EE 597 February 16, 2012

Power control.

- Independent links, fixed threshold/rate radios

  $SINR_i \geq \theta$

  $0 \leq P_i \leq P_{max}$

- Parallel links, rate adaptive radios

  $\max \sum \log(1 + SNR_i)$ s.t. $\sum P_i \leq P_T$, $P_i \geq 0$

$$\text{Max} \sum \log\left(1 + \frac{P_i g_i}{N_i}\right)$$

$$\text{s.t.} \quad \sum P_i \leq P_T \quad , \quad P_i \geq 0$$

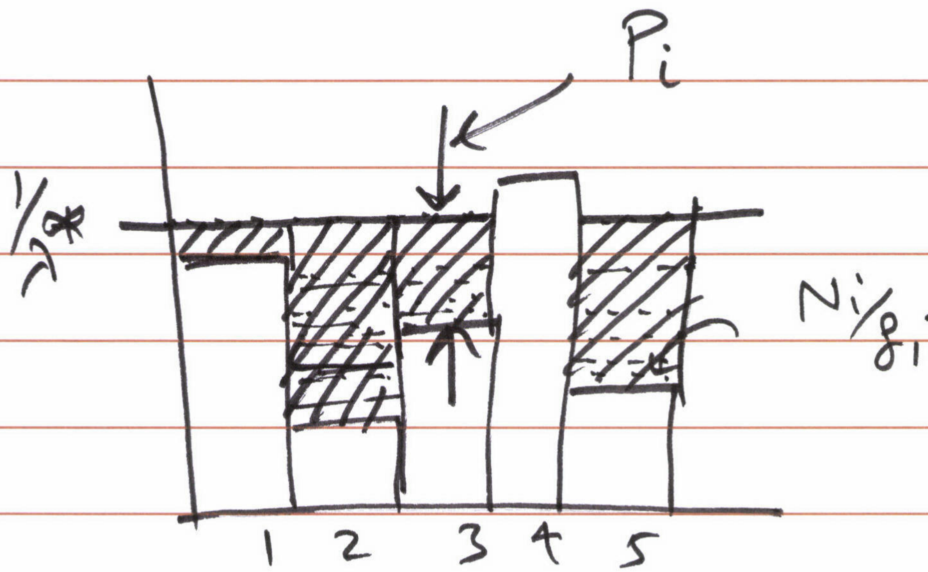
$$P_i^* = \left(\frac{1}{\lambda^*} - \frac{N_i}{g_i}\right)^+$$

$$= \max\left(\frac{1}{\lambda^*} - \frac{N_i}{g_i}, 0\right) \quad \text{(1)} \quad \forall i$$

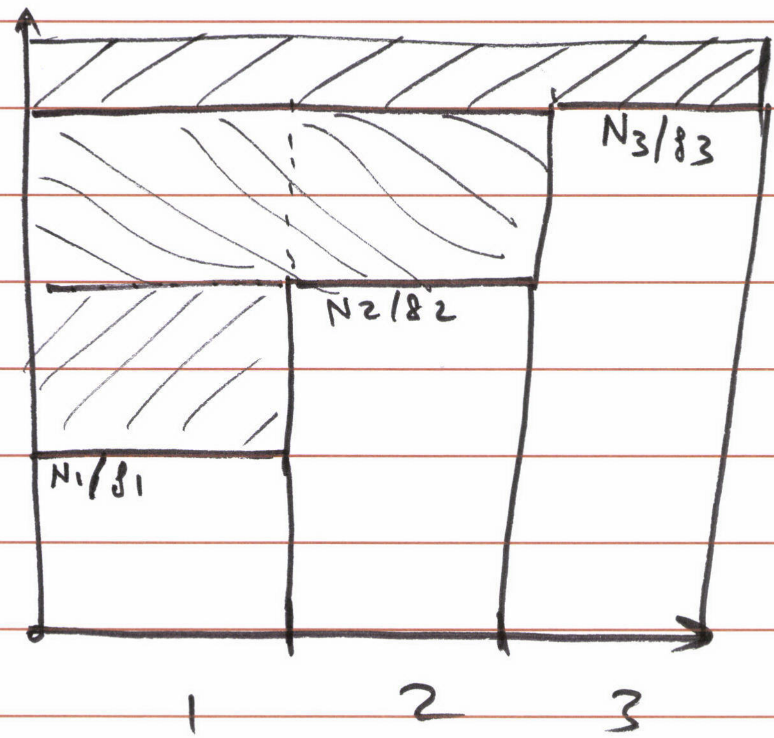
to figure out what λ^* is, use

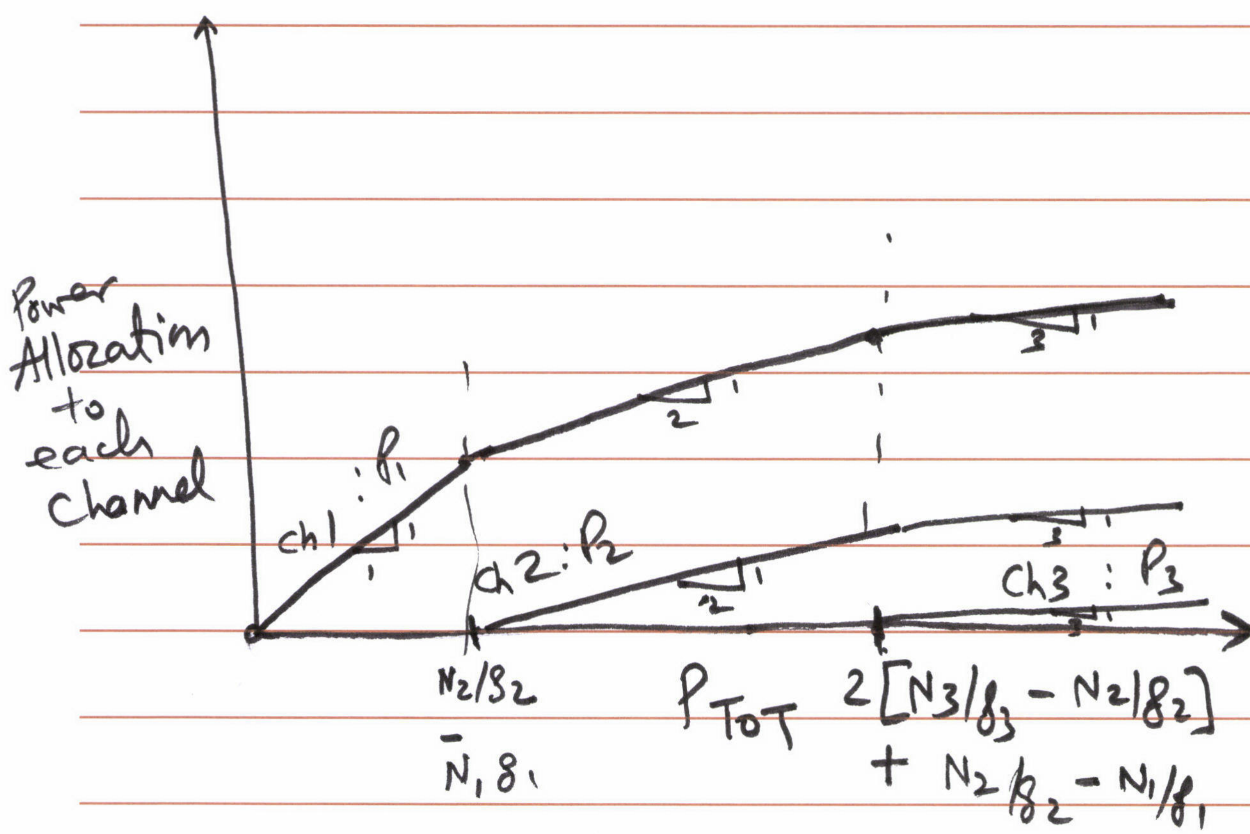
$$\sum_{i=1}^{M} P_i = \sum_{i=1}^{M} \left(\frac{1}{\lambda^*} - \frac{N_i}{g_i} \right)^+ = P_{TOT} \quad \text{—(2)}$$

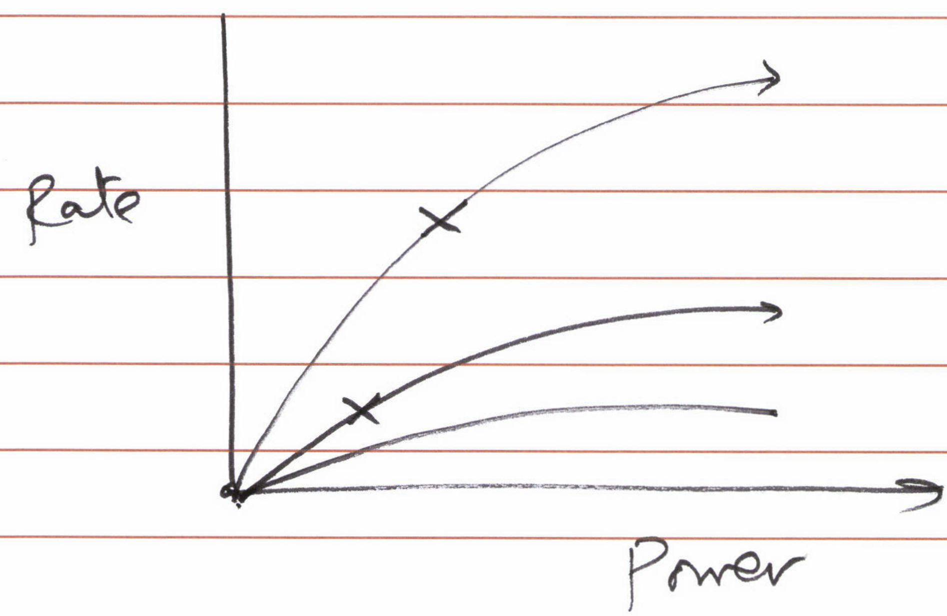
notice that λ^* is indep. of i.



Note: power P_i is being measured in Watts (not dB).


N3/g3
N2/g2
N1/g1
1
2
3


Power Allocation to each Channel
Ch1 : P1
Ch2 : P2
Ch3 : P3
N2/g2 − N1 g1
P TOT
2[N3/g3 − N2/g2] + N2/g2 − N1/g1

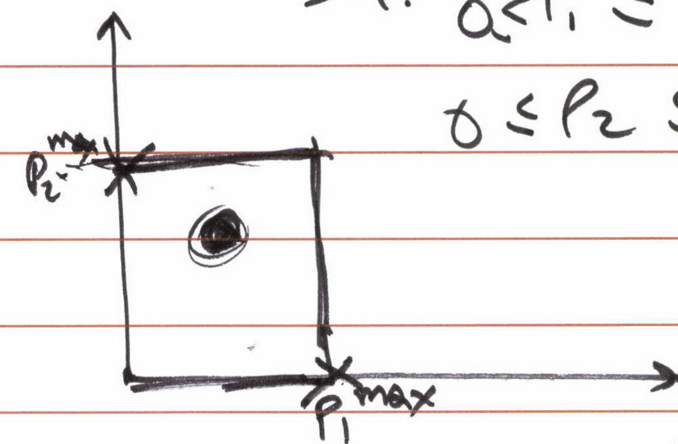


	Fixed Rate radio	Rate Adaptive radio
Independent Links (interference)	×	? ✓
Parallel Links (power budget)	✓? =	×

$$\max_{P_1,P_2} \overbrace{\log\left(1+\frac{P_1 g_{11}}{P_2 g_{21}+N}\right)+\log\left(1+\frac{P_2 g_{22}}{P_1 g_{12}+N}\right)}^{R(P_1,P_2)}$$

$$\text{s.t.}\quad 0 < P_1 \le P_1^{max}$$

$$0 \le P_2 \le P_2^{max}$$



Note: in the following, we are ignoring constraints.

$$\frac{\partial R}{\partial P_1}=0 \Rightarrow \frac{g_{11}/(P_2 g_{21}+N)}{1+\frac{P_1 g_{11}}{P_2 g_{21}+N}} + \frac{1}{1+\frac{P_2 g_{22}}{P_1 g_{12}+N}} \cdot$$

$$\left[-\frac{P_2 g_{22}}{(P_1 g_{12}+N)^2}\right]\cdot g_{12} = 0$$

$$g_{11}(P_1 g_{12} + N + P_2 g_{22})(P_1 g_{12} + N)$$
$$= P_2 g_{22} g_{12}(P_2 g_{21} + N + P_1 g_{11})$$

$$(g_{11} P_1 g_{12} + N g_{11} + g_{11} P_2 g_{22})(P_1 g_{12} + N)$$
$$= P_2^2 g_{22} g_{12} g_{21} + N P_2 g_{22} g_{12} + \cancel{P_1 P_2 g_{22} g_{12} \cdot g_{11}}$$
$$= P_1^2 g_{11} g_{12}^2 + P_1 N g_{11} g_{12} + \cancel{P_1 P_2 g_{11} g_{22} g_{12}}$$
$$+ g_{11} P_1 g_{12} N + N^2 g_{11} + g_{11} P_2 g_{22} \cdot N$$

$$P_2^2 g_{22} g_{12} g_{21} - P_1^2 g_{11} g_{12}^2$$
$$+ P_2 \left[N g_{22} g_{12} - g_{11} g_{22} \cdot N \right]$$
$$+ P_1 \left[-2N g_{11} g_{12} \right]$$
$$- N^2 g_{11} = 0$$

<u>Simplify by assuming</u> $g_{12} = g_{21} = h$, $g_{11} = g_{22} = g$.

$$P_2^2 g h^2 - P_1^2 g h^2$$

$$+ \cancel{P_2}(0) - P_1 2Ngh$$

$$- N^2 g = 0$$

$$P_2^2 g h^2 = P_1^2 g h^2 + P_1 \cdot 2Ngh + N^2 g$$

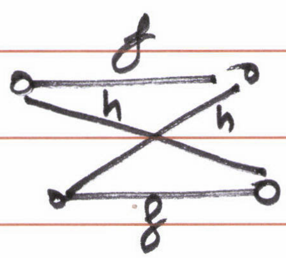
$$P_2 = \sqrt{P_1^2 + 2N\frac{P_1}{h} + N^2/h^2}$$

$$\Rightarrow \quad P_2 = P_1 + N/h$$

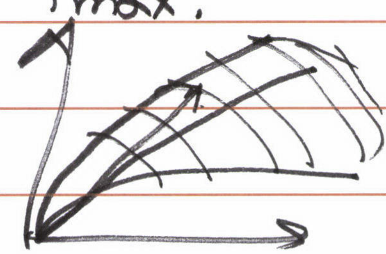
$$P_1 = P_2 + N/h$$

(i.e. no critical point!)

$\rightarrow (\infty, \infty)$



$P_1 = 0 \qquad P_2 \rightarrow P_{max}.$



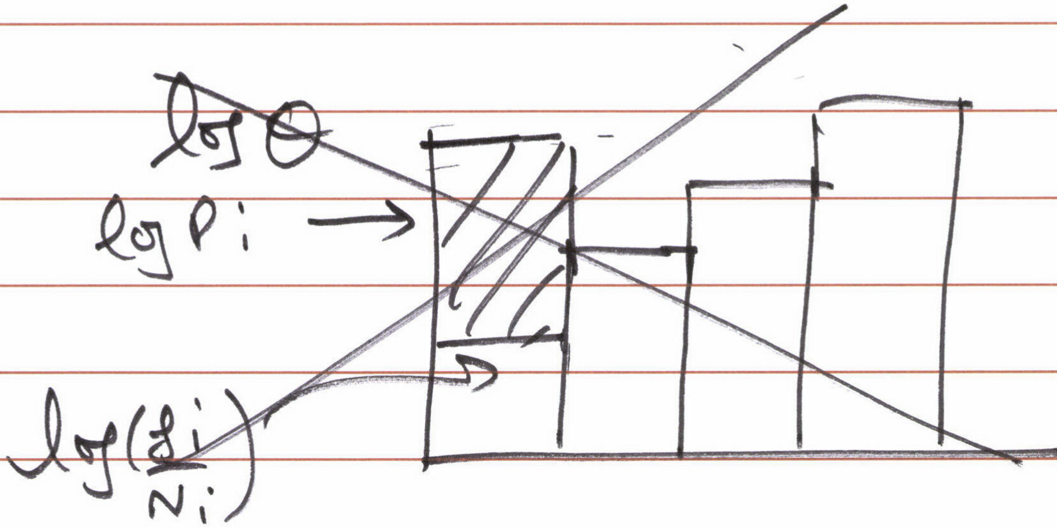
What about Parallel links w/ power budget & fixed rate radio?

$$\frac{P_i g_i}{N_i} > \theta$$

$$\sum P_i \leq P_{TOT}$$

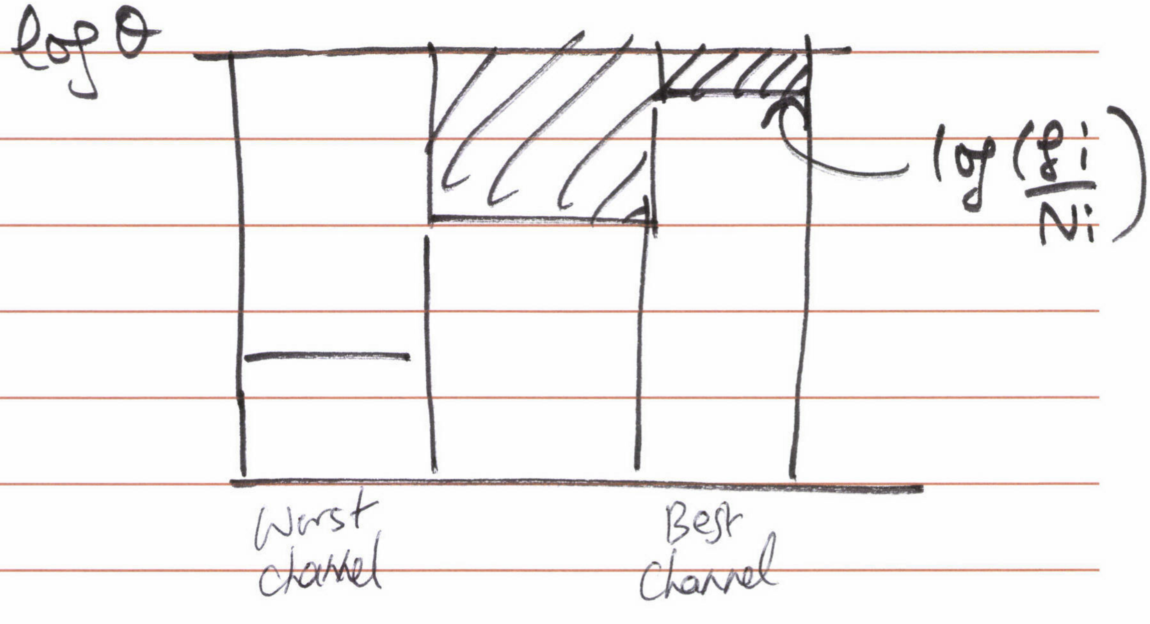
max # channels satisfying threshold

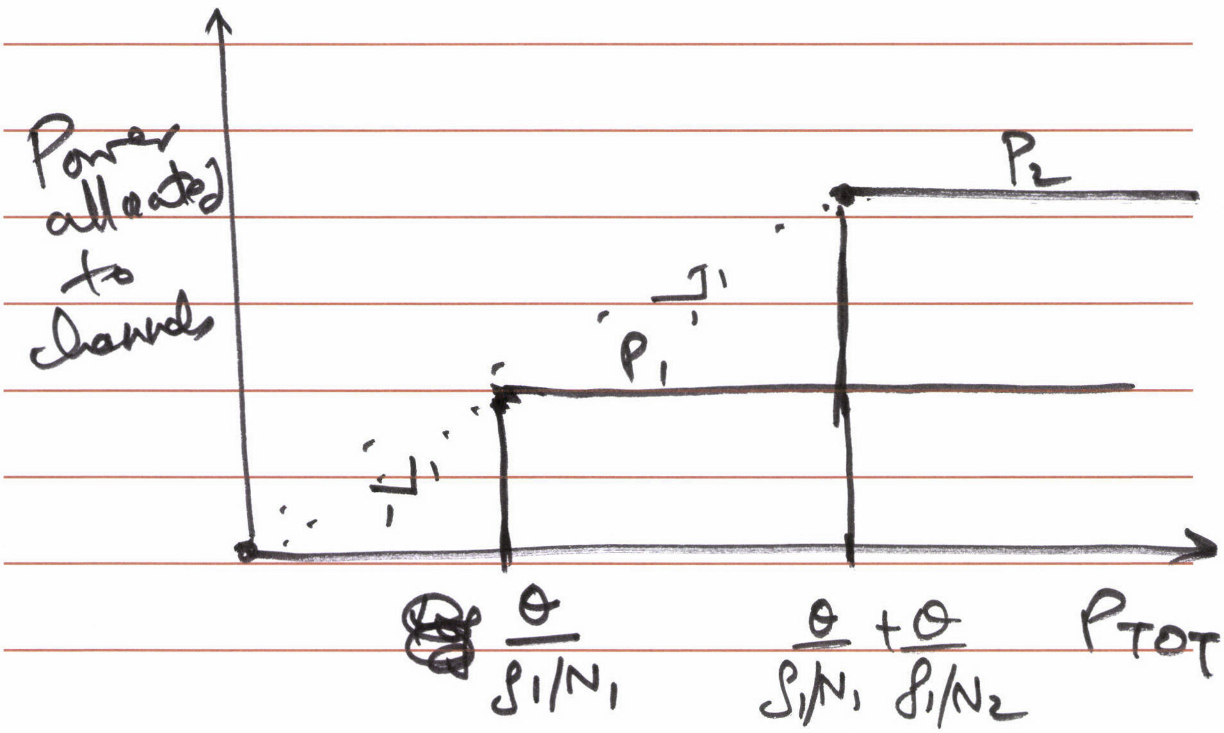
greedy solution: sort channels in increasing order of N_i/g_i, & put in enough power on each channel in order to meet threshold, until you run out of power.



$$P_i \frac{g_i}{N_i} = \Theta$$

$$\log P_i + \log \frac{g_i}{N_i} = \log \Theta$$





Coming Up:

Medium Access & Resource Allocation

- Power ✓
- Time ← Random Access
  - Scheduled Access
- Frequency, "Channels" } channel Allocation

Random Access:

- Aloha / Slotted Aloha
- CSMA
  - p-CSMA
  - 802.11 DCF
  - RTS/CTS

Throughput Analysis.

Protocol issues

Recall: Indep. links with fixed rate radio:

$$SINR_i \geq \theta_i$$

$$\frac{P_i g_{ii}}{\sum_{j \neq i} P_j g_{ji} + N} \geq \theta_i$$

What about if we have rate adaptive radio?

$$\max \sum \log(1 + \underline{\underline{SINR_i}})$$

$$\max \sum_{i=1}^{M} \log\left(1 + \frac{P_i g_{ii}}{\sum_{\substack{j=1 \\ j \neq i}}^{M} P_j g_{ji} + N}\right)$$

$P_1 \ldots P_M$ $\quad X$ power levels, M links $\quad X^M$

$$\frac{g_{11}/(\cancel{P_2 g_{21}} + N)}{\dfrac{P_2 g_{21} + N + P_1 g_{11}}{\cancel{P_2 g_{21} + N}}} + \frac{\cancel{P_1 g_{12}} + N}{P_1 g_{12} + N + P_2 g_{22}} \cdot \frac{-P_2 g_{22} \cdot g_{12}}{(P_1 g_{12} + N)^{\cancel{2}}} = 0.$$

$$\frac{g_{11}}{P_2 g_{21} + N + P_1 g_{11}} = \frac{P_2 g_{22} g_{12}}{(P_1 g_{12} + N + P_2 g_{22}) \cdot (P_1 g_{12} + N)} \quad — (1)$$

$$\partial R / \partial P_1 = 0 \quad ; \quad \frac{\partial R}{\partial P_2} = 0$$

$$\frac{g_{22}}{P_1 g_{12} + N + P_2 g_{22}} = \frac{P_1 g_{11} g_{21}}{[P_2 g_{21} + N + P_1 g_{11}] \cdot [P_2 g_{21} + N]} \quad — (2)$$

in principle, (1) & (2) represent 2 eqns in 2 unknowns, if $\exists$ a solution, this will be yielded by solving these two non-linear equations.