Phy Layer

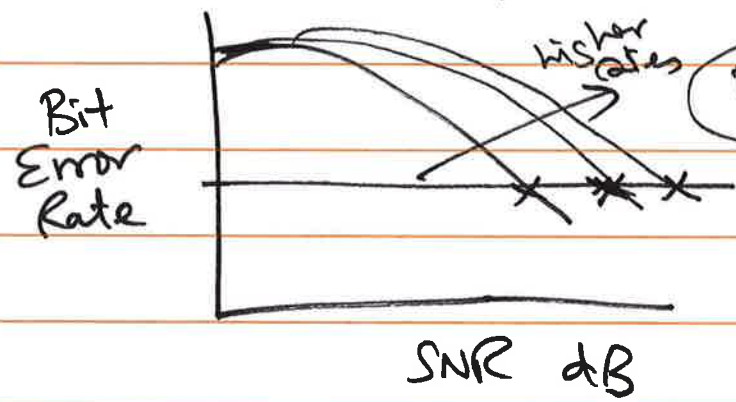
Lecture 2
1/12/12

Last class:

- modulation schemes

  constellation diagrams

  BPSK performance under AWGN



Power-Rate-Error tradeoffs.

Shannon's Capacity Law for AWGN

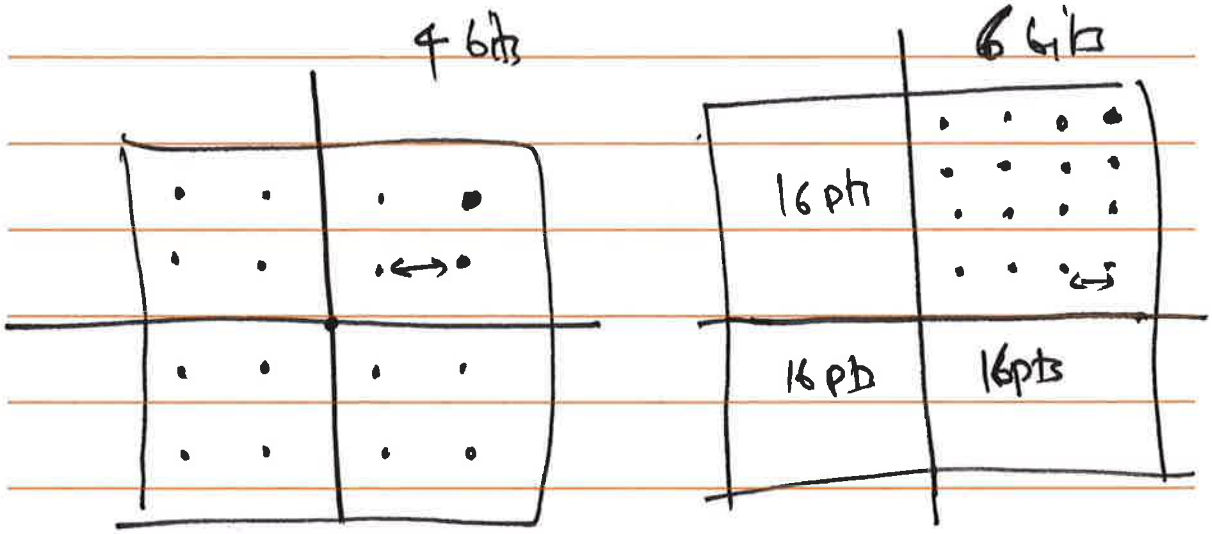
$$R = W \cdot \log(1 + \underbrace{SNR})$$

R: throughput / Rate

SNR: Received Signal Power to Noise Ratio

Under idealized settings, error $\rightarrow 0$

16 QAM 64 QAM



for the same inter-symbol distance, need 4 times the area.

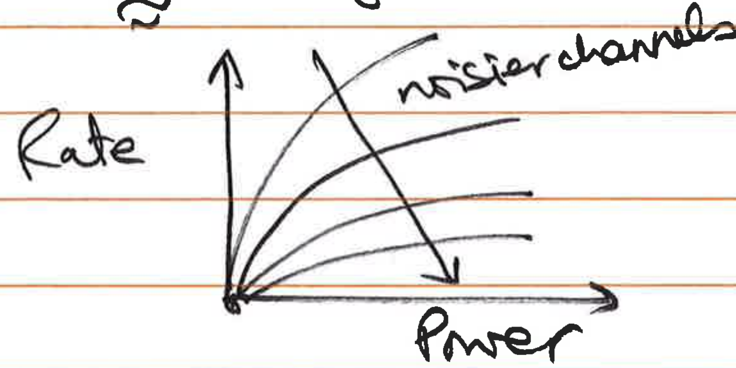
Power expended $\propto$ Area $\propto$ 4 times increase

Eg

Power $\propto$ # of symbols

Rate $\propto$ # bits / symbol $\propto$ log (# of symbols)

Rate $\underset{\sim}{\propto}$ log (Power)



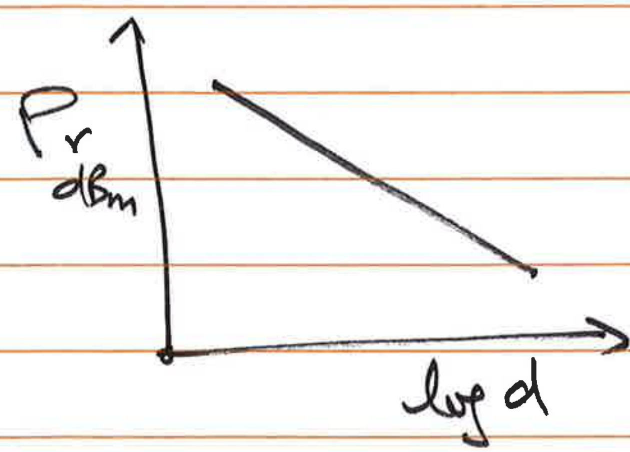
· Radio Propagation

Simplified Path loss model

$$P_r = P_t K \left[\frac{d_0}{d}\right]^\eta$$

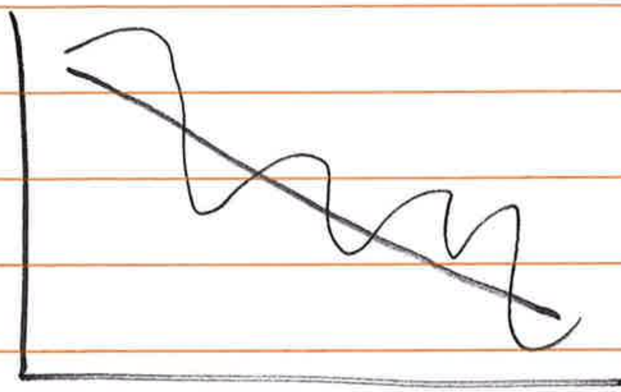
first order idea of how the signal strength (in terms of received power) scales w/ distance

$$P_{r_{dBm}} = P_{t_{dBm}} + K_{dB} - \eta \log_{10}[d/d_0]$$

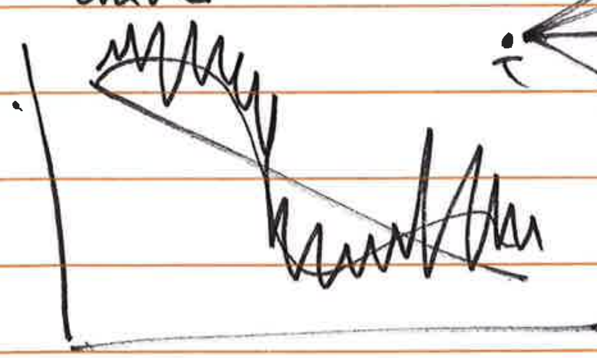


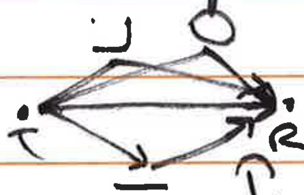
this shows the <u>mean</u> path loss, in practice there are further variations.

Shadowing: slow variation due to large scale obstructions & reflections e.g. trees/house/buildings/lake etc.



Multipath fading: short (sub-wavelength-order) variations in power due to constructive & destructive superposition of multipath reflections of the radio wave





combination of these signals affects the received power

We need statistical models for shadowing & fading.

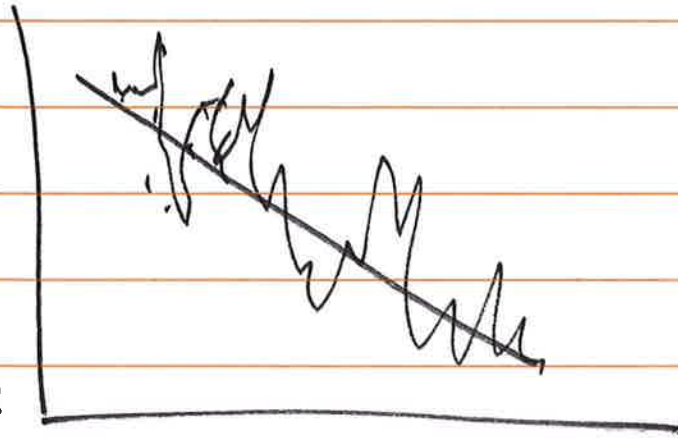
1. Log-normal distribution
   - shadowing in large-scale networks
   - also used for fading in small-scale networks

2. Rayleigh distribution
   - fading
   - a "worst case" non line-of-sight model

3. Nakagami-m

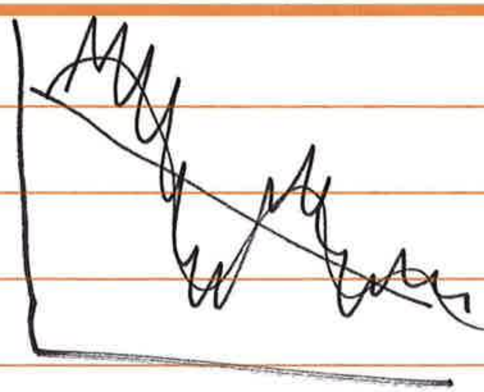
   a general parameterized fading distribution, such that Rayleigh is a special case, as is no fading (also Ricean)

Small-scale:



Received Power = mean received power + fading

Large-Scale



Received Power = mean Power + shadowing + fading

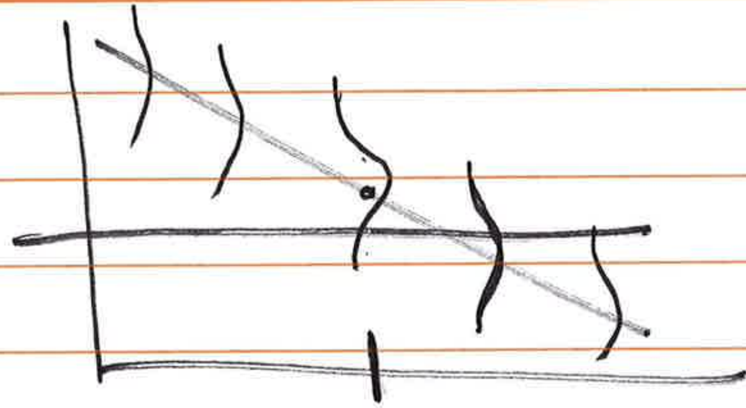
Log-normal shadowing / fading model

$$P_r = P_t K \left[\frac{d_0}{d}\right]^n \cdot \psi$$

a random variable with a log-normal distribution

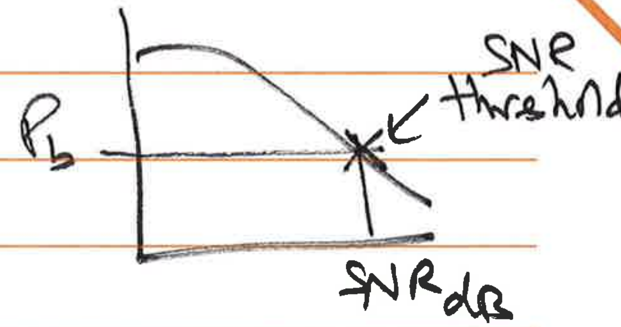
i.e. $\psi_{dB} \sim \mathcal{N}(0, \sigma^2)$

fading/shadowing variance

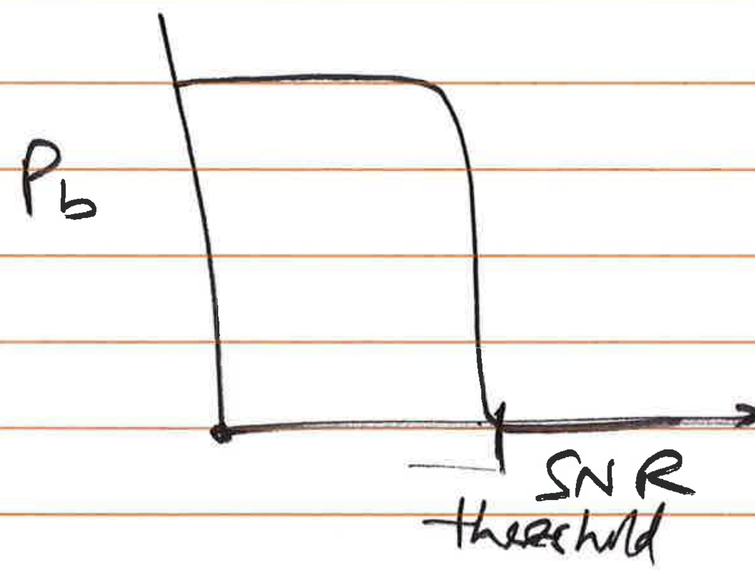


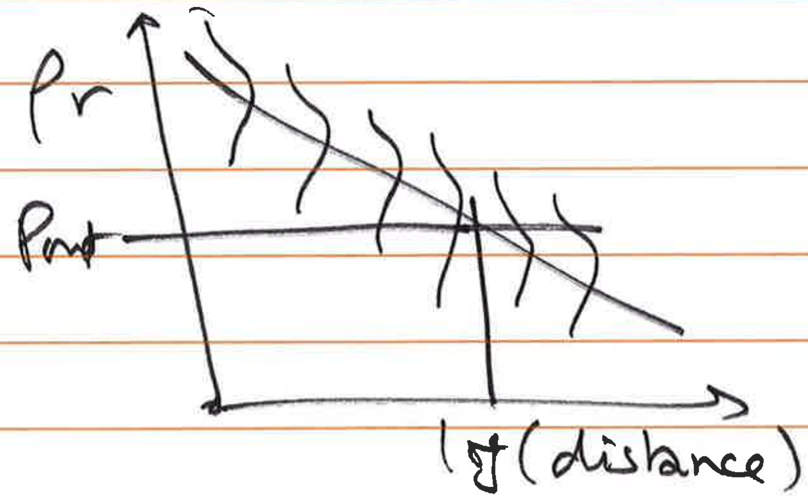
for large scale networks, this is typically used to model 'just' shadowing
but could model both shadowing & fading
for small-scale networks, used to model fading

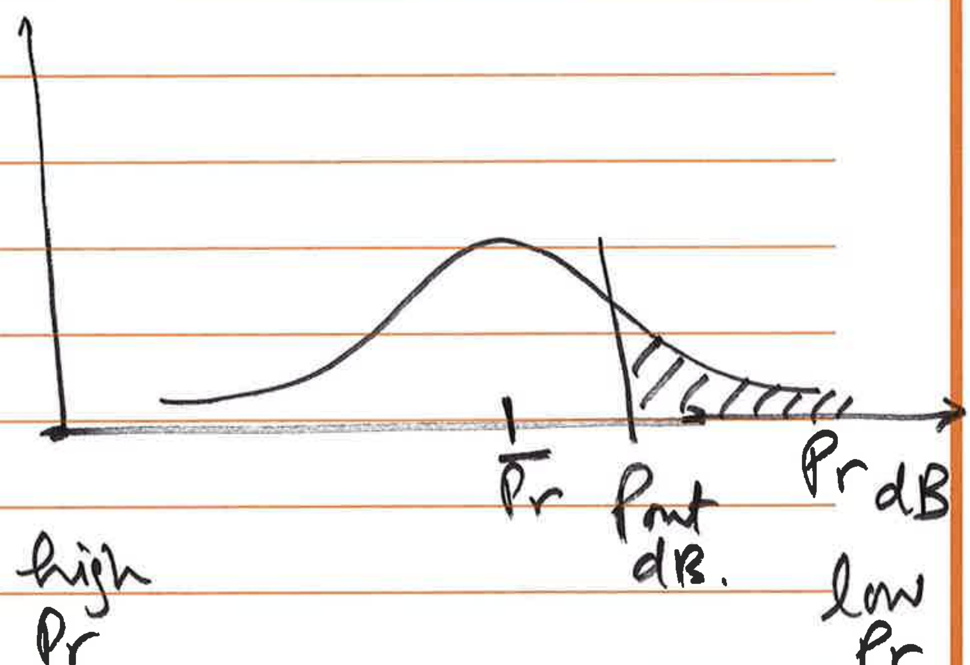
Outage

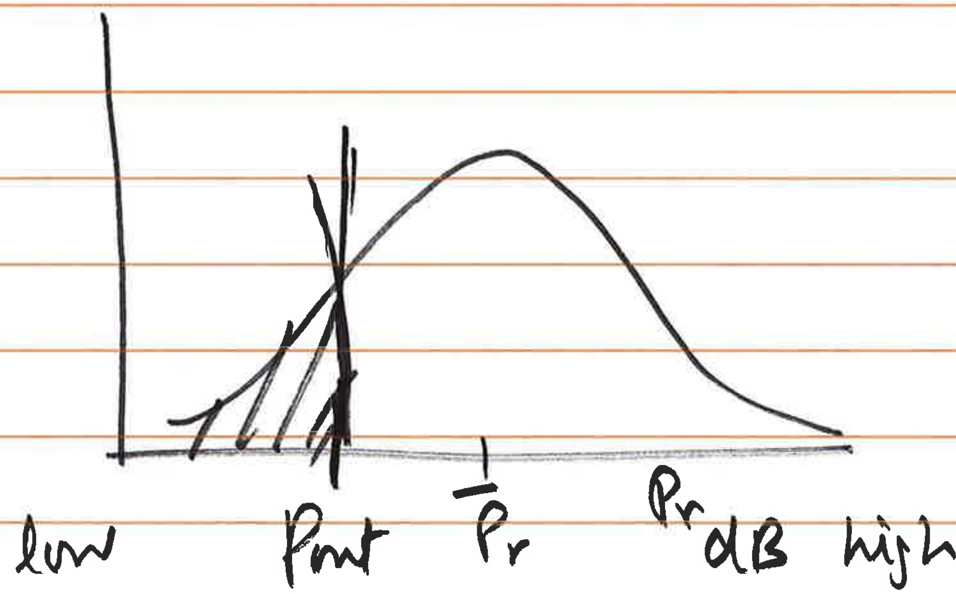


if my noise power is a constant,
then if $P_r < P_{out}$
then the communication
does not meet the
specified error performance.
"outage"




Pr
Pout
lg(distance)


Pr
Pout dB.
Pr dB
high Pr
low Pr



outage Probability =

$$\text{Probability}\left(P_{r_{dB}} < P_{out_{dB}}\right)$$

$$P_{r_{dB}} = P_{t_{dB}} + K_{dB} - \eta \log\left[\frac{d}{d_0}\right] + \psi_{dB}$$

$$\psi_{dB} \sim \mathcal{N}(0, \sigma^2)$$

$$\mathrm{Prob}\left[P_{t\,dB} + K_{dB} - \eta \log\left(\frac{d}{d_0}\right) + \psi_{dB} < P_{out} \right]$$

$$= \mathrm{Prob}\left[\psi_{dB} < \underbrace{P_{out\,dB} - P_{t\,dB} - K_{dB} + \eta \log\left(\frac{d}{d_0}\right)} \right]$$

$$1 - \mathrm{Prob}\left[\psi_{dB} > \quad \cdot \quad \right]$$

Given an arbitrary Gaussian random variable $Y \sim N(\mu, \sigma^2)$

$$\frac{Y - \mu}{\sigma} \sim N(0, 1)$$

$\therefore$ Outage Probability $= 1 - Q\left(\dfrac{P_{out} - P_t - K + \eta \log\left(\frac{d}{d_0}\right)}{\sigma}\right)$

Rayleigh Distribution for fading

$$f_{P_r}(x) = \frac{1}{\bar{P}_r} \exp\left(\frac{-x}{\bar{P}_r}\right) \quad x \geq 0$$

Power distribution for Rayleigh fading is exponential (pdf)

here Power is in Watts $P_r, \bar{P}_r, P_{out}$

Outage Probability:

$$\rho = \text{Prob}[P_r < P_{out}]$$

$$\int_0^{P_{out}} \frac{1}{\bar{P}_r} \exp\left(\frac{-x}{\bar{P}_r}\right) dx$$

$$= -\exp\left(\frac{-P_{out}}{\bar{P}_r}\right) - - \exp(0)$$

1

$$\rho = 1 - \exp\left(\frac{-P_{out}}{\bar{P}_r}\right)$$

$$\rho = 1 - \exp\left(-\frac{P_{out}}{\bar{P}_r}\right)$$

$$\frac{-P_{out}}{\bar{P}_r} = \ln(1-\rho)$$

if $$\bar{P}_r \geq \frac{P_{out}}{-\ln(1-\rho)}$$

then outage prob. is below ρ

$$\underbrace{10\log_{10}\bar{P}_r - 10\log_{10}P_{out}}_{\text{Fading margin}}$$

$$\bar{P}_{r\,dB} = 10\log_{10}\left(\frac{P_{out}}{\ln\left(\frac{1}{1-\rho}\right)}\right)$$

$$= 10\log_{10}(P_{out}) - 10\log_{10}\ln\left(\frac{1}{1-\rho}\right)$$

$$\bar{P}_{r\,dB} - P_{out,dB} = 10\log_{10}\underbrace{\ln(1-\rho)}_{\text{F.M for Rayleigh fading.}}$$

$$\overline{P_r}_{dB} = 10\log_{10}\left(\frac{P_{out}}{\ln\left(\frac{1}{1-\rho}\right)}\right)$$

$$\overline{P_r}_{dB} = P_{out\,dB} - 10\log_{10}\left(\ln\left(\frac{1}{1-\rho}\right)\right)$$

$$F.M. = \overline{P_r}_{dB} - P_{out,dB} = -10\log_{10}\left(\ln\left(\frac{1}{1-\rho}\right)\right).$$

Nakagami-m fading

Γ function

power distribution:

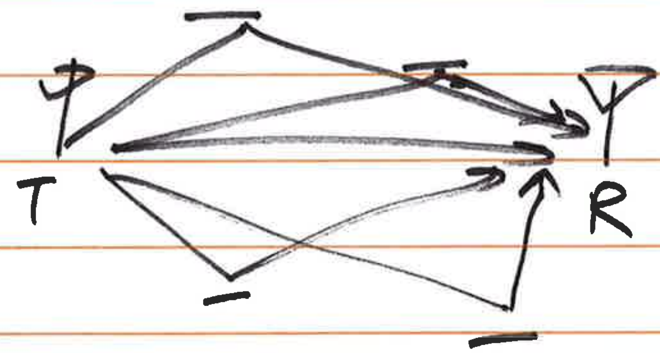
$$f_{P_r}(x) = \left(\frac{m}{\overline{P_r}}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left[\frac{-mx}{\overline{P_r}}\right]$$

$m=1$ This is exactly Rayleigh

$m=\infty$ there is no fading

other values of m capture other fading distributions including Ricean.

Communication in the presence of fading



If there is <u>no</u> change in either the wireless nodes or the environment, the received power is constant over time.

"slow-fading"

Consider an env. with static TX & RX node but with changing/moving objects.

The receiver power will fluctuate over time. if the dynamics are high, we will have "fast fading"

Performance on a link w/ fading:

Expected prob of error.

$$E[B] = \int_0^\infty \underbrace{B(x)}_{\text{Bit error probability for received power } x \text{ under AWGN channel}} \cdot f_{P_r}(x)\, dx$$

expected prob. of error