Lecture 13 EE597

Feb 24, 2012

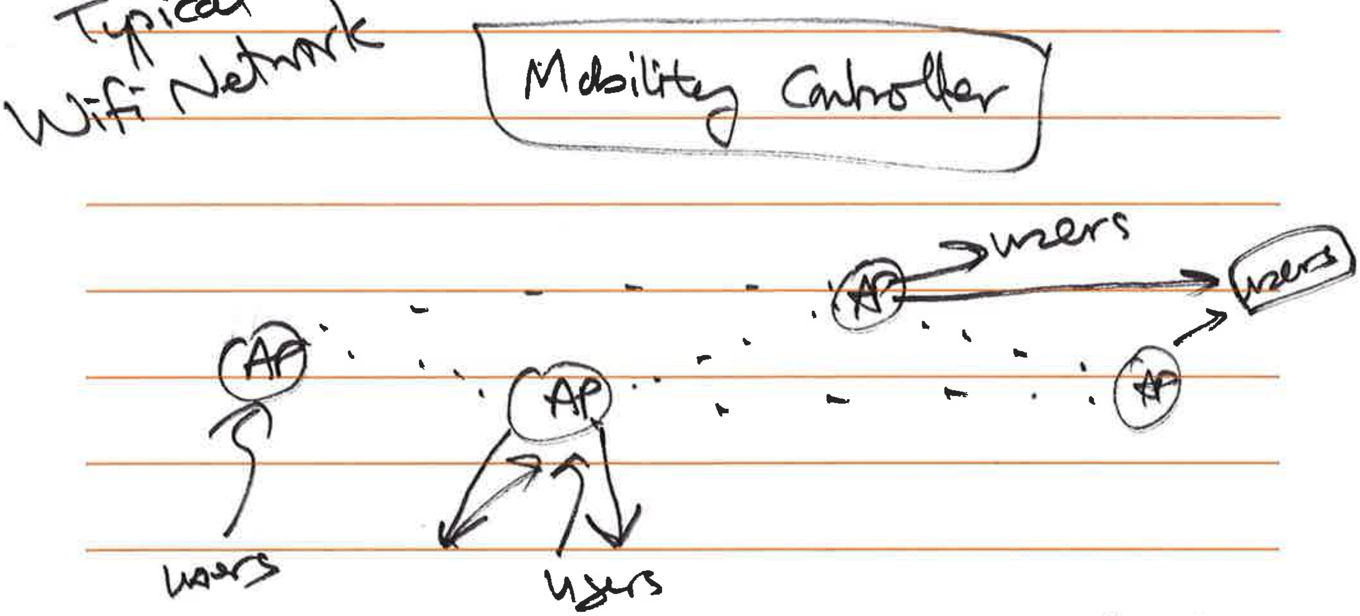
Recap:

Medium Access & Resource Allocation

Power Control

- Independent &
- Parallel Links

Typical Enterprise-class Wifi Network



Resource Allocation:
- Allocate channels to each access point
- Power Control

Random Access

Aloha & slotted Aloha

- send when you have a pkt
- send on the next slot boundary

p-slotted Aloha

- send w/ probabity p when you have a pkt, at the next slot.

p-slotted Aloha with **saturated** users

backlogged users that always have a packet.

Total throughput = $Np(1-p)^{N-1} \cdot R_{max}$

maximized when $p^* = 1/N$

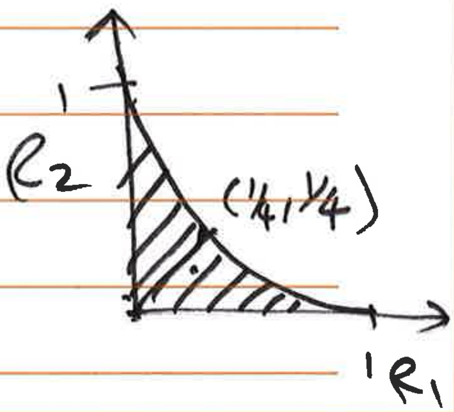
$\therefore$ max ach. throughput = $\left(1-\frac{1}{N}\right)^{N-1} \cdot R_{max}$

tends to $e^{-1} \approx 36\%$ as $N \to \infty$

special case of 2 users that are heterogeneous

p_i - Slotted Aloha w/ saturated users.

$$R_1 = p_1(1-p_2)$$
$$R_2 = p_2(1-p_1)$$



Rate Region Boundary is described by the condition: $p_1 + p_2 = 1$

i.e. $R_1 = p_1^2$, $R_2 = p_2^2$

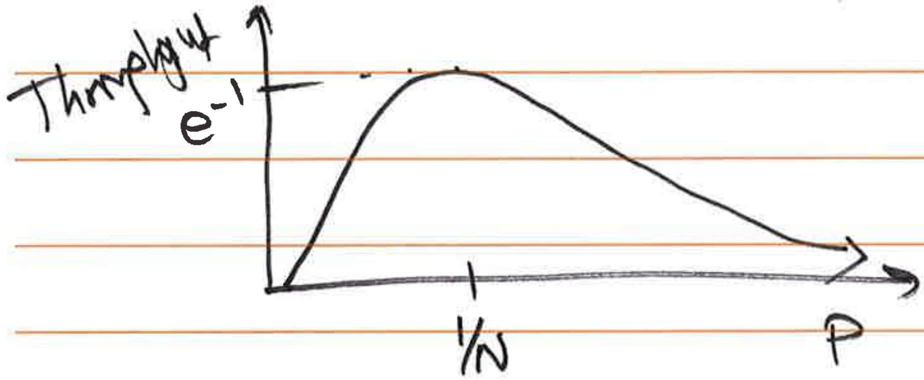
$$\sqrt{R_1} + \sqrt{R_2} = 1$$

for N-users, shown by Massey & Mathys in 1985 that $\sum_{i=1}^{N} p_i = 1$ defines the boundary, with

(normalized)

corresponding rates:

$$R_i = p_i \prod_{j \neq i} (1 - p_j)$$



There is a tradeoff between fairness & throughput. Best sum rate is obtained when only one node transmit.

p-CSMA

avoid transmitting when channel is already busy.

contention slots of duration σ:

- if no one sends, move to next slot after σ s.
- if exactly one transmits, the channel is busy for T_s seconds.
- if more than one collide the channel is busy (wasted) for T_c seconds

$$E[\text{Throughput}] = \frac{E\left[\begin{array}{c}\text{Success duration}\\ \text{per virtual}\\ \text{contention}\\ \text{period}\end{array}\right]}{E\left[\begin{array}{c}\text{duration of the}\\ \text{virtual period}\end{array}\right]}$$

(based on the Key Renewal Theorem,
Renewal Theory)
(see EE512)

Event	Prob.	Duration
Idle	$(1-p)^N$	σ
Success	$Np(1-p)^{N-1}$	T_s
Collision	$1-[(1-p)^N + Np(1-p)^{N-1}]$	T_c

p-CSMA throughput :

$$\frac{Np(1-p)^{N-1}T_s}{(1-p)^N\sigma + Np(1-p)^{N-1}T_s + \left(1-[(1-p)^N + Np(1-p)^{N-1}]\right)T_c}$$

Observations

- when $\sigma = T_s = T_c$
  p-CSMA reduces to
  p-slotted Aloha.

- Another typical setting is
  when $T_s = T_c = T > \sigma$

$$\text{Throughput} = \frac{Np(1-p)^{N-1}\,T}{(1-p)^N \sigma + [1-(1-p)^N]T}$$

$N = 2.$ $\qquad T/\sigma = \hat{T}$

$$\frac{2p(1-p)\hat{T}}{(1-p)^2 + [1-(1-p)^2]\hat{T}}$$

$T \gg \sigma$

as $\hat{T} \to \infty$: $\dfrac{2p(1-p)}{1-(1-p)^2}$

(i.e. larger pkts)

1 0. 1.0

$$\boxed{R = \text{Throughput} = \frac{2p(1-p)}{1-(1-p)^2}} = \frac{2(1-p)}{2-p} = \frac{2-2p}{2-p}$$

$\cancel{1} - \cancel{1} - p^2 + 2p$

$$\frac{\partial R}{\partial p} = (2-2p)(2-p)^{-1} - 2\,\cancel{(2-p)^{-1}} = 0$$

$$\cancel{2}\,1 = \frac{\cancel{2}^{1} - 2p}{2-p}$$

$$2-p = 1-p. \quad \times$$

$p=0$: Throughput $= 0$ ✗

$p=1$: Throughput $= 0$

$$R = (2p-2p^2)(2p-p^2)^{-1}$$

$$\frac{\partial R}{\partial p} = 0 \Rightarrow (2p-2p^2) - (2p-p^2)^{-2} \cdot (2-2p)$$

$$+ (2-4p)(2p-p^2)^{-1} = 0$$

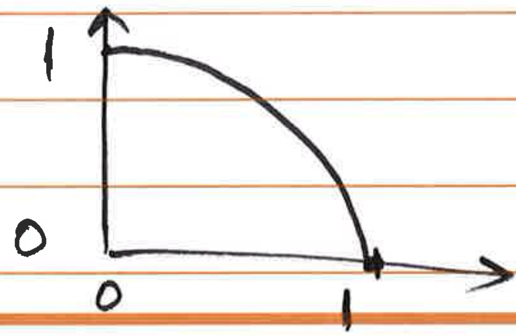
$$\Rightarrow (2p-2p^2)(2-2p) = (2-4p)(2p-p^2)$$

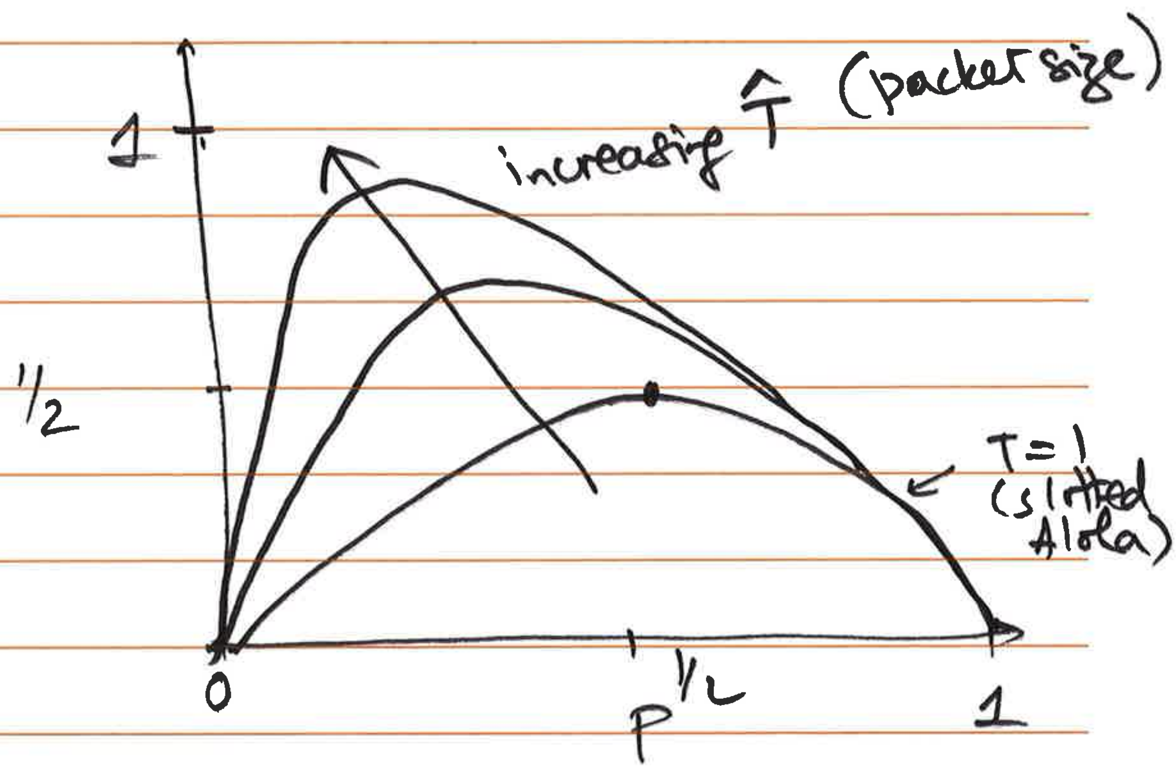
$$4p - 4p^2 - 4p^2 + 4p^3 = 4p - 2p^2 - 8p^2 + 4p^3$$

$$4p - 8p^2 + 4p^3 = 4p - 10p^2 + 4p^3$$

$$\cancel{4p^3} + 2p^2 = 0$$

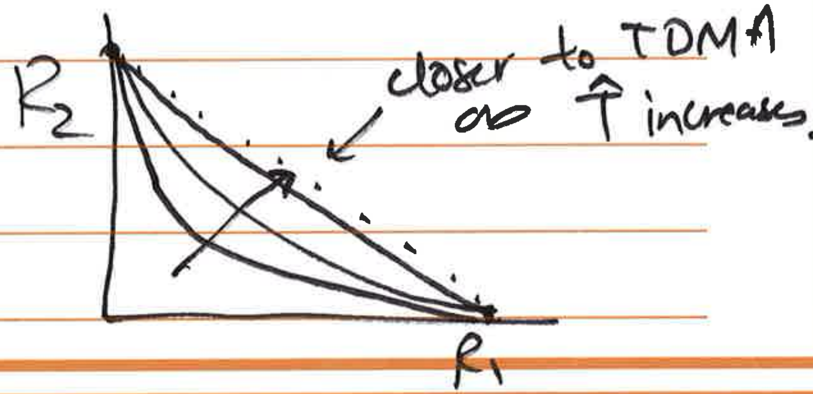
$$p^2 \cancel{(4p+2) = 0} = 0.$$





What about p_i - CSMA ?

$$R_i = \frac{p_i \prod_{j \neq i} (1-p_j)\, \hat{T}}{\prod_{j=1}^{N} (1-p_j) + \left(1 - \prod_{j=1} (1-p_j)\right) \cdot \hat{T}}$$



Condition at the Boundary:

$$1 - \prod_j (1-p_i) + \hat{T}\left(\sum p_i + \prod (1-p_i) - 1\right) = 1$$

as $\hat{T}$ gets larger,
p_i's will be smaller.