Lecture 12

EE 597

2/21/2012

Announcements

- Exam on 3/8 in-class

- HW 2 posted this week, due back on 3/6

- no class on thursday 2/23 rescheduled taping TBA.

- Tomorrow : talk by Arinash Sridharan, Aruba Networks 3:30 pM 2/22 SGM 101

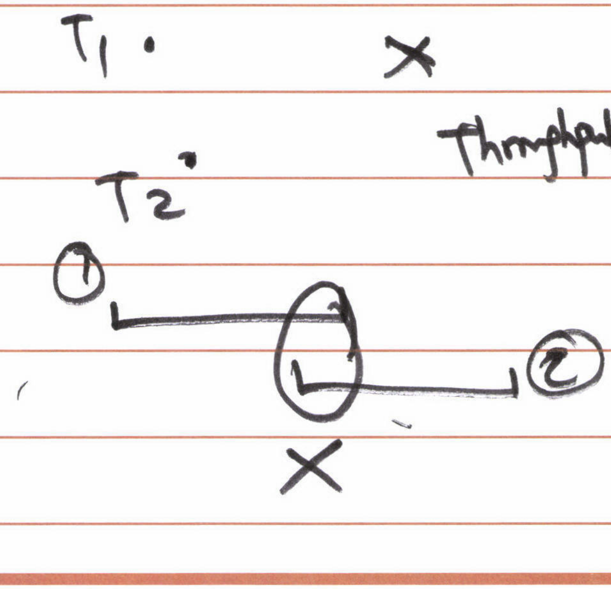
Medium Access & Resource Allocation

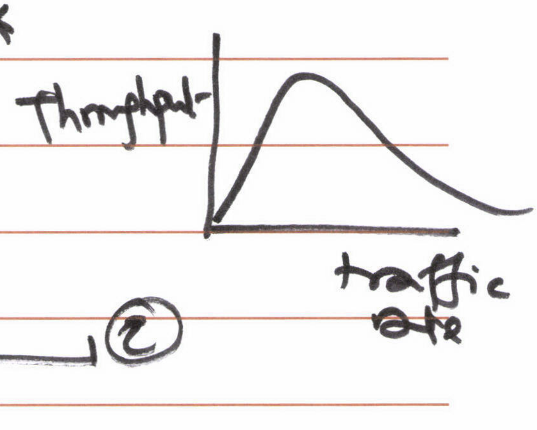
Random Access Techniques

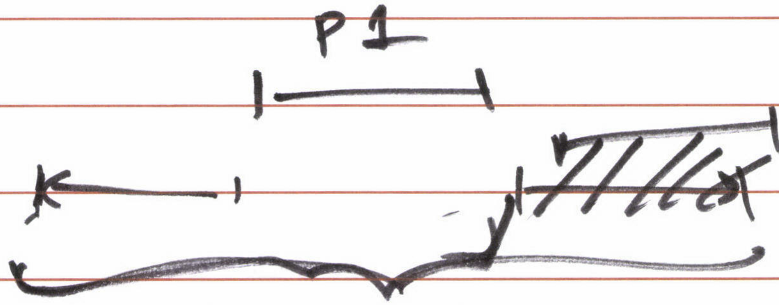
Aloha (dates back to the early 1970's. Abramson @ U. Hawaii)

Simplest form: send when ready.

Unslotted Aloha.



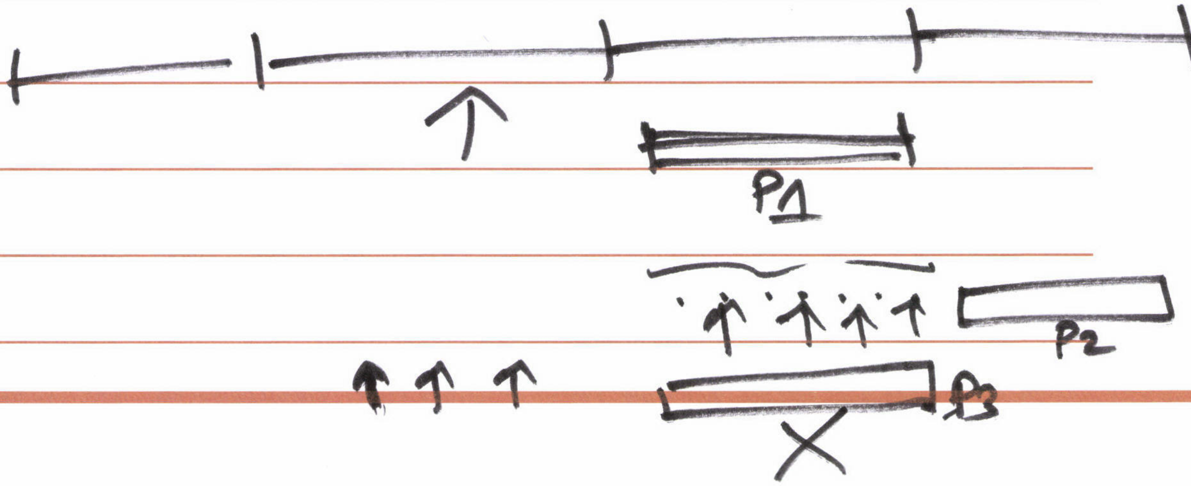




Arrival at node 2 in this time causes a collision

SLOTTED ALOHA

Time is discretized.
Transmit only at the start of an interval (next interval)



The collision rate in slotted Aloha is halved because transmissions are deferred to the next slot beginning.

model slotted Aloha:

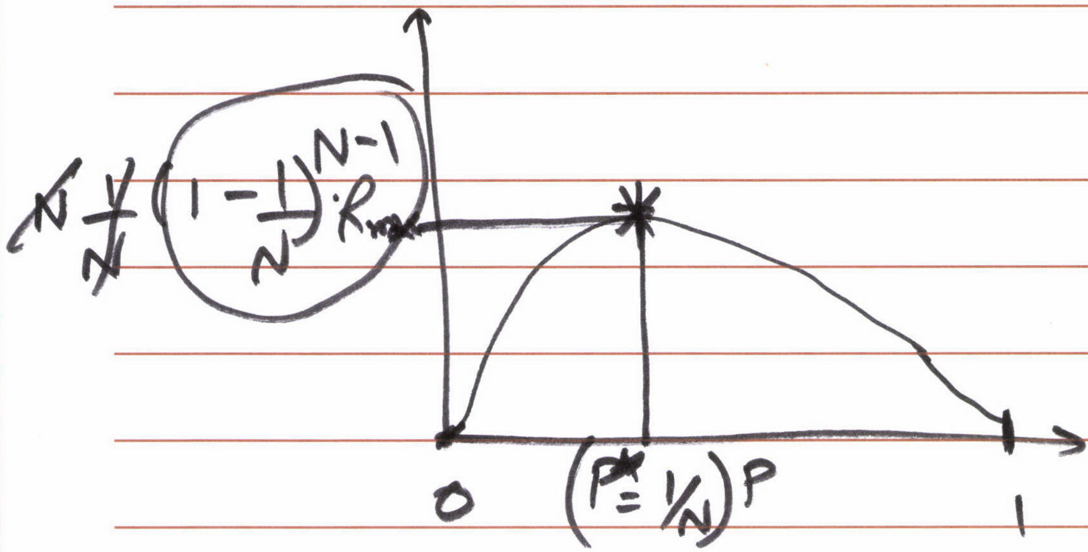
Packets arrive at each slot for each of N transmitters w/ probability p. (i.i.d.)

all senders are xmitting to same receiver.

Events	Prob
IDLE	$(1-p)^N$
SUCCESS	$Np(1-p)^{N-1}$
COLLISION	$1-(1-p)^N - Np(1-p)^{N-1}$

nominal throughput : $NP(1-P)^{N-1} \cdot R_{max}$

max link rate



$$T = \text{max throughput} = \max_{P} NP(1-P)^{N-1}$$

$$\frac{\partial T}{\partial P} = 0 \Rightarrow (N-1)(1-P)^{N-2} \cdot P - (1-P)^{N-1} = 0$$

$$(1-P)^{N-2}\left[(N-1)P - (1-P)\right] = 0$$

$$NP - \cancel{P} - 1 + \cancel{P} = 0$$

$$\Rightarrow P^* = 1/N.$$

max throughput is

$$\left(1-\frac{1}{N}\right)^{N-1} R_{max}$$

$N=2$ $\qquad$ $1/2\ R_{max}$

$N=3$ $\qquad$ $4/9\ R_{max}$

$\vdots$

$$\lim_{N\to\infty} \left(1-\frac{1}{N}\right)^{N-1} = e^{-1} R_{max}$$

maximum achievable throughput in the limit.

$\sim 36\%$ efficiency.

per-source max throughput $\sim \frac{e^{-1}}{N}$

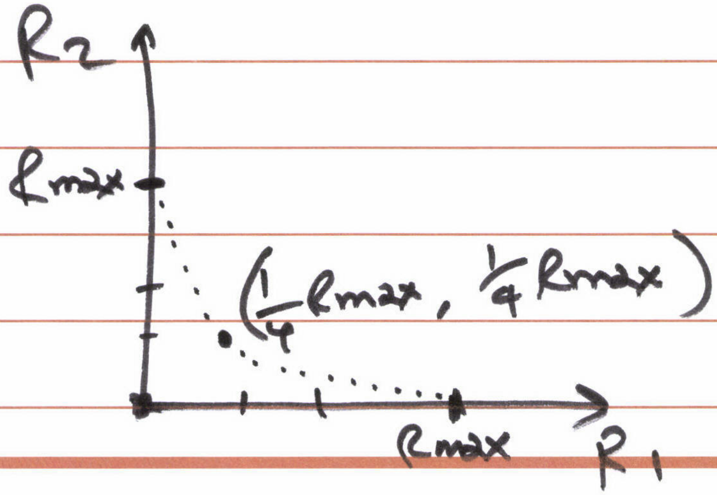
Note:

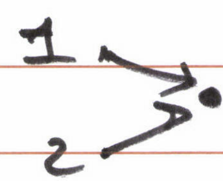
same analysis holds, if we have backlogged / saturated users (who always have a pkt) but they transmit w/ iid probability p at each time.

Heterogeneous users case

Consider 2 BACKLOGGED users:

$$\underbrace{p_1, \; p_2}_{\text{different transmit prob. for each.}}$$





What is the Throughput region for slotted Aloha with 2 saturated users?

$$R_1 = P_1(1 - P_2) \cdot R_{max}$$

$$R_2 = P_2(1 - P_1) \cdot R_{max}$$

Keep R_1 constant & maximize R_2

$R_1 = c.$

for simplicity, let's normalize our rates by R_{max}.

$$\max_{(P_1, P_2)} P_2(1 - P_1)$$

$$\text{s.t. } P_1(1 - P_2) = c \quad \text{— } 2$$

$$\Rightarrow P_2 = 1 - \frac{c}{P_1}$$

$$\Leftrightarrow \quad \max_{p_1} \left(1 - \frac{c}{p_1}\right) \overbrace{\left(1 - p_1\right)}^{\tilde{R}_2}$$

$$\frac{\partial \tilde{R}_2}{\partial p_1} = 0 \Rightarrow \frac{\partial}{\partial p_1}\left[1 - \frac{c}{p_1} - p_1 + c\right] = 0$$

$$\Rightarrow \frac{c}{p_1^{*2}} = 1$$

$$p_1^{*2} = c \quad \text{—— (1)}$$

$$p_1^*(1 - p_2^*) = c \quad \text{—— (2)}$$

$$\therefore \quad p_1^* = 1 - p_2^* \qquad p_1^* + p_2^* = 1$$

Rates at the boundary $\longrightarrow \left[p_1^*(1-p_2^*) \, , \; p_2^*(1-p_1^*) \right]$

$$p_1^{*2} \, , \quad (1 - p_1^*)^2$$

$$\left[R_1 \, , \quad (1 - \sqrt{R_1})^2 \right]$$

$$R_1 \in [0, 1]$$

bdry of the Rate region $\hookrightarrow$

$$\boxed{R_2 = \left(1 - \sqrt{R_1}\right)^2}$$

In general for n users:

$$R_i = p_i \prod_{j \neq i} (1 - p_j) \quad \forall i$$

- The boundary of this rate region is describe by all vectors $\vec{p}$ s.t.

$$\sum p_i = 1.$$

A result due to Massey & Mathys, 1983

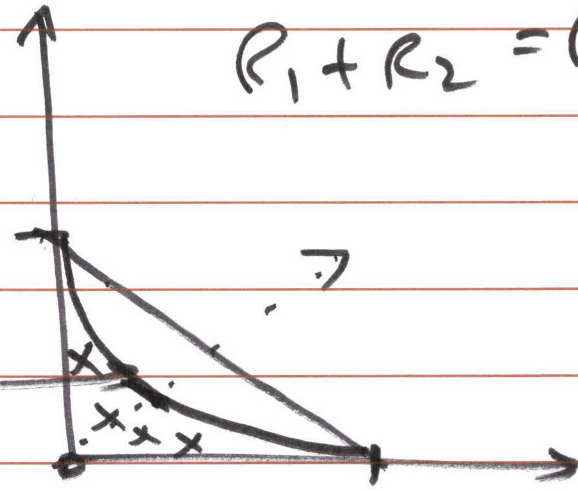
Note: NOT an obvious statement

If $p_1 + p_2 \neq 1$

the corresponding rates will lie in the interior of the rate region

TDMA:

$R_1 + R_2 = R_{max}$



slotted Aloha

then why bother?

TDMA gives full utilization
but requires coordination

incurs overhead when traffic
is sporadic / random / non-stationary

∴ rarely used for stochastic
data arrivals.

e.g: $p_1 = 3/4$, $p_2 = 1/2$

$$R_1 = 3/4 \cdot 1/2 = 3/8$$

$$R_2 = 1/2 \cdot 1/4 = \frac{1}{8}$$

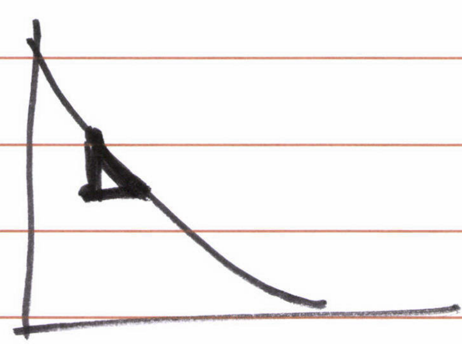
$$R_2^* = (1 - \sqrt{R_1})^2$$

could have had

$$R_2^* = \left(1 - \frac{\sqrt{3}}{2\sqrt{2}}\right)^2$$

$$> 1/8$$

(check!)



$p_1 = \sqrt{3/8}$

$p_2 = 1 - \sqrt{\frac{3}{8}}$

CSMA - Carrier sence
multiple/medium access

first sense the channel; if busy, wait, else transmit.

p- CSMA

sensing slots of length σ

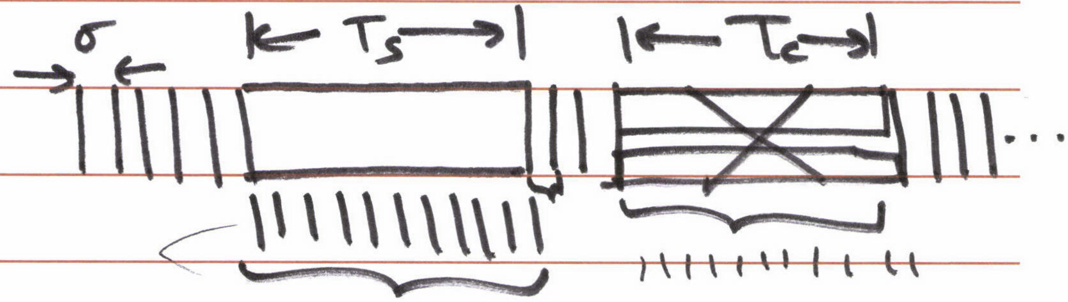
if no pkt, send w/ probability p

packet duration :

T_s if success

T_c if collision

$T_c \leq T_s$



At every idle slot, there is contention for transmission w/ probability p

If successful (i.e. only one of the n contenders transmits) the channel is utilized for duration T_s.

If unsuccessful w/ collision (i.e. more than transmit simultaneously) channel is wasted for T_c duration.

If idle, the next contention starts after σ duration

Expected throughput of p-CSMA

= $\dfrac{\text{Expected time that a virtual contention slot is successful}}{\text{Expected duration of a virtual contention slot.}}$

At each virtual contention period

EVENT	PROBABILITIES	DURATIONS
idle	$(1-p)^N$	σ
success	$Np(1-p)^{N-1}$	T_s
collision	$1-(1-p)^N-Np(1-p)^{N-1}$ $=\sum_{i=2}^{N}\binom{N}{i}p^i(1-p)^{N-i}$	T_c

Expected Duration of VCS :

$$(1-p)^N \sigma + N p (1-p)^{N-1} \cdot T_s +$$
$$1 - (1-p)^N - N p (1-p)^{N-1} T_c$$

Expected success Duration :

$$N p (1-p)^{N-1} T_s$$

$$\text{Rate}_{p\text{-CSMA}} = \frac{N p (1-p)^{N-1} T_s}{\left[(1-p)^N \sigma + N p (1-p)^{N-1} T_s + \left(1 - (1-p)^N - N p (1-p)^{N-1}\right) T_c\right]} \cdot R_{max}$$

a generalization of p-slotted Aloha

this is when
$(\sigma = T_s = T_c)$