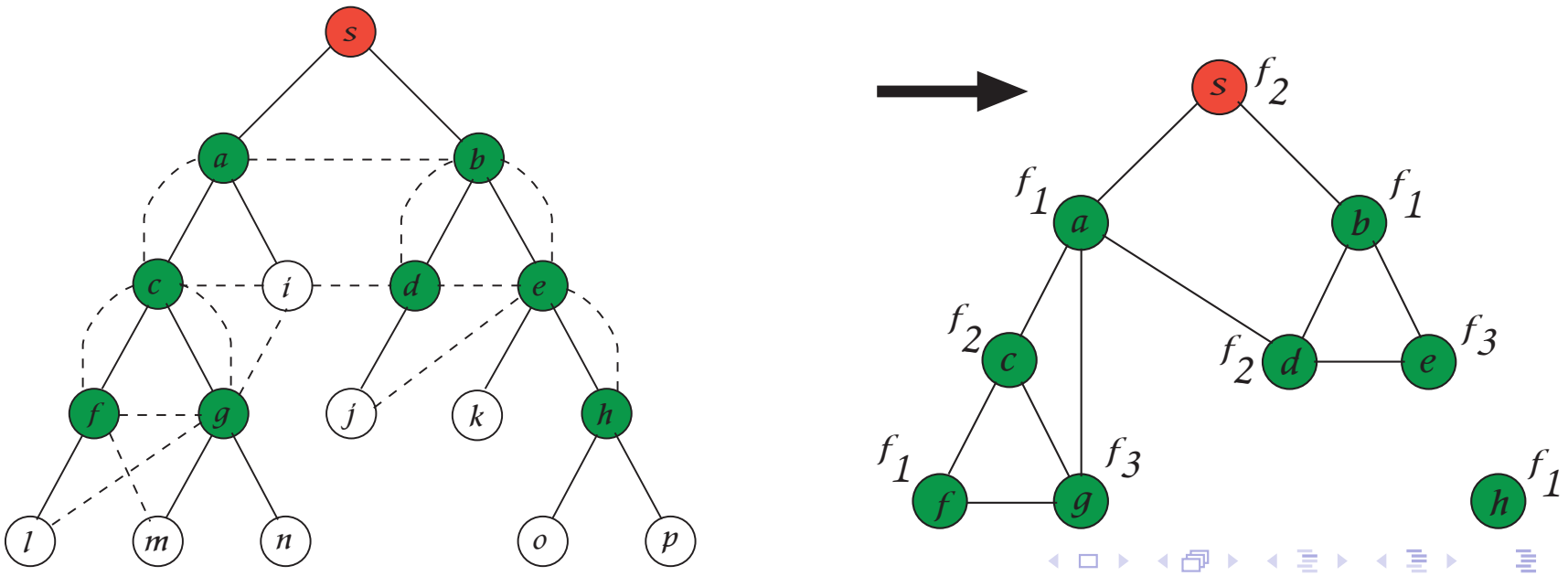
Theorem

MFAP is NP-complete.
Proof: Reduction from Vertex Color.

An Upper Bound

Lemma

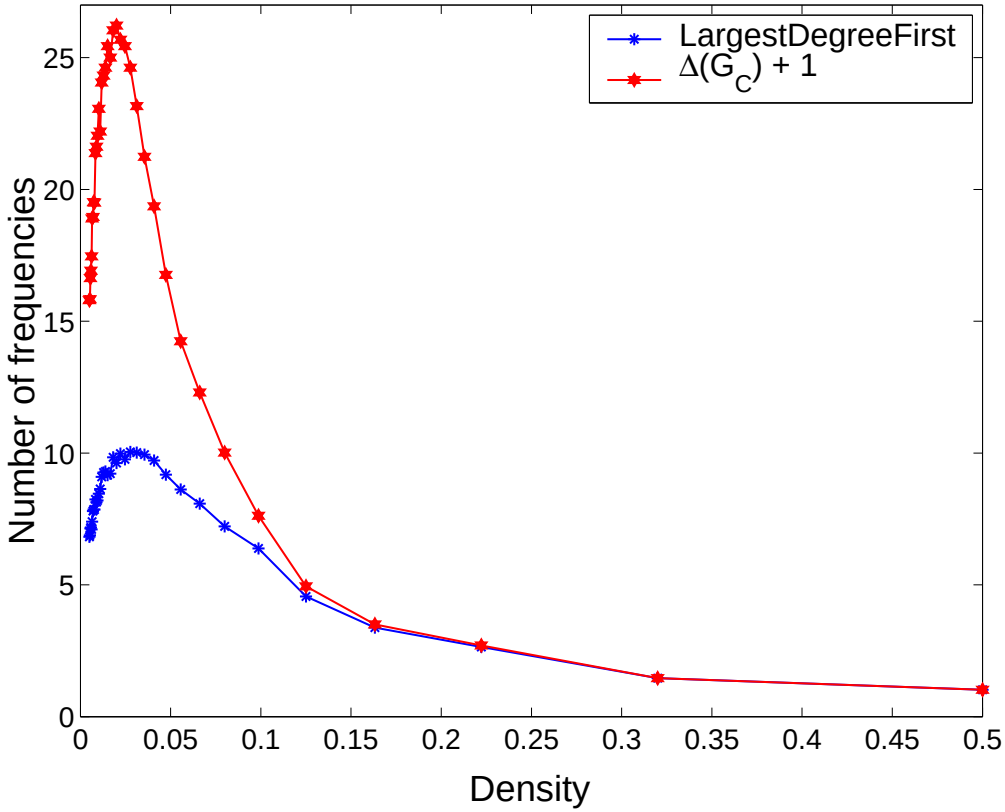
Construct a constraint graph $G_C = (V_C, E_C)$ from $G = (V, E)$ as follows:
For each receiver in G, create a vertex in G_C. Create an edge between two such vertices in G_C if their corresponding receivers in G are part of an interfering edge structure.
Then, the number of frequencies required to remove all the interfering link constraints is: $K_{max} \leq \Delta(G_C) + 1$, where $\Delta(G_C)$: max degree in G_C.



Evaluation: Frequency Bounds

Largest Degree First (LDF)

1. while $V_C \neq \phi$ do
2. $u \leftarrow$ max degree in V_C
3. Assign the first available frequency to u different from its neighbors
4. $V_C \leftarrow V_C \setminus \{u\}$



BFS Time Slot Assignment

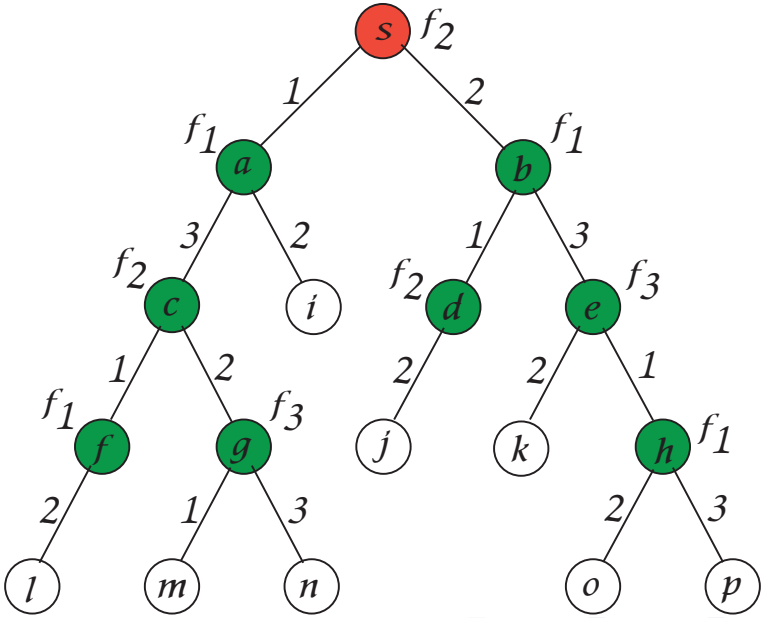
Algorithm

Input: $T = (V, E_T)$
while $E_T \neq \phi$
    $e \leftarrow$ next edge from T in BFS order
    Assign minimum time slot to e
    $E_T \leftarrow E_T \setminus \{e\}$

Theorem

Algorithm BFS-TimeSlotAssignment *on a tree gives the minimum schedule length equal to* $\Delta(T)$*, where* $\Delta(T)$ *is the maximum node degree in* T*.*

Proof by induction on i*:*
$N(i+1) = \max\{N(i), N(i)+1\}$



Multiple-Frequency Minimum Time Scheduling Problem

MFMTSP

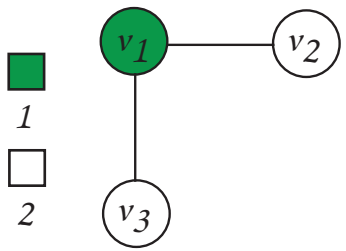
Given a spanning tree T on an arbitrary graph $G = (V, E)$ and q frequencies, find an assignment of frequencies and time slots such that the schedule length is minimized.

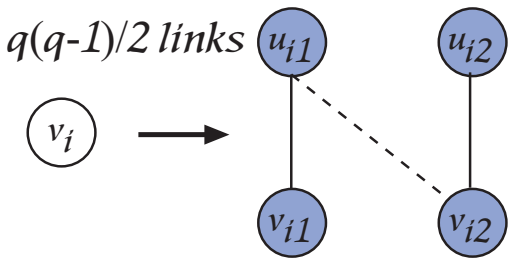
Theorem

MFMTSP is NP-complete.
Proof: Reduction from Vertex Color.

Multiple-Frequency Minimum Time Scheduling Problem

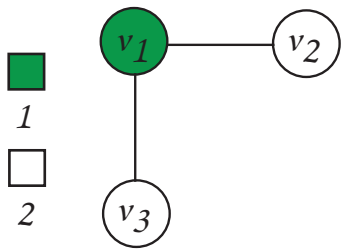
Reduction from Vertex Color.

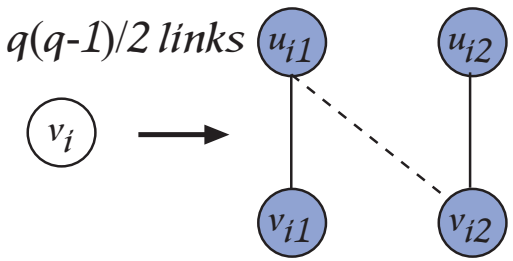




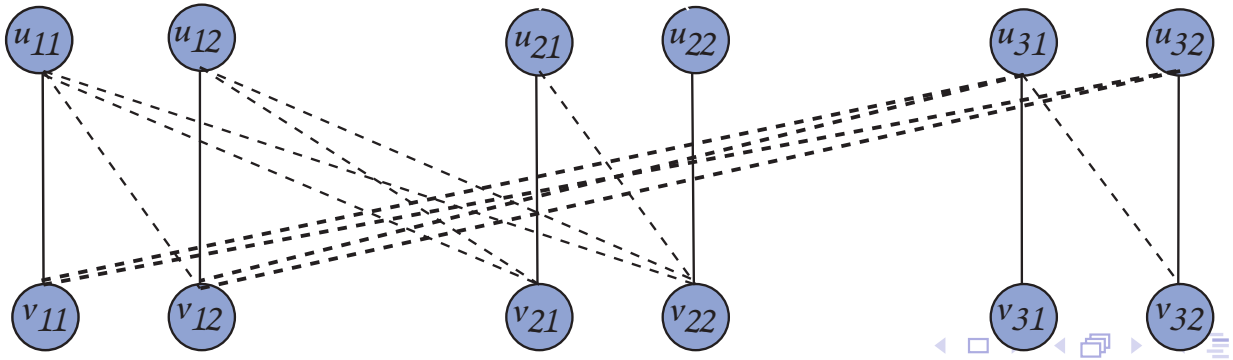
Multiple-Frequency Minimum Time Scheduling Problem

Reduction from Vertex Color.





q^2 *interfering links*



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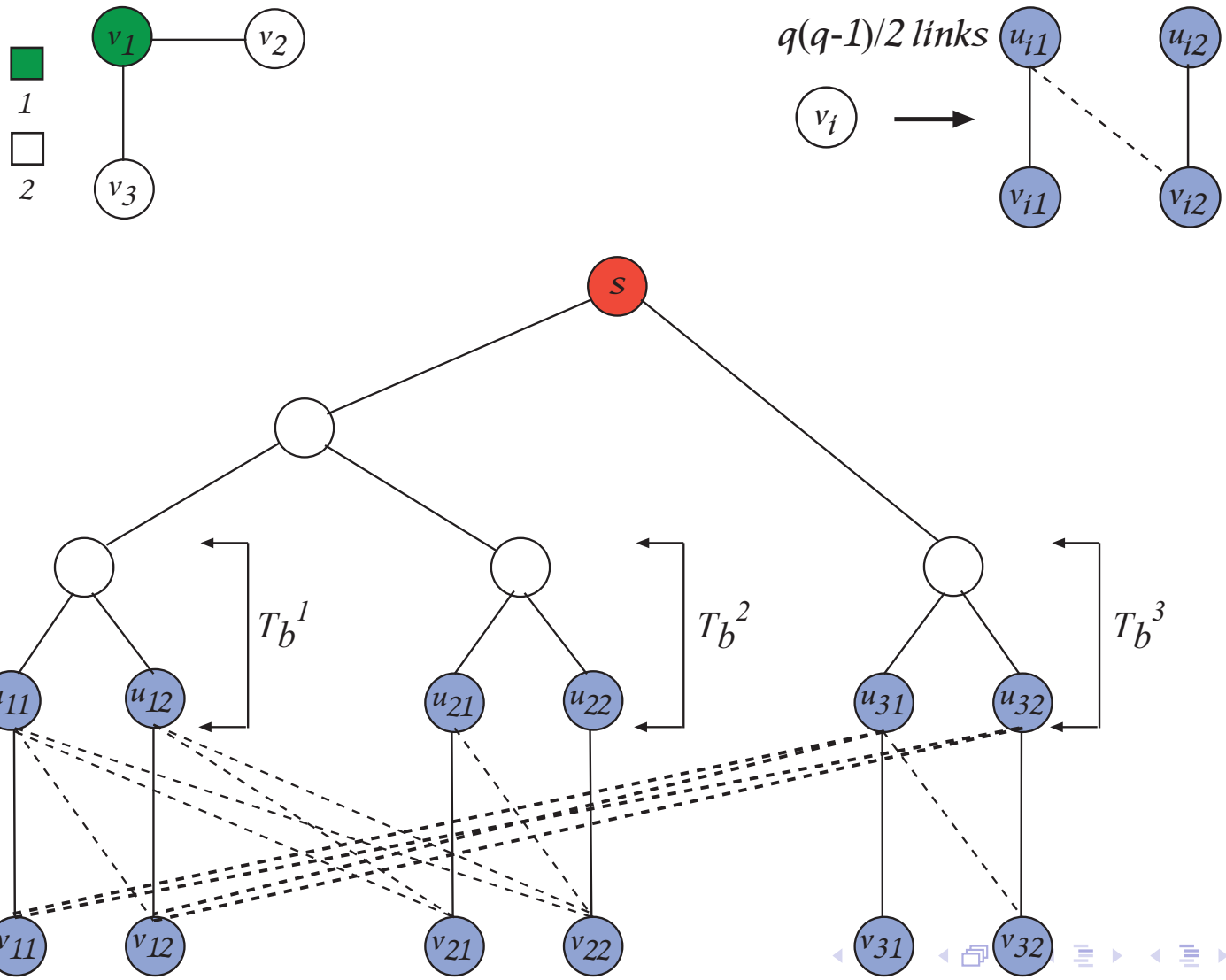

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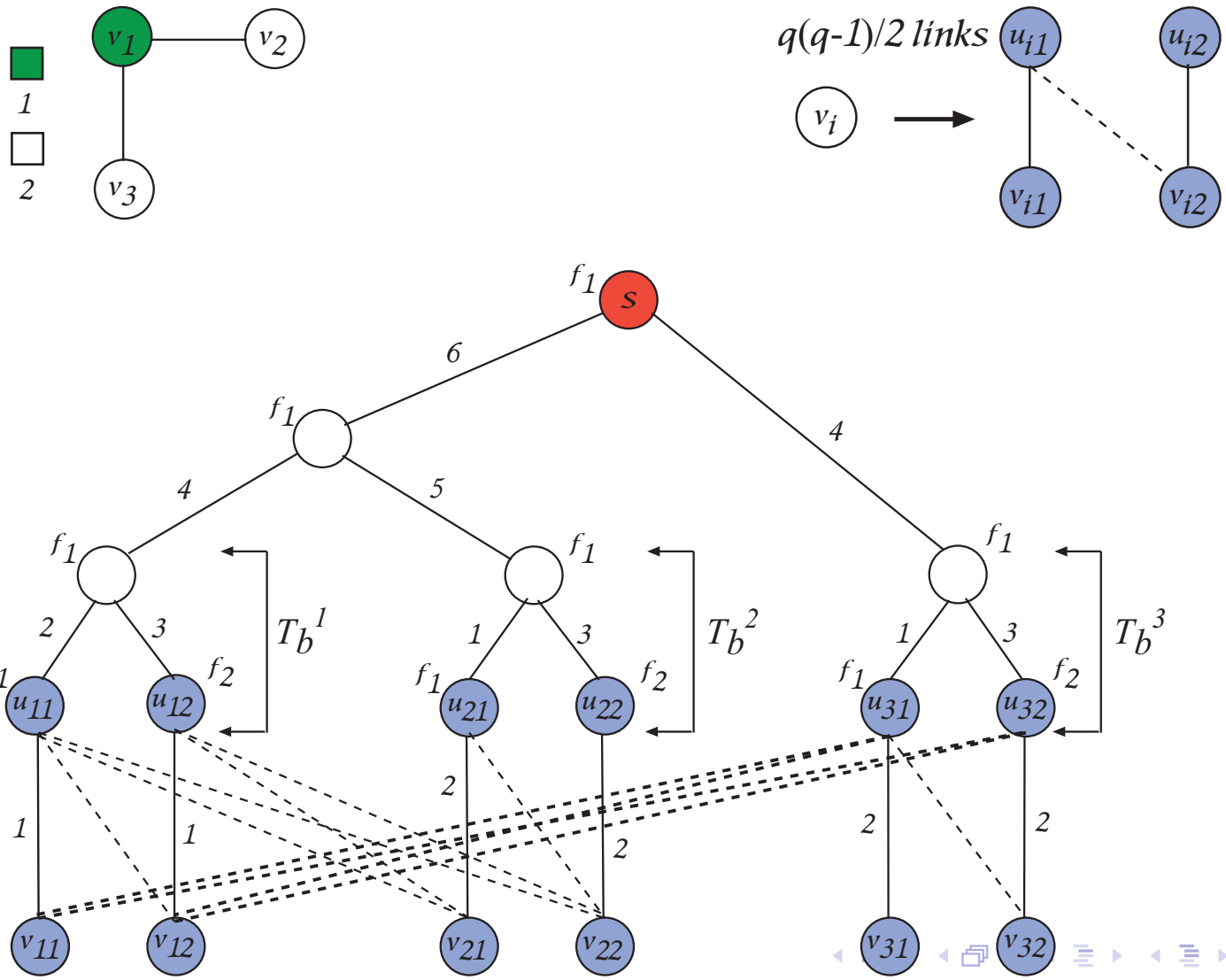
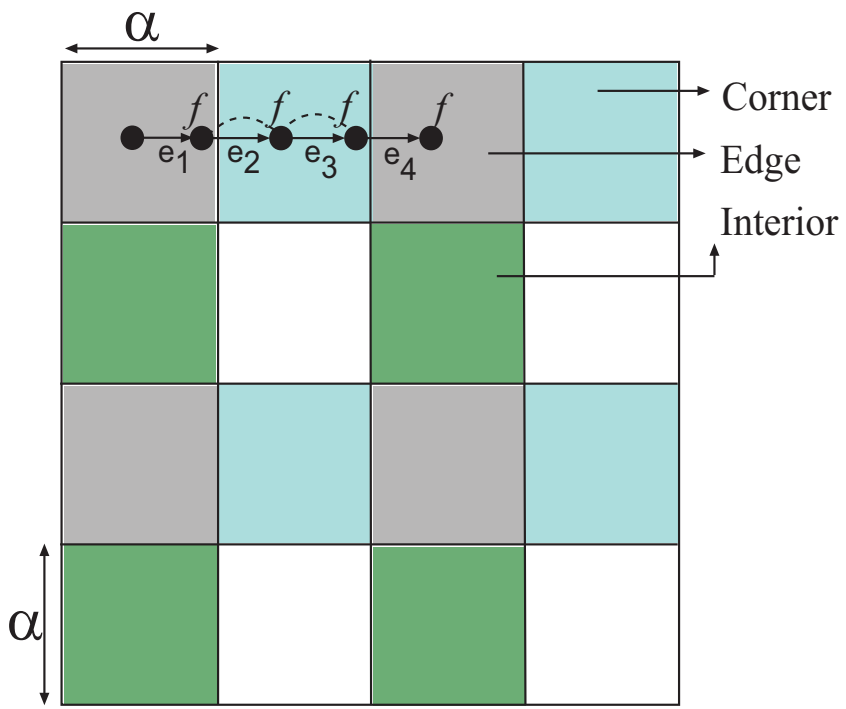


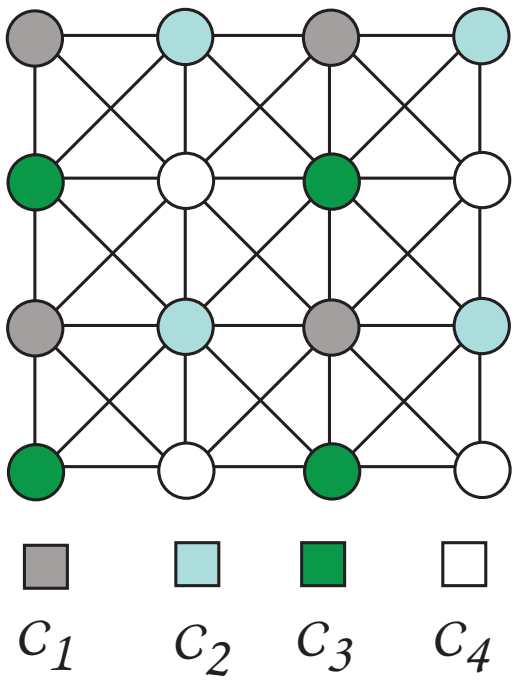
Time Slot Assignment on UDG

Lemma

Suppose γ_c denote the set of time slots to schedule the edges in cell c. Then, the minimum schedule length Γ for the whole network is:

$$\Gamma \leq 4 \cdot \max_c \{|\gamma_c|\}, \forall \alpha \geq 2.$$





Frequency Assignment on UDG

Load-Balanced Frequency Assignment

$R = \{v_1, \ldots, v_n\}$: receivers on T
$m : R \rightarrow \{f_1, \ldots, f_K\}$

If $m(v_j) = f_k$, then children of v_j transmit on f_k

Define load on f_k under m as:

$$l^m(f_k) = \sum_{m(v_j)=f_k} deg^{in}(v_j)$$

Then, a load-balanced frequency assignment m^* is:

$$m^* = \arg\min_m \max_{f_k} \{l^m(f_k)\}$$

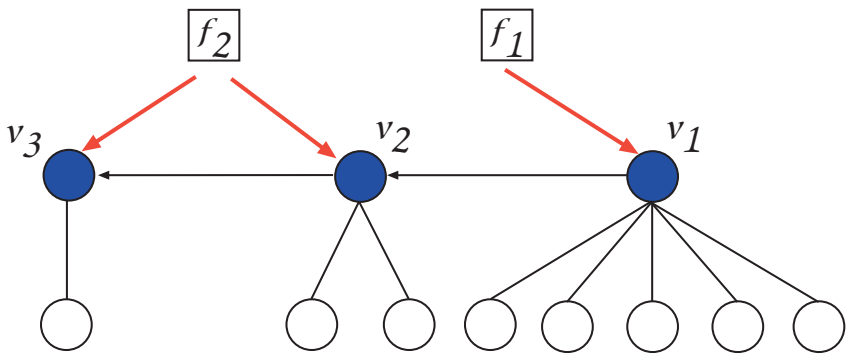


Figure: $l(f_1) = 5$, $l(f_2) = 5$

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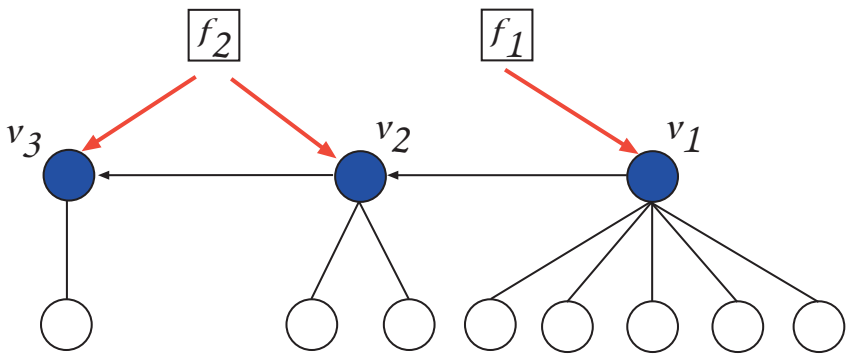


Figure: $l(f_1) = 5$, $l(f_2) = 5$

Lemma

Load-balanced frequency assignment is NP-complete.
OPT (min-max) load is L^{m^*}.

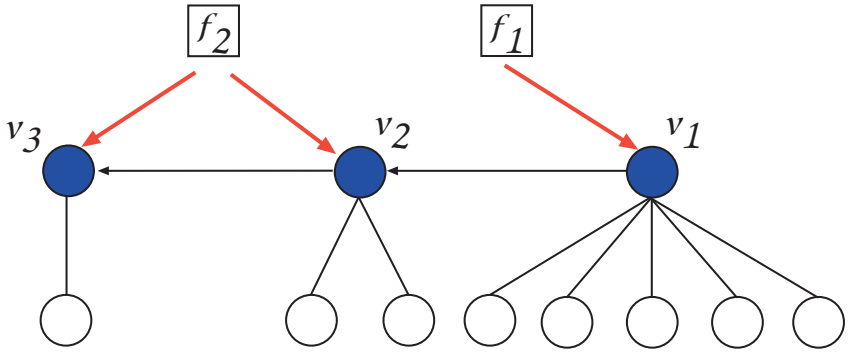
Frequency Assignment on UDG

Algorithm FREQUENCYGREEDY (ϕ)

(1) Sort the receivers in non-increasing order of in-degrees.

$$deg^{in}(v_1) \geq deg^{in}(v_2) \geq \ldots \geq deg^{in}(v_n).$$

(2) Starting from v_1, assign each successive node a frequency from $\{f_1, \ldots, f_K\}$ that has the least load, breaking ties arbitrarily.



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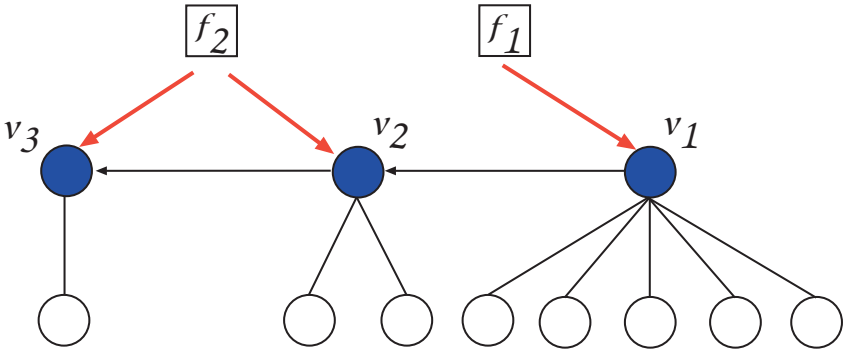

Figure: $l(f_1) = 5$, $l(f_2) = 5$

Lemma

Algorithm FREQUENCYGREEDY *gives a* $(4/3 - 1/3K)$ *approximation on* L^{m^*}.

Proof: Follows from Graham's algorithm for scheduling jobs on identical parallel machines according to longest processing time first (LPT).

An Upper Bound on γ_c

Lemma

If L_c^ϕ denote the load on the maximally loaded frequency in cell c achieved by FrequencyGreedy*, then any greedy time slot assignment can schedule all the edges in c within $2L_c^\phi$ time slots, i.e.,*

$$|\gamma_c| \leq 2L_c^\phi.$$

A Constant Factor Approximation

Theorem

Given a tree T on a UDG G, and K frequencies, there exists an algorithm $\mathcal{G}$ that achieves a constant factor $8\mu_\alpha\,(4/3 - 1/3K)$ approximation on the optimal schedule length, where $\mu_\alpha > 0$ is a constant for a given cell size $\alpha \geq 2$, i.e.,

$$\Gamma_{\mathcal{G}} = O(\Gamma_{\mathcal{OPT}})$$

Proof Sketch

$\mathcal{G}$ consists of 2 phases:

(1) Run FREQUENCYGREEDY in each cell c_i
(2) Run any greedy time slot assignment scheme
e.g., schedule a maximal number of edges in each slot

Lower bound on OPT: $\Gamma_{OPT} \geq \max_c \left\{ L_c^{m^*} \right\} / \mu_\alpha$

$$\begin{aligned} \Gamma_{\mathcal{G}} &\leq 4 \cdot \max_c \{|\gamma_c|\} \leq 8 \cdot \max_c \{L_c^{\phi}\} \\ &\leq 8 \cdot \max_c \{(4/3 - 1/3K) \cdot L_c^{m^*}\} \\ &\leq 8\mu_\alpha (4/3 - 1/3K) \cdot \Gamma_{OPT} \end{aligned}$$

Arbitrary Trees Under UDG

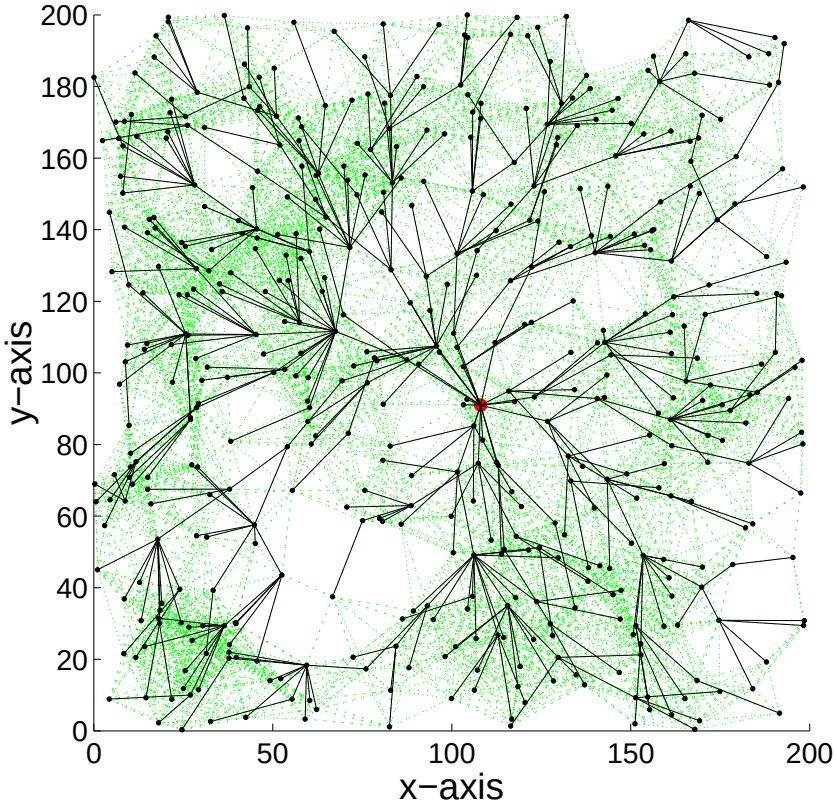

Figure: Shortest Path Tree

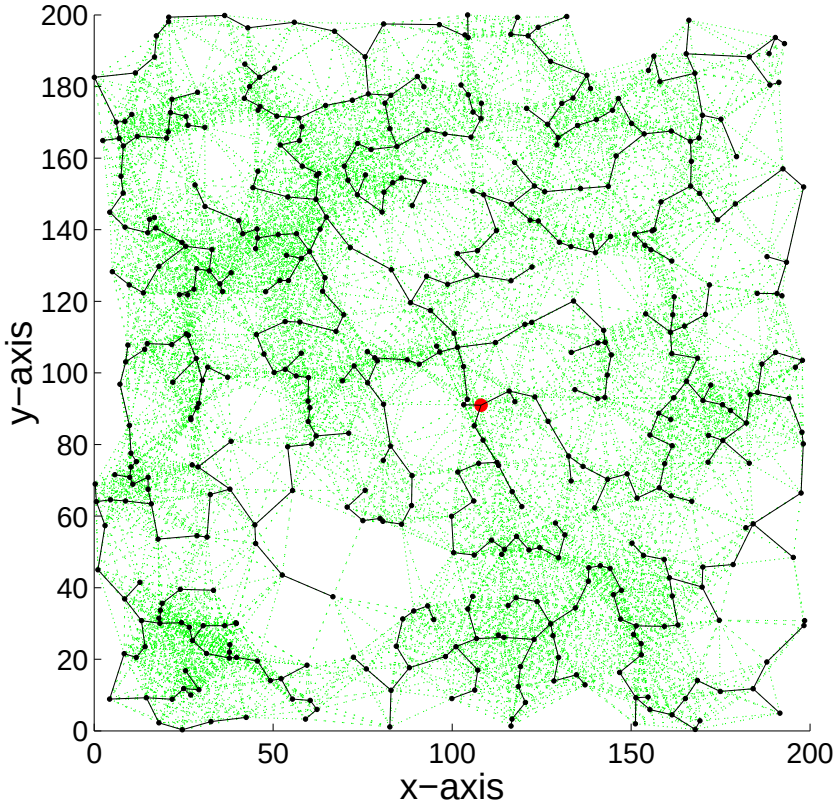

Figure: Minimum Spanning Tree

Arbitrary Trees Under UDG

Theorem

Given a UDG G and K frequencies, there exists an algorithm $\mathcal{H}$ that achieves a constant factor $8\mu_\alpha\Delta_C$ approximation on the optimal schedule length, where $\mu_\alpha > 0$, $\Delta_C > 0$ are constants for a given cell size $\alpha \geq 2$, i.e.,

$$\Gamma_{\mathcal{H}} = O(\Gamma_{\mathcal{OPT}})$$

Proof Sketch

V_c: set of nodes in cell c
$R_c = \{v_1, \ldots, v_n\}$: set of receivers in c on arbitrary T
$\Delta^{in}(T)$: max in-degree in T

Lower bound on OPT: $\Gamma_{OPT} \geq \max_c \left\{ \left\lceil \frac{|V_c|}{K} \right\rceil \right\} / \mu_\alpha$

Then

$$\begin{aligned} \Gamma_{\mathcal{H}} &\leq 4 \cdot \max_c \{|\gamma_c|\} \leq 8 \cdot \max_c \{L_c^\phi\} \\ &\leq 8 \cdot \max_c \{\lceil |V_c|/K \rceil\} \cdot \Delta^{in}(T) \\ &\leq 8\mu_\alpha \Delta^{in}(T) \cdot \Gamma_{OPT} \end{aligned}$$

Bounded-degree spanning tree always exists on UDG. Therefore, proved.

Schedule Length

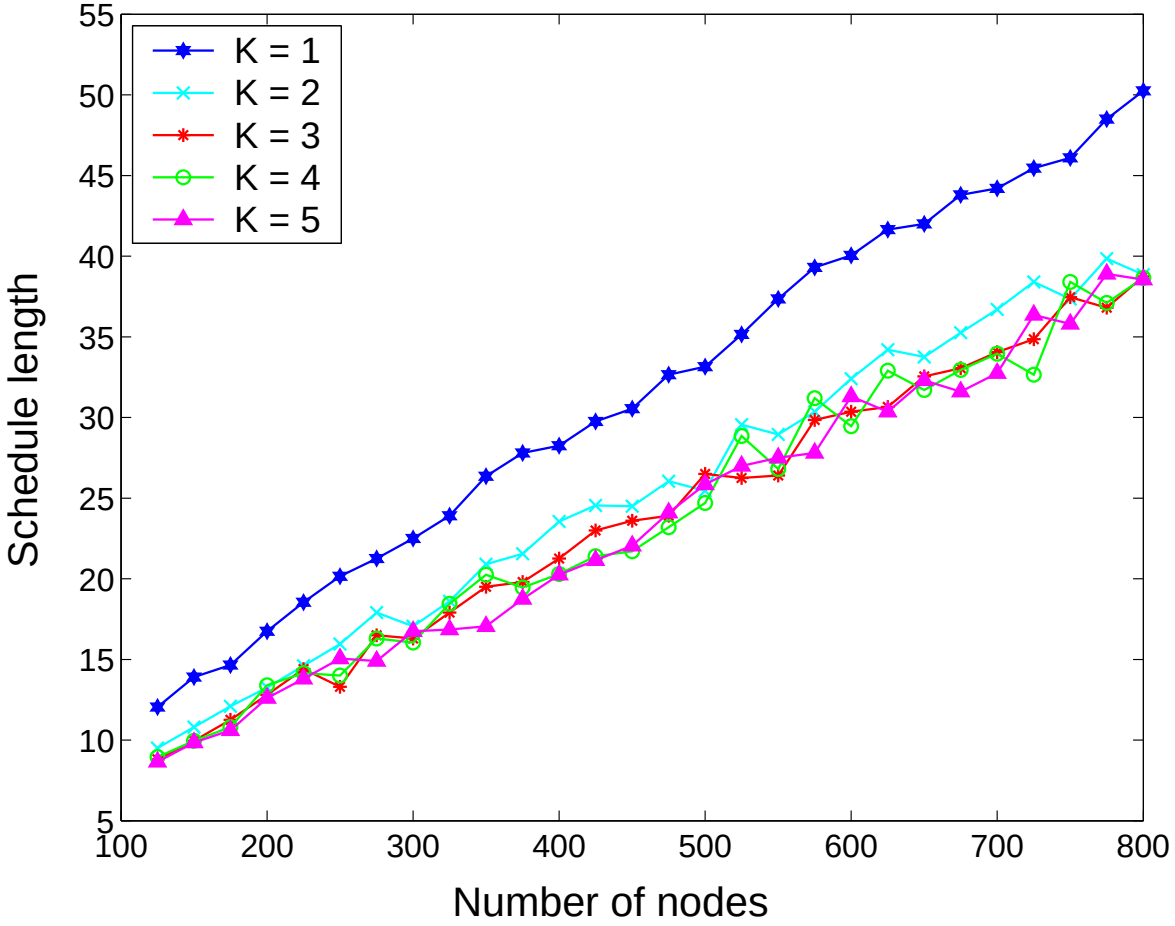

Figure: Algorithm $\mathcal{G}$ on Shortest Path Tree for Different Network Sizes

Schedule Length

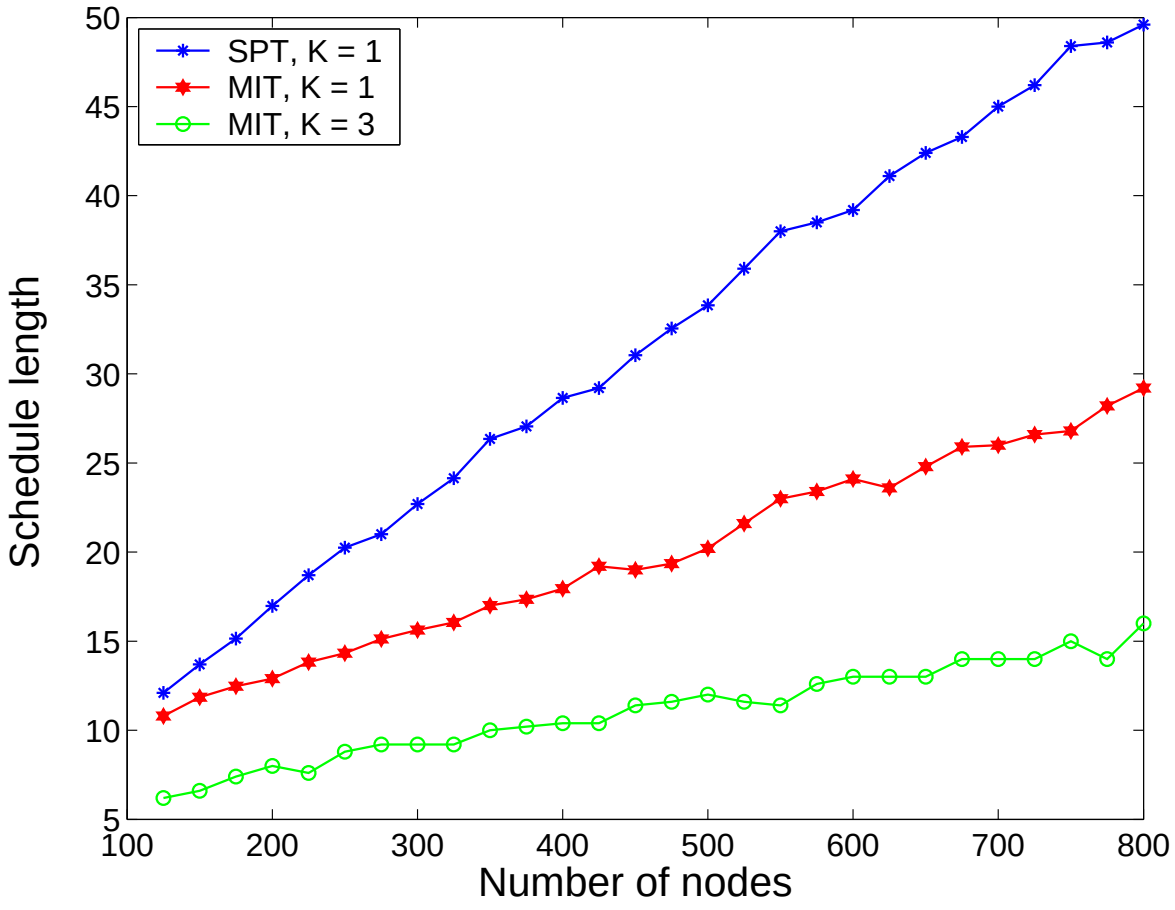

Figure: Algorithm $\mathcal{G}$ on Minimum Interference Tree for Different Network Sizes

Conclusions

Summary

(1) Addressed a scheduling problem under aggregated convergecast using multiple channels.

(2) Proved a NP-completeness result on finding the minimum number of channels required to remove all the interfering links in an arbitrary wireless network.

(3) Proved a NP-completeness result on minimizing the schedule length for a given number of channels in an arbitrary wireless networks.

(4) Proposed an optimal time slot scheduling scheme when enough frequencies are available.

(5) Proposed constant factor approximation algorithms to minimize the schedule length on unit disk graphs.

(6) Evaluated algorithms using simulations: most of the times 3 to 4 frequencies are enough all the interfering links.