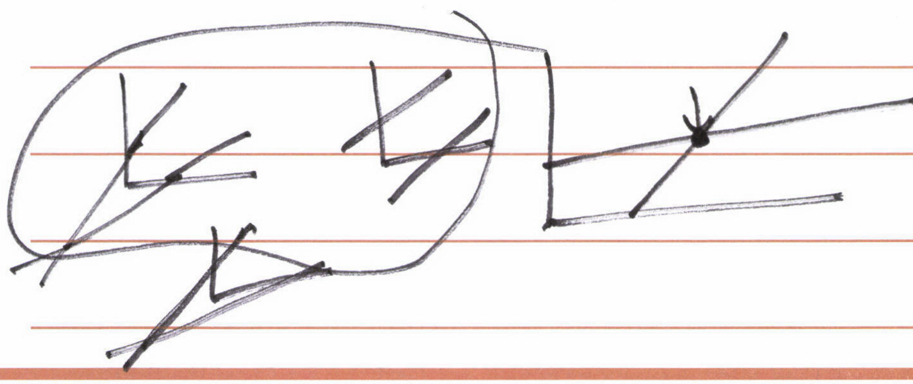
EE 597

Lecture 10 Feb 14, 2012

Power Control

Independent channels, allocating power levels to satisfy SINR threshold in fixed rate radios.



Corrected expressions for optimal power allocation

$$P_1^* = \frac{N\left(\frac{1}{g_{21}} + \frac{\theta}{g_{22}}\right)}{\left(\frac{g_{11}}{g_{21}\,\theta} - \frac{g_{12}}{g_{22}}\right)}$$

$$P_2^* = \frac{N\left(\frac{1}{g_{12}} + \frac{\theta}{g_{11}}\right)}{\left(\frac{g_{22}}{g_{12}\,\theta} - \frac{g_{21}}{g_{11}}\right)}$$

Generalize to M links

$$\frac{P_i\, g_{ii}}{\sum_{\substack{j=1 \\ j\neq i}}^{M} P_j\, g_{ji} + N} \geq \Theta \qquad \forall i$$

$$P_i\, g_{ii} \geq \sum_{\substack{j=1 \\ j\neq i}}^{M} \Theta\, g_{ji}\, P_j + N\Theta$$

$$P_i\, g_{ii} - \sum_{j=1}^{M} \Theta\, g_{ji}\, P_j \geq N\Theta \qquad \forall i = 1{:}M$$

At the minimum power vector solution (if it exists), all of these become equalities.

$$\underset{M \times M}{A}\;\underset{M \times 1}{\vec{x}} = \underset{M \times 1}{\vec{b}}$$

$\vec{p}$

$\vec{x}$

$$A = \begin{array}{c c} & \begin{array}{ccccc} & & & j^{th} & \end{array} \\ i^{th} & \begin{bmatrix} g_{11} & & & & \\ & g_{22} & & & \\ & & g_{ii} & \cdots \; -\theta g_{ji} & \cdots \\ & & & \ddots & \\ & & & \cdots & g_{MM} \end{bmatrix} \end{array} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_i \\ p_j \\ p_M \end{bmatrix}$$

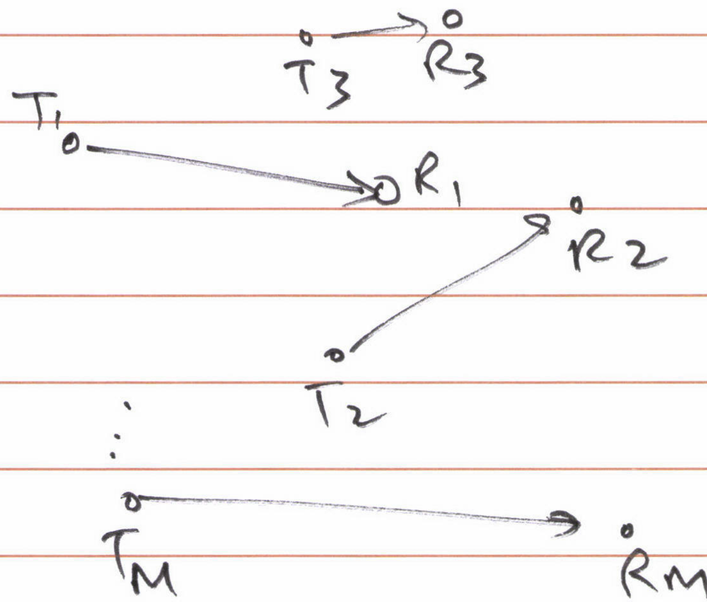
$$\vec{b} = \begin{bmatrix} N_0 \\ \vdots \\ N_0 \end{bmatrix}$$

$$A\vec{p} = \vec{b}$$

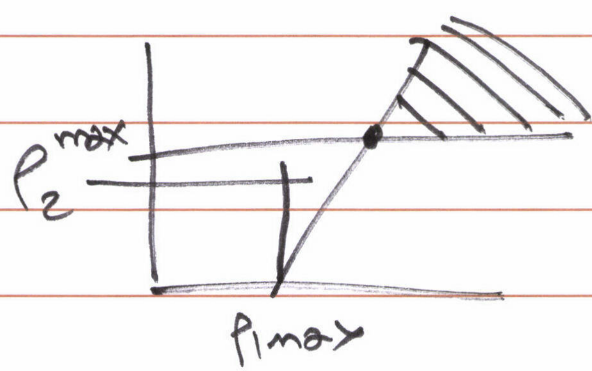
$$A^{-1}A\vec{p} = A^{-1}\vec{b}$$

$$\boxed{\vec{p} = A^{-1}b}$$

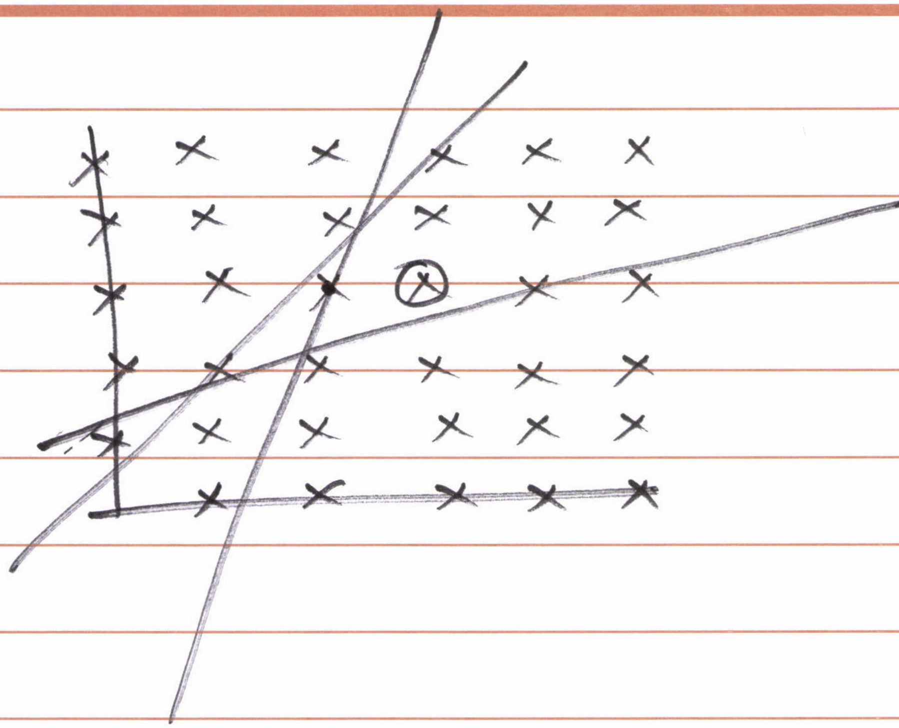
- A should be invertible i.e have nonzero determinant
- $\vec{p}^* \geq 0$ elementwise



Additional conditions typical in real radios:



- $P_i^* \leq P_i^{max}$
- P_i may only be allowed to take on discrete values.



Two issues to consider

- Distributed implementation
- What to do w/ infeasible situations?

There are 2^M possible configurations of M links.

In general finding the maximum sized configuration is computationally hard (NP-hard). Typically some kind of a greedy heuristic such as "turn off most-interfering transmitter" is used.

A related problem is finding the minimum # of time rounds / frequencies to schedule a given set of M links.

Foschini - Miljanic Algorithm for Iterative, Distributed Power Control

$$P_i^{t+1} = P_i^t \cdot \frac{\theta}{SINR_i(t)}$$

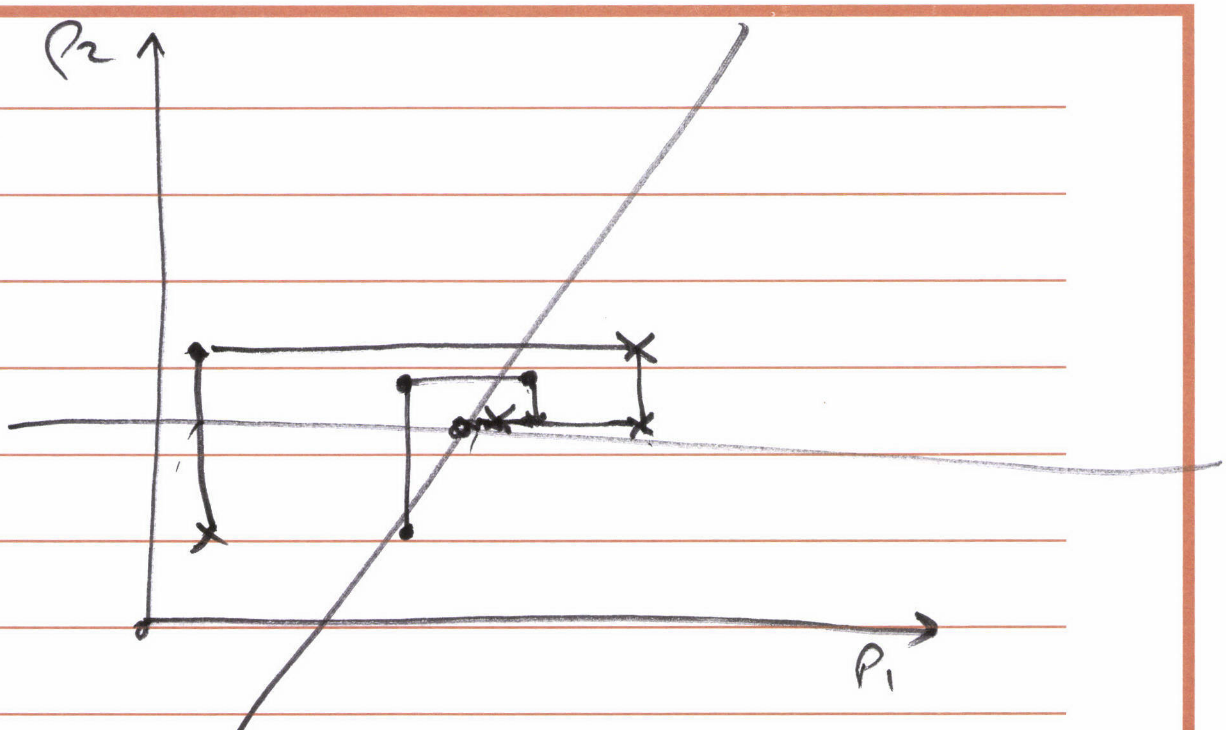
if $SINR_i(t) = \dfrac{P_i^t g_{ii}}{\sum_{j \neq i} P_j^t \cdot g_{ji} + N} > \theta$

the (link) node i will reduce its transmit power at time t+1.

If $SINR_i(t) < \theta$

the transmitting node i will increase its power at time t+1.

The only quantity that needs to be estimated is $SINR_i^t$, a local quantity at the receiver (communicated to the xmitter).



The algorithm works well even if links are not synchronized

Can prove that this algorithm <u>converges</u> to the global optimum power vector $\vec{p}^*$

as $t \to \infty$

In practice 2 to 3 iterations often suffice to get near-optimal, feasible power levels.

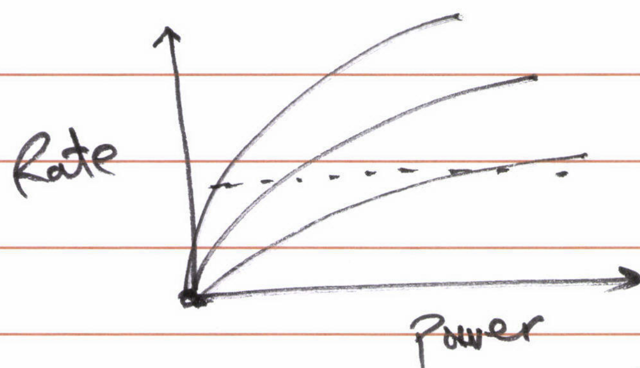
Power Allocation for Maximizing sum rate over parallel channels (assuming a rate-adaptive radio model).

$$\max \sum_{i=1}^{M} \log(1 + SNR_i)$$

$$\text{s.t.} \quad \sum_{i=1}^{M} P_i \leq P_{TOT}.$$

$$P_i \geq 0 \quad \forall i$$

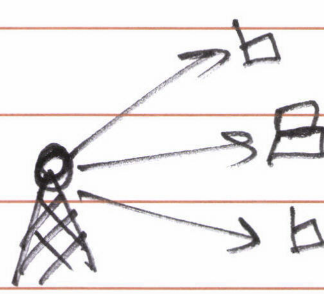
(no interference)



$$SNR_i = P_i \cdot \frac{g_i}{N_i}$$

gain to noise ratio.

e.g. Multiuser OFDM in WiMax



This is a convex optimization problem.

Solving this problem requires the use of Lagrange multipliers to handle the constraints.

Can simplify the problem to

$$\max \sum_{i=1}^{M} \log\left(1 + P_i\, g_i/N_i\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{M} P_i = P_{TOT}$$

$$\max f(\vec{x}) \quad \text{s.t.} \quad g_j(\vec{x}) \le c_j$$

$$\mathcal{L}(\vec{x}, \bar{\lambda}) = f(\vec{x}) - \sum_j \lambda_j (g_j(\bar{x}) - c_j)$$

find $\bar{x}^*, \bar{\lambda}^*$ that satisfies

KKT conditions: $\lambda_j^* \geq 0$

$g_j(\bar{x}^*) \le c_j$

complimentary slackness

$$\left.\frac{\partial L}{\partial x_i}\right|_{\bar{x}^*, \bar{\lambda}^*} = 0 \qquad \lambda_j^* (g_j(\bar{x}^*) - c_j) = 0$$

$-P_i \le 0 \Leftrightarrow P_i \ge 0 \; \forall i$

$$\max \sum \log(1 + P_i g_i / N_i) \quad \text{s.t.} \quad \sum P_i \le P_{TOT}$$

$$\boxed{\mathcal{L}(\bar{P}, \lambda) = \sum_{i=1}^{M} \log(1 + P_i g_i / N_i) - \lambda\left(\sum P_i - P_{TOT}\right) + \sum \nu_i P_i}$$

$\lambda \geq 0 \qquad \sum P_i \le P_{TOT} \qquad \nu_i \geq 0$

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{g_i / N_i}{\left(1 + P_i^* \frac{g_i}{N_i}\right)} - \lambda^* + \nu_i^* = 0$$

$$\lambda^* \left(\sum P_i^* - P_{TOT}\right) = 0 \qquad \nu_i^* \cdot P_i^* = 0$$

$\sum P_i^* \le P_{TOT}$ complementary slackness conditions

$P_i^* \geq 0$

for any given channel, either $\nu_i^* = 0$ or $P_i^* = 0$.

focus on a channel i that gets non-zero power, its $\nu_i^* = 0$

$$\therefore \frac{g_i/N_i}{(1 + P_i g_i/N_i)} - \lambda^* + \cancel{\nu_i^*}^{\,0} = 0$$

$$\lambda^* = g_i/N_i \Big/ \left(1 + P_i^* g_i/N_i\right)$$

$$P_i^* = \left(\frac{g_i/N_i}{\lambda^*} - 1\right) \cdot \frac{N_i}{g_i}$$

$$P_i^* = \left(\frac{1}{\lambda^*} - \frac{N_i}{g_i}\right)$$

$$P_i^* = \left(\frac{1}{\lambda^*} - \frac{N_i}{g_i}\right)^+ \quad \forall i \quad \text{—— (1)}$$

$$\sum_{i=1}^{M} P_i^* = P_{TOT} \quad \text{—— (2)}$$

$$\sum_{i=1}^{M} \left(\frac{1}{\lambda^*} - \frac{N_i}{g_i} \right)^+ = P_{TOT}$$

an implicit equation that defines what λ^* should be.

This solution is called waterfilling