Lecture 12

Announcements

• Exam on 3/8 in-class

• HW 2 posted this week, due back on 3/6

• No class on Thursday 2/23, rescheduled timing TBA.

• Tomorrow: talk by Aninath Sridharan, Aruba Networks
  3:30 pm 2/22 SGM 101
Medium Access & Resource Allocation

Random Access Techniques

Aloha (dates back to the early 1970's. Abramson @ U. Hawaii)

Simplest form: Send when ready.

Unslotted Aloha.

\[ T_1 \times \]

\[ T_2' \text{ traffic} \]

\[ \text{Thought} \]
Arrival at node 2 in this time causes a collision

SLOTTED ALOHA

Time is discretized.
Transmit only at the start of an interval (next interval)

P1

K

P2
The collision rate in slotted Aloha is halved because transmissions are deferred to the next slot beginning.

model slotted Aloha:
packets arrive at each slot for each of \( n \) transmitters with probability \( p \) (i.i.d.)

all senders are simultaneous

\[
\begin{align*}
\text{Events} & \quad \text{Idle} & \quad (1 - p)^n \\
& \quad \text{Success} & \quad NPC(1 - p)^{n-1} \\
& \quad \text{Collision} & \quad 1 - (1 - p)^n - NPC(1 - p)^{n-1}
\end{align*}
\]
nominal throughput: $N P (1 - p)^{N-1}$

$R_{\text{max}}$

max.

link rate

$N \left(1 - \frac{1}{N} p \right)^{N-1}$

$0 \quad (p^* = \frac{1}{N}) p \quad 1$

$T: \text{max throughput} = \max_p NP (1 - p)^{N-1}$

$\frac{\partial T}{\partial p} = 0 \Rightarrow (N-1)(1-p)^{N-2} p + (1-p)^{N-1} = 0$

$(1-p)^{N-2} \left[ (N-1)p - (1-p) \right] = 0$

$NP - p + 1 + p^* = 0$

$\Rightarrow p^* = 1/N.$
max throughput is
\[ (1 - \frac{1}{N})^{N-1} R_{\text{max}} \]

N = 2: \[ \frac{1}{2} R_{\text{max}} \]
N = 3: \[ \frac{4}{9} R_{\text{max}} \]

\[ N \to \infty \quad \sum (1 - \frac{1}{N})^{N-1} = e^{-1} R_{\text{max}} \]

maximum achievable throughput in the limit.
\[ \sim 36\% \] efficiency.

per-source throughput \[ \sim \frac{e^{-1}}{N} \]
**Note:**

Same analysis holds, if we have backlogged/saturated users (who always have a pkt) but they transmit iid probability \( p \) at each time.

**Heterogeneous users case**

Consider 2 **backlogged users**:

\[ P_1, P_2 \]

Different transmit prob. for each.

\[ \text{Graph with points at } (\frac{1}{4} R_{\text{max}}, \frac{1}{4} R_{\text{max}}), (0, R_{\text{max}}), \text{ and } (R_{\text{max}}, 0) \]
What is the Throughput region for slotted Aloha with 2 saturated users?

\[ R_1 = P_1 (1 - P_2) \cdot R_{\text{max}} \]

\[ R_2 = P_2 (1 - P_1) \cdot R_{\text{max}} \]

Keep \( R_1 \) constant & maximize \( R_2 \)

\[ R_1 = c. \]

For simplicity, let's normalize our rates by \( R_{\text{max}} \).

\[ \max_{(P_1, P_2)} P_2 (1 - P_1) \]

s.t. \( P_1 (1 - P_2) = c \quad - (2) \]

\[ \Rightarrow \quad P_2 = 1 - \frac{c}{P_1} \]
\[ \max_{p_i} \left( 1 - c \right) \left( 1 - p_i \right) \]

\[ \frac{\partial R_2}{\partial p_i} = 0 \Rightarrow \frac{2}{\partial p_i} \left[ 1 - c - \frac{p_i}{p_i^*} + c \right] = 0 \]

\[ \Rightarrow \frac{c}{p_i^*} = 1 \]

\[ p_i^* = c \quad \text{(1)} \]

\[ p_i^* \left( 1 - p_i^* \right) = c \quad \text{(2)} \]

\[ p_i^* = 1 - p_i^* \quad p_i^* + p_i^* = 1 \]

Rates at the boundary

\[ \begin{bmatrix} p_i^* \left( 1 - p_i^* \right) \quad p_i^* \left( 1 - p_i^* \right) \end{bmatrix} \]

\[ R_i^*, \quad \left( 1 - p_i^* \right)^2 \]

\[ \begin{bmatrix} R_1^* \quad \left( 1 - \sqrt{R_1^*} \right)^2 \end{bmatrix} \]

\[ R_1 \in (0, 1) \]

\[ R_2 = \left( 1 - \sqrt{R_1^*} \right)^2 \]
In general for $n$ users:

$$R_i = p_i \prod (1-p_j) + i$$

The boundary of this rate region is described by all vectors $\frac{1}{n} p$ s.t.

$$\sum p_i = 1.$$
TOMA: \[ P_1 + P_2 = 1 \]

then why bother?

TOMA gives full utilization but requires coordination

\[ \text{incurs overhead when traffic is \underline{spread/}random/\underline{nonstationary}} \]

\[ \therefore \text{rarely used for stochastic data arrivals.} \]
Example:

\[ p_1 = \frac{3}{4}, \quad p_2 = \frac{1}{2} \]

\[ R_1 = \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8} \]

\[ R_2 = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \]

\[ R_2^* = \left(1 - \sqrt{R_1}\right)^2 \]

Could have had:

\[ R_2^* = \left(1 - \frac{\sqrt{3}}{2\sqrt{2}}\right)^2 \]

\[ > \frac{1}{8} \quad \text{(check!)} \]

\[ p_1 = \sqrt{3/8} \]

\[ p_2 = 1 - \sqrt{3/8} \]
CSMA - Carrier sense multiple/medium access

first sense the channel; if busy, wait, else transmit.

P- CSMA

sensing slots of length $\delta$

if no pkt, send with probability $p$

packet duration:

$T_s$ if success

$T_c$ if collision

$T_c \leq T_s$
At every idle slot, there is contention for transmission with probability $p$.

If successful (i.e., only one of the $n$ contenders transmits), the channel is utilized for duration $T_c$.

If unsuccessful or collision (i.e., more than one transmit simultaneously), channel is wasted for $T_c$ duration.

If idle, the next contention starts after $\delta$ duration.
Expected throughput of p. CSMAs

= Expected time that a virtual contention slot is successful

Expected duration of a virtual contention slot.

At each virtual contention period

EVENT | PROBABILITIES | DURATIONS
idle \( (1 - p)^N \) \( \sigma \)
success \( Np \(1 - p)^{N-1} \) \( t_s \)
collision \( 1 - \(1 - p)^N - Np(1 - p)^{N-1} \) \( T_c \)

\[ T_e = \sum_{i=2}^{N} (\binom{N}{i} p^i (1 - p)^{N-i}) \]
Expected Duration of vCS:

\[(1-p)^N_\sigma + NP(1-p)^{N-1} Ts + 1 - (1-p)^N - NP(1-p)^{N-1} T_c\]

Expected success duration:

\[NP(1-p)^{N-1} Ts\]

Rate \(\text{PCSMA}\):

\[
\text{Rate} = \frac{NP (1-p)^{N-1} Ts}{\max\left[(1-p)^N + NP(1-p)^{N-1} Ts + (1 - (1-p)^N - NP(1-p)^{N-1} T_c\right]}
\]

a generalization of \(p\)-slotted Aloha

this is when

\[\sigma = Ts = T_c\]