Last topic in PHY layer communication:

MIMO communications

\[ \begin{array}{c}
\text{T} \\
\text{R}
\end{array} \]

Single input Single output (SISO)

\[ \begin{array}{c}
\text{T} \\
\text{R}
\end{array} \]

Single input multiple output (SIMO)

\[ \begin{array}{c}
\text{L} \\
\text{P}
\end{array} \]

MISO
Multiple inputs multiple outputs

MIMO

Two possible sources of gain:
more independent paths

\( \Rightarrow \) - greater rate (multiplexing gain)
- lower error due to diversity (diversity gain)
Receive Diversity. Combining

\[ r_1 \cdot e^{j\theta_1} = r_1(a_1 e^{j\theta_1}) \]

\[ r_2 \cdot e^{j\theta_2} \]

\[ r_3 \cdot e^{j\theta_3} \]

\[ \alpha_1 r_1 \cdot a_1 e^{j\theta_1} = r_1(a_1 e^{j\theta_1}) \]

\[ \alpha_2 r_2(a_2) \]

\[ \alpha_3 r_3(a_3) \]

\[ \text{Combining output} \]

\[ E_{r_1}(a_1, 5^3) \]

\[ E_{r_2}(a_2) \]

\[ E_{r_3}(a_3) \]

\[ \sum a_i \cdot r_i = \sum a_i \cdot \frac{E_{a_i}r_i}{N_0} \]

\[ \alpha_i = a_i e^{-j\theta_i} \]

\[ \text{to be designed} \]

SNR at the output:

\[ \frac{\left( \sum_{i=1}^{M} a_i r_i \right)^2}{\left( \sum a_i^2 : N_0 \right)} = \frac{\left( \sum E_{a_i} r_i \right)^2}{N_0 \left( \sum E_{a_i} r_i \right)} \]
How to choose $a_i$ to maximize the output SNR?

i.e., find $a_i$ to

$$\max \quad \frac{\left(\sum_{i=1}^{M} a_i r_i \right)^2}{\sum_{i=1}^{M} a_i^2} = r$$

\[
\frac{\partial Y}{\partial a_j} \bigg|_{a_i=a_i^*+\theta} = 0
\]

\[
2 \left( \left( \sum_{i=1}^{M} a_i r_i \right)^2 \left( \sum_{i=1}^{M} a_i^2 \right)^{-1} \right) = 0
\]

\[
\frac{\partial Y}{\partial a_j} = \left( \sum_{i=1}^{M} a_i r_i \right)^2 \left( \sum_{i=1}^{M} a_i^2 \right)^{-2} 2a_j
\]

\[
+ \left( \sum_{i=1}^{M} a_i^2 \right)^{-1} 2 \left( \sum_{i=1}^{M} a_i r_i \right) r_j = 0
\]
\[(\sum_{i=1}^{M} a_i \cdot r_i)^2 \cdot x_j = 2 r_j \cdot \Sigma a_i^2 (\sum_{i=1}^{M} a_i \cdot r_i)\]

\[
\Rightarrow \quad (\sum_{i=1}^{M} a_i \cdot r_i) \cdot x_j = (\sum_{i=1}^{M} a_i^2) \cdot r_j
\]

Toy:
\[x_j = r_j \quad \iff \quad \text{this satisfies all equations}\]

Output SNR:
\[
\text{output SNR} = \frac{(\sum_{i=1}^{M} r_i \cdot a_i)^2}{\sum_{i=1}^{M} a_i^2 \cdot N_0}
\]

\[
= \frac{(\sum_{i=1}^{M} a_i^2)^2}{\sum_{i=1}^{M} a_i^2 \cdot N_0}
= \frac{\sum_{i=1}^{M} r_i^2}{N_0}
\]

Equivalent to \[\sum_{i=1}^{M} \text{SNR}_i\]

This is called \textit{Array Gain}.
With a fading channel, we have another source of gain.

\[ P_{emr} = \int P_e(x) f_{\text{SNR}}(x) \, dx \]

If fading

\[ f_{\text{SNR}}(\cdot) \]

is the SNR distribution at the output of the channel,

\[ e \]

is more favorable than the distribution for a single path.

\[ E[\text{output SNR}] = M \cdot E[\text{SNR per path}] \]

the improvement in error performance due to this more favorable distribution is called diversity gain.
It can be shown that:

for high SNR (for different distributions such as Rayleigh)

\[ P_{\text{error}} \approx \tilde{R} - M \text{ diversity order} \]

average SNR per branch
\[ x_1 = a_1 \cdot e^{-j\theta_1} \]

If the channel state information (magnitude/phase of each path) is known at the transmitter, can correct at the transmitter.

Correct for phase & set magnitude of weight proportional to the gain (MRC)

This also yields a diversity gain of order \( M \) for \( M \) transmit antennas.
In a MIMO system with $M_T$, $M_R$ transmit, receive antennas respectively, it is possible to achieve a maximum diversity gain of $(M_T \cdot M_R)$.

If the channel state information is not known at the transmitter, one can still get a diversity order of $M_T \cdot M_R$ using "space-time codes."
Multiplexing over a MIMO channel

\[ H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \]

Matrix representation of the 2x2 MIMO channel.
\[ y = Hx + n \]
\[ 2 \times 1 = 2 \times 2 \times 2 \times 1 = 2 \times 1 \]

\[ y_1 = h_{11} \cdot x_1 + h_{21} \cdot x_2 + n_1 \]
\[ y_2 = h_{12} \cdot x_1 + h_{22} \cdot x_2 + n_2 \]

**SVD - Singular Value Decomposition**

\[ \text{svd}(H) : H = U \Sigma V^H \]

\[ U^H U = I \]
\[ V^H V = I \]

- Unitary matrix
- Diagonal matrix

At the decoder, multiply by \( U^H \)
At the encoder, multiply \( x \) by \( V \)
Sending: \( \hat{x} = v x \)

At receiver: \( U^H (H x^2 + n) = y \).

\[
y = u^H U \Sigma V^H V x + u^H n
\]

\[
y = \Sigma^2 \mathbf{x} + u^H n
\]

\[
y = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}
\]

\( y_1 = \sigma_1 x_1 + n_1 \)
\( y_2 = \sigma_2 x_2 + n_2 \)

This is like having two parallel virtual channels, doubling the rate.
In general, with \( M_T \) transmit antennas & \( M_R \) receive antennas, it is possible to have a maximum multiplexing gain of

\[
\min (M_T, M_R).
\]
In general, under low SNR conditions, it's preferable to use the multiple antenna resources to obtain a diversity gain. Conversely, at high SNR, it is preferable to obtain a multiplexing gain to boost the data rates in a MIMO system.

Directional Antennas

typically, omnidirectional antennas are used
In cellular communications, Base stations typically use 120° sector antennas.

In this example node b does not experience interference from B.S. A.
NetBlagr.com
- startup in Boston
providing mesh-network based wireless Broadband

This boosts the SNR significantly.

Networked MIMO / distributed MIMO/
Cooperative networking/communications/
Distributed input Distributed output
4G/LTE standards talk about CoMP - coordinated multi point transmissions