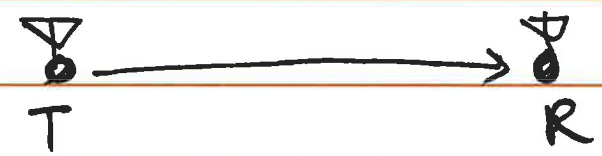
Lecture 7 EE 597

1/31/12

Last topic in

PHY layer communication:

MIMO communications



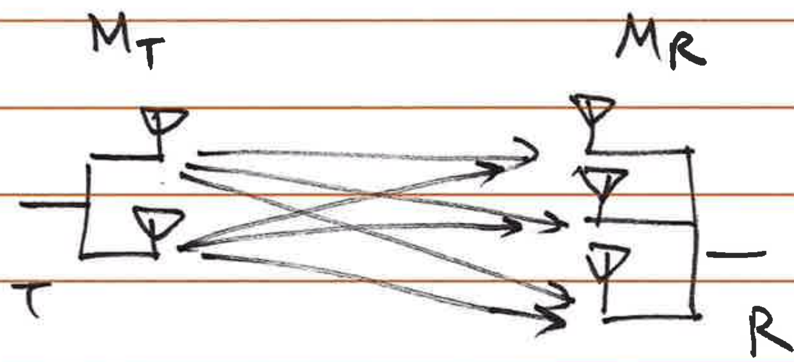
Single input single output

(SISO)

* Single input multiple output

(SIMO)

MISO



Multiple inputs multiple outputs

MIMO

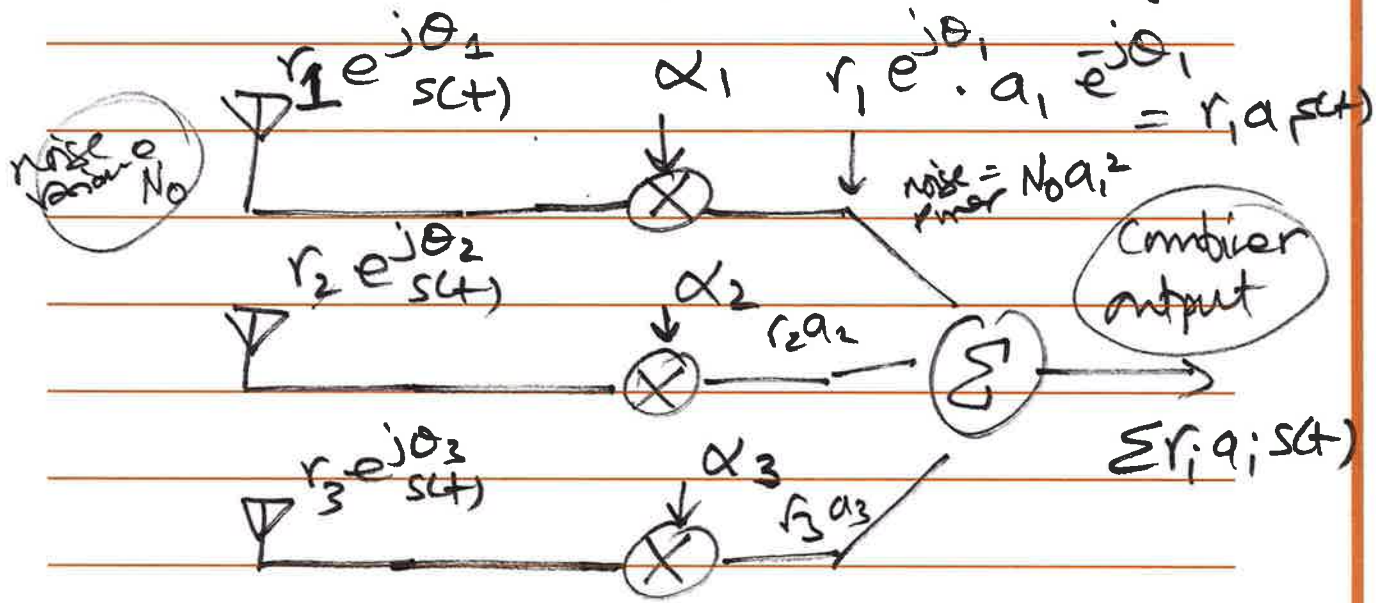
Two possible sources of gains:

more independent paths

$\Rightarrow$ - greater rate (multiplexing gain)

- lower error due to diversity (diversity gain)

Receive Diversity. $\underline{\text{Combining}}$



$$\alpha_i = \underline{\underline{a_i}}\, e^{-j\theta_i}$$

to be designed.

SNR at the output :

$$\frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{\left(\sum a_i^2 \cdot N_0\right)} = \frac{\left(\Sigma a_i r_i\right)^2}{N_0\left(\Sigma a_i^2\right)}$$

How to choose a_i to maximize the output SNR?

i.e. find a_i to

$$\max \frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{\sum_{i=1}^{M} a_i^2} = \gamma$$

$$\left.\frac{\partial \gamma}{\partial a_j}\right|_{a_j = a_j^* \ \forall j} = 0$$

$$\frac{\partial}{\partial a_j}\left(\left(\sum_{i=1}^{M} a_i r_i\right)^2 \left(\sum_{i=1}^{M} a_i^2\right)^{-1} \right) = 0$$

$$-\left(\sum_{i=1}^{M} a_i r_i\right)^2 \left(\sum_{i=1}^{M} a_i^2\right)^{-2} \cdot 2a_j$$

$$+ \left(\sum_{i=1}^{M} a_i^2\right)^{-1} 2\left(\sum_{i=1}^{M} a_i r_i\right) r_j = 0$$

$$\left(\sum_{i=1}^{M} a_i r_i\right)^{\cancel{2}} \cdot \cancel{2} a_j = \cancel{2} r_j \cdot \sum a_i^2 \cancel{\left(\sum_{i=1}^{M} a_i r_i\right)}$$

$$\Rightarrow \quad \left(\sum_{i=1}^{M} a_i r_i\right) \cdot a_j = \left(\sum_{i=1}^{M} a_i^2\right) \cdot r_j$$

$$\forall j$$

try: $\boxed{a_j = r_j}\ \forall j$ this satisfies all equations

output SNR $= \dfrac{\left(\sum r_i a_i\right)^2}{\sum a_i^2 \cdot N_0}$

$$= \frac{\left(\sum a_i^2\right)^2}{\sum a_i^2 \cdot N_0} = \frac{\sum r_i^2}{N_0}$$

equivalent to $\sum SNR_i$

This is called <u>Array Gain</u>.

With a fading channel, we have another source of gain.

$$\overline{P_{err}} = \int_0^\infty P_e(\gamma)\, f_\Sigma(\gamma)\, d\gamma$$

w/ fading

$f_\Sigma(\cdot)$ is the SNR distribution at the output of the combiner,

& is more favorable than the distribution for a single path.

$$E[\text{output SNR}] = M \cdot E[\text{SNR for each path}]$$

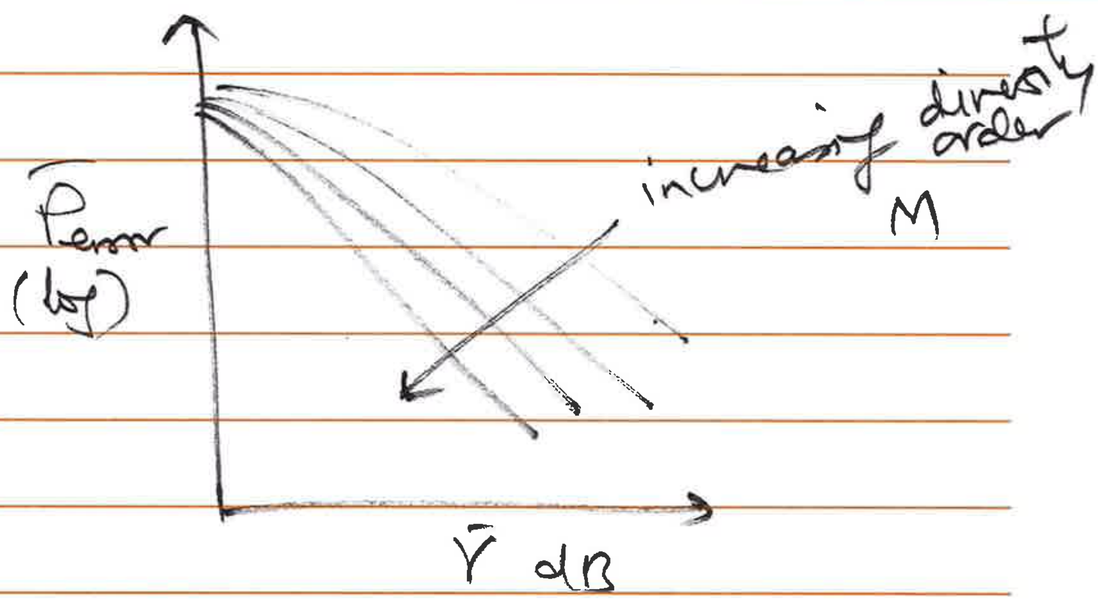
the improvement in error performance due to this more favorable distribution is called diversity gain.

It can be shown that
for high SNR (for different distributions such as Rayleigh)

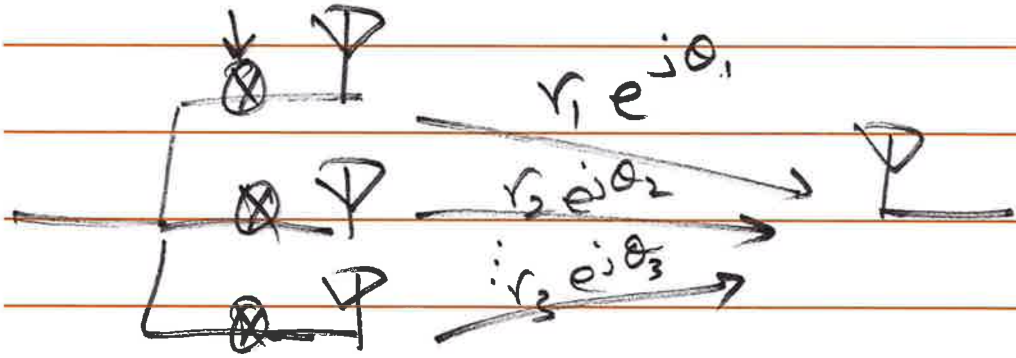
$$\overline{P_{error}} \approx \bar{\gamma}^{-M}$$

M — diversity order.

$\bar{\gamma}$ — average SNR per branch



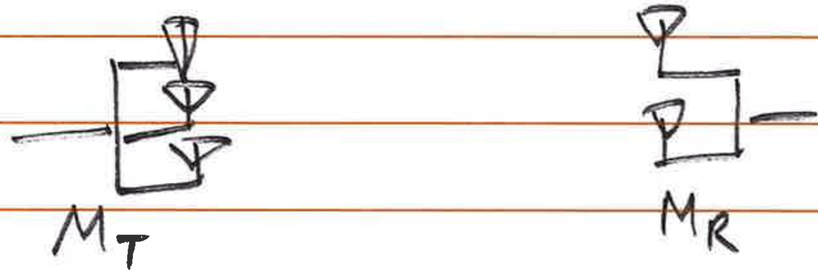
$\alpha_1 = a_1 \cdot e^{-j\theta_1}$



If the channel state information (magnitude / phase of each path) is known at the transmitter,

Can correct at the transmitter (correct for phase & set magnitude of weight proportional to the gain (MRC)

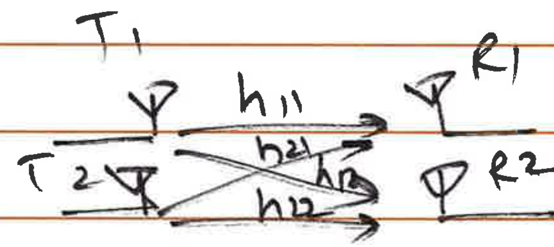
This also yields a max diversity gain of order M for M transmit antennas



In a MIMO system w/ M_T, M_R transmit, receive antennas respectively, it is possible to achieve a maximum diversity gain of $(M_T \cdot M_R)$

If the channel state information is not known at the transmitter, can still get a diversity order of $M_T \cdot M_R$ using "space-time codes".

Multiplexing over a MIMO channel



$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

matrix representation of the 2x2 MIMO channel

$$\underset{2\times1}{y} = \underset{2\times2}{H}\,\underset{2\times1}{x} + \underset{2\times1}{n}$$

$$y_1 = h_{11}\cdot x_1 + h_{21}\cdot x_2 + n_1$$
$$y_2 = h_{12}\cdot x_1 + h_{22}\cdot x_2 + n_2$$

SVD — Singular vector decomposition

$$\text{svd}(H): H = U \Sigma V^H$$

diagonal matrix

$U^H U = I$

$V^H V = I$

unitary matrix

at the decoder, multiply by U^H

at the encoder multiply x by V

sending. $\tilde{x} = Vx$

at receiver: $U^H(H\tilde{x} + n) = y$.

$$y = \underbrace{U^H U}_{I} \Sigma \underbrace{V^H V}_{I} x + U^H n$$

$$y = \underbrace{\Sigma}_{2\times 2} \underbrace{x}_{2\times 1} + \underbrace{U^H n}_{\tilde{n}}$$

diagonal matrix column vector.

$$y = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{pmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{pmatrix}$$

$$y_1 = \sigma_1 x_1 + \tilde{n}_1$$

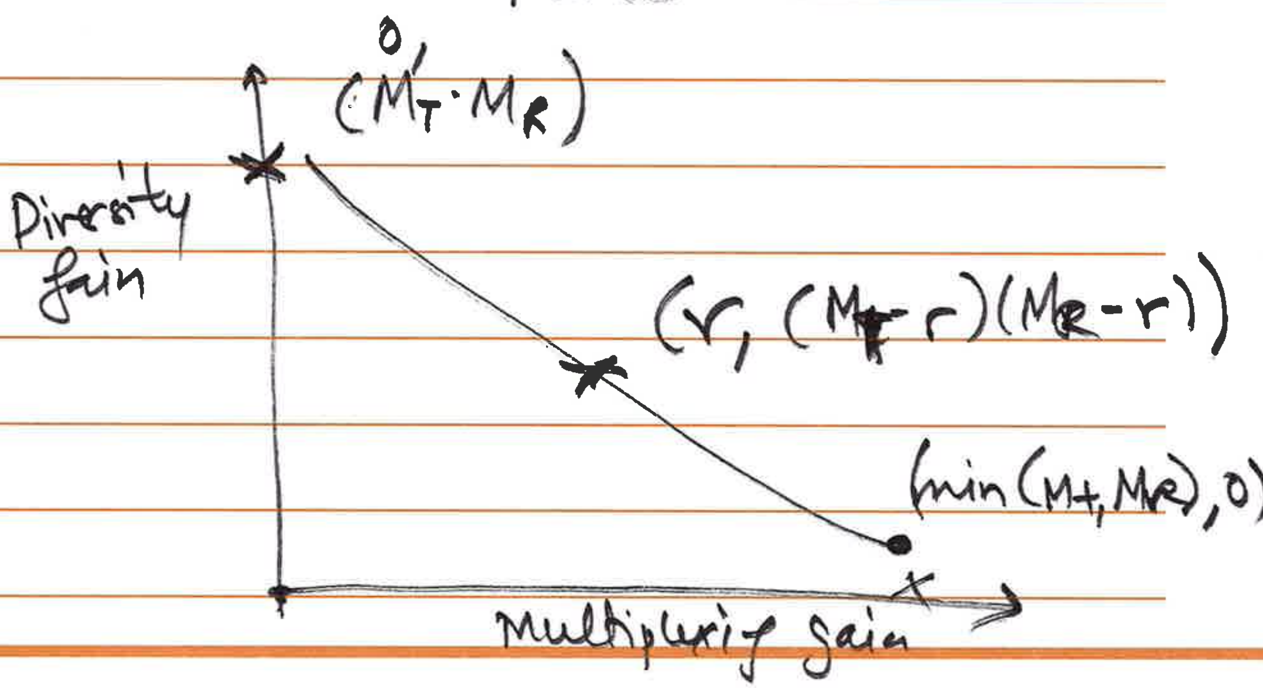
$$y_2 = \sigma_2 x_2 + \tilde{n}_2$$

This is like having two parallel virtual channels, doubling the rate.

In general, with M_T transmit antennas & M_R receive antennas, it is possible to have a maximum multiplexing gain of

$$\min(M_T, M_R).$$

Theoretical tradeoff between Diversity & multiplexing in MIMO



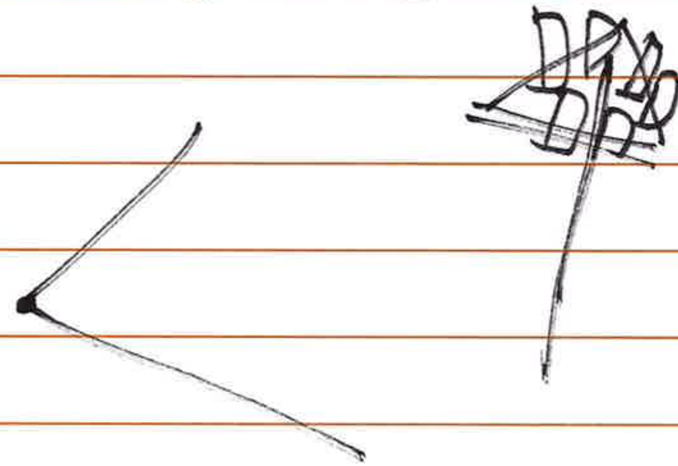
In general, under low SNR conditions, it's preferable to use the multiple antenna resources to obtain a diversity gain. Conversely, at high SNR, it is preferable to obtain a multiplexing gain to boost the data rates in a MIMO

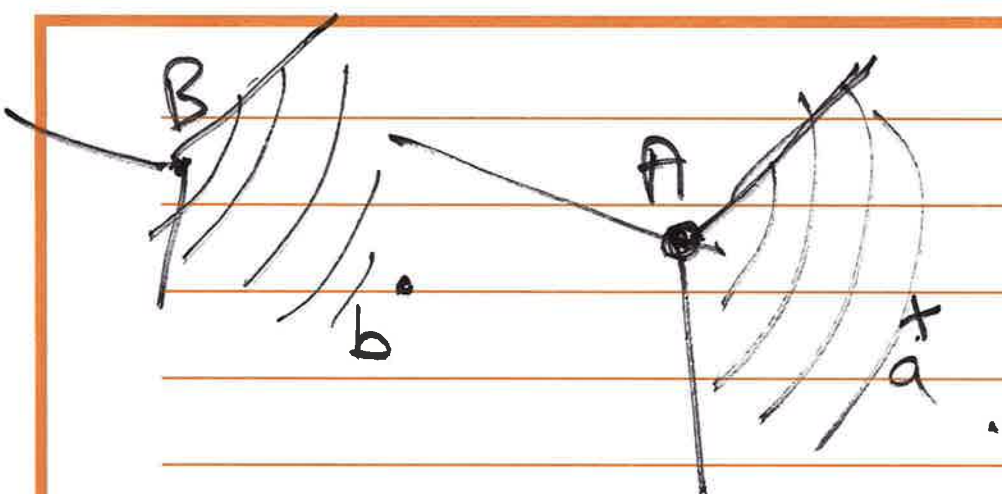
System.

Directional Antennas

typically, omnidirectional antennas are used

In cellular communications, Base Stations typically use 120° sector antennas



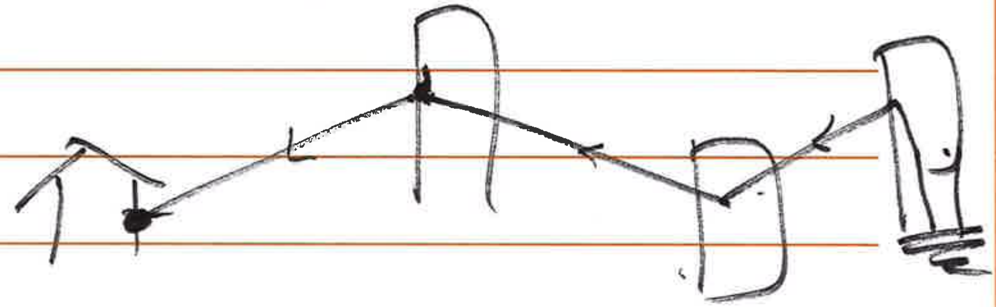


in this example node b does not experience interference from B.S. A.

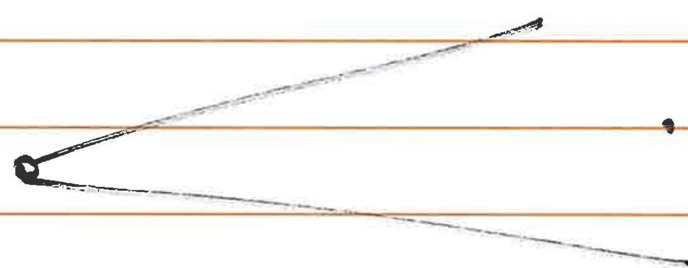
NetBlazr.com

L startup in Boston

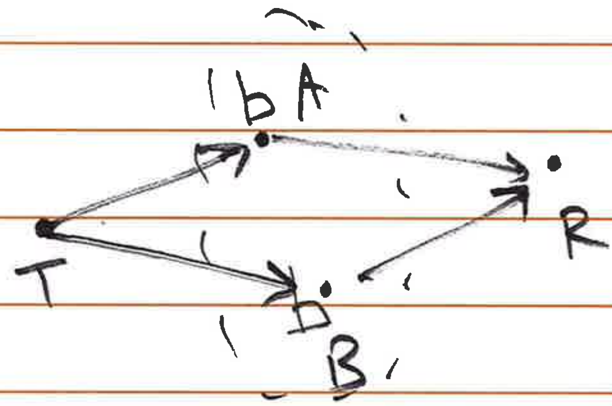
providing mesh-network based wireless Broadband



this boosts the SNR significantly.



Networked MIMO / distributed MIMO /
Cooperative networking / communications /
Distributed input Distributed output



4G/LTE standards talk about
CoMP — coordinated
multi point transmissions