(1)

5/23/2016

Recap

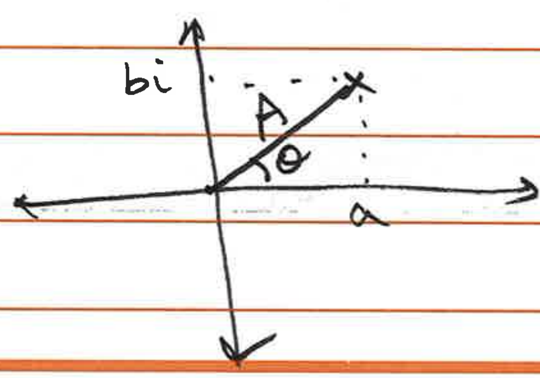
· Phy Layer for Digital communication

all digital information is encoded on the sinusoidal EM-wave.

Single-frequency carrier.

∴ can change amplitude & phase

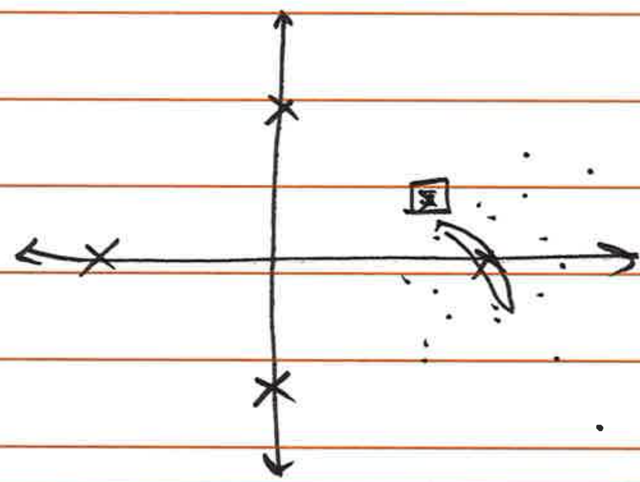
mathematically convenient to represent these as complex numbers.



$a + bi$
$\Updownarrow$
(A, θ)

AWGN channel **model**

noise itself can be modelled as a random complex number.



$r = s + w$

complex circularly symmetric Gaussian noise

$w = w_{re} + i \cdot w_{im}$

w_{re} & w_{im} · are indep. of each other

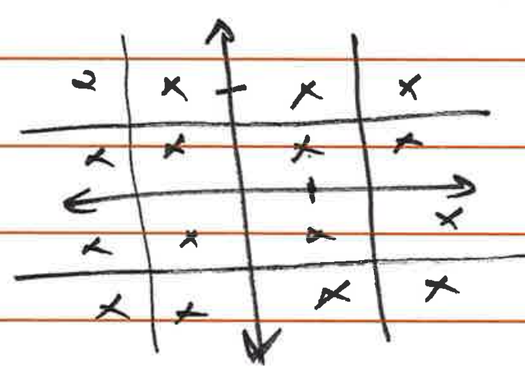
· both Gaussian random variables $\sim N(0, \frac{N_0}{2})$

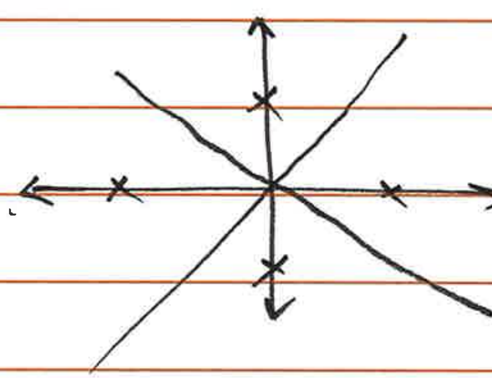
Can represent single-freq. modulation schemes using a constellation diagram, where each point represent a possible symbol.

BPSK, QPSK: 4PSK
8PSK ...

QAM - quadrature amplitude modulation

4-QAM, 8-QAM, 16QAM.





can map bits to symbols arbitrarily

Performance of Digital modulation under AWGN.

Probability of symbol error

= Prob. that ~~detected~~ received symbol lies outside the decision region of the sent symbol.

error probability depends on signal-to-noise ratio SNR.

e.g. BPSK : $P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

Q: 1 − cdf of a std. $N(0,1)$ Gaussian r.v.

E_b: received energy / bit.

N_0: noise variance

a monotonically decreasing function ↓

$$P_b = f(\gamma)$$

γ = SNR at the receiver.

f depends on the modulation scheme

$$SNR = \frac{P_{received}}{Noise}$$

$P_{received}$ ← received power

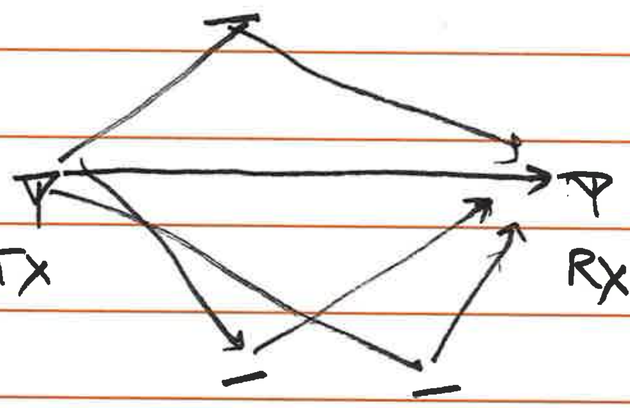
Noise ← noise power.

Typically, higher rate constellation will show worse BER (P_b) for the same SNR.

Tradeoff between

Power (SNR) - Error Rate (P_b) - Throughput (# symbols in constellation or # of bits/symbol)

How does a wireless channel behave?



multipath reflections.

constructive or destructive depending on phase shift.

$$P_{received} \propto \frac{1}{(\text{distance})^{\eta}}$$

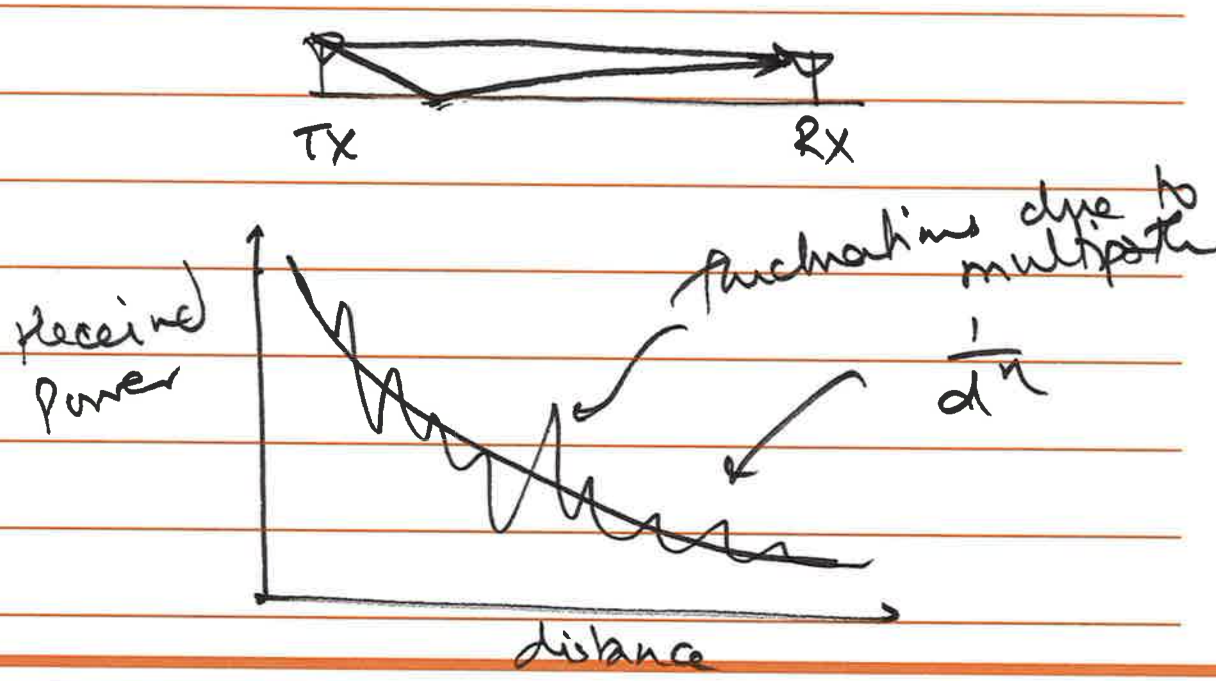
$\eta \sim 1$ to 5

for ideal comm in a vacuum

$\eta = 2$

Surface area of spherical wavefront $\propto d^2$

energy captured at receiver $\propto \frac{1}{d^2}$



Multipath fading

shadowing — large-scale fading phenomenon due to obstructions.

A common fading model for wireless:

Log Normal fading

or

Simple Path Loss model with log-normal fading.

$$P_r = P_T \cdot K \cdot \left(\frac{d}{d_0}\right)^{-\eta} \cdot \psi$$

- P_r: received power
- P_T: transmitted power
- K: path loss at reference distance
- d: distance between xmtr & receiver
- d_0: reference distance
- η: path loss exponent
- ψ: log-normal r.v.

if $d = d_0$ & $\psi = 1$

then $P_r = P_T \cdot K$

P_r, P_t typically measured
in Watts (cellular)
or mW (Wifi, BT etc.)

more often, we use a
log-scale called dB-scale.

$$\text{Power (dBm)} = 10 \log_{10} \text{Power}_{(mW)}$$

$$SNR = \frac{P_{received}}{\text{Noise power}} \quad \text{— unitless.}$$

$$10 \log_{10}(SNR)$$

$$= 10 \log_{10} \left(\frac{P_{received}}{N} \right)$$

$$= \underbrace{10 \log_{10} P_{received}} - \underbrace{10 \log_{10} N}$$

$$= SNR_{dB} = P_{received\,(dBm)} - N_{(dBm)}$$

SIMPLE PATH-LOSS MODEL WITH LOG-NORMAL FADING

$$P_r = P_T \cdot K \left(\frac{d}{d_0}\right)^{-\eta} \cdot \psi$$

in dB-scale

$$P_{r_{dBm}} = P_{T_{dBm}} + K_{dB} \underbrace{- \eta \, 10 \log_{10}\left(\frac{d}{d_0}\right)}_{dB} + \psi_{dB}$$

where

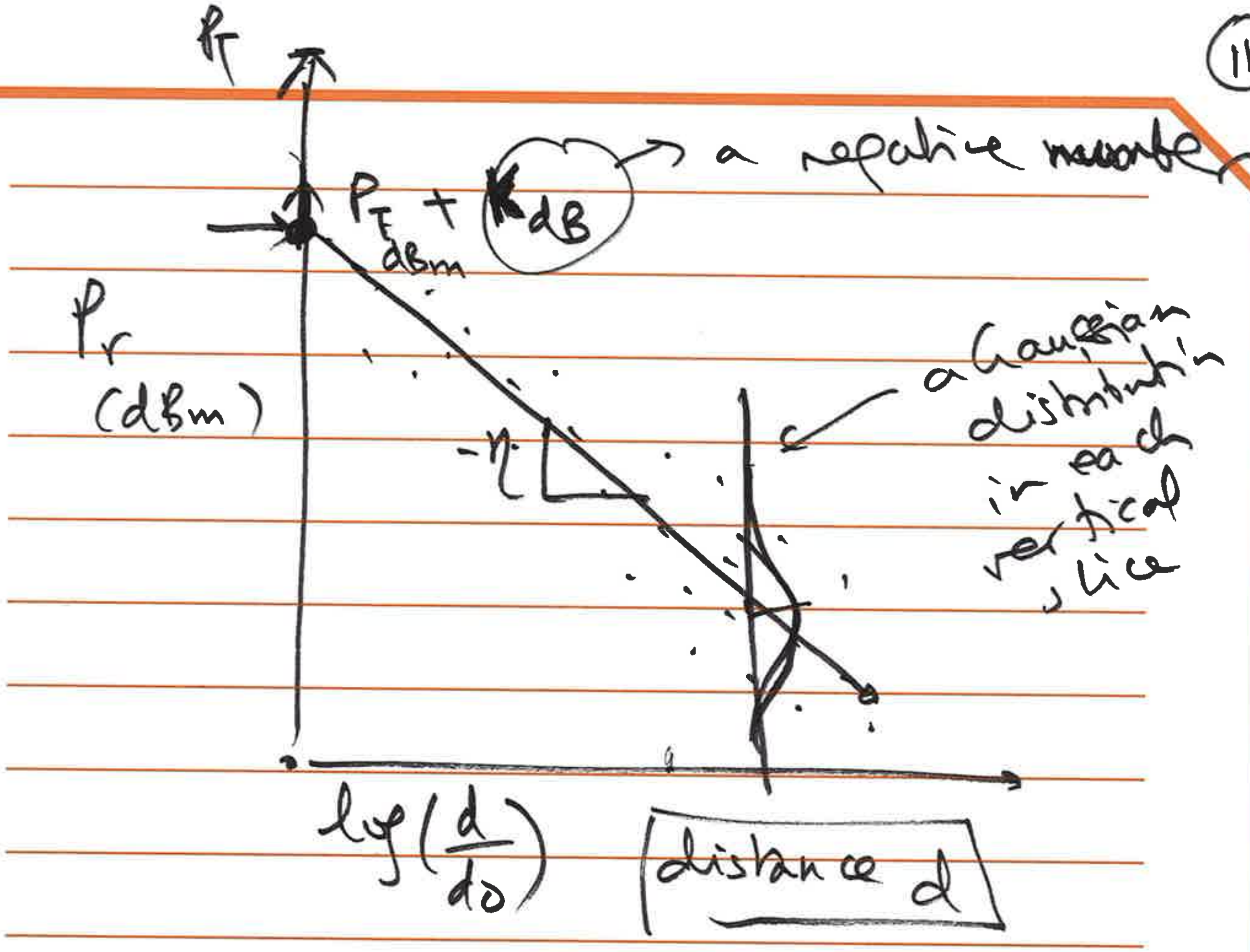
$$P_{T_{dBm}} = 10 \log_{10} P_{T\,(mW)}$$

$$K_{dB} = 10 \log_{10} K$$

$$\psi_{dB} = 10 \log_{10} \psi$$

$$\psi_{dB} \sim N(0, \sigma_{dB}^2)$$

This is a Gaussian r.v. (in the dB-scale).

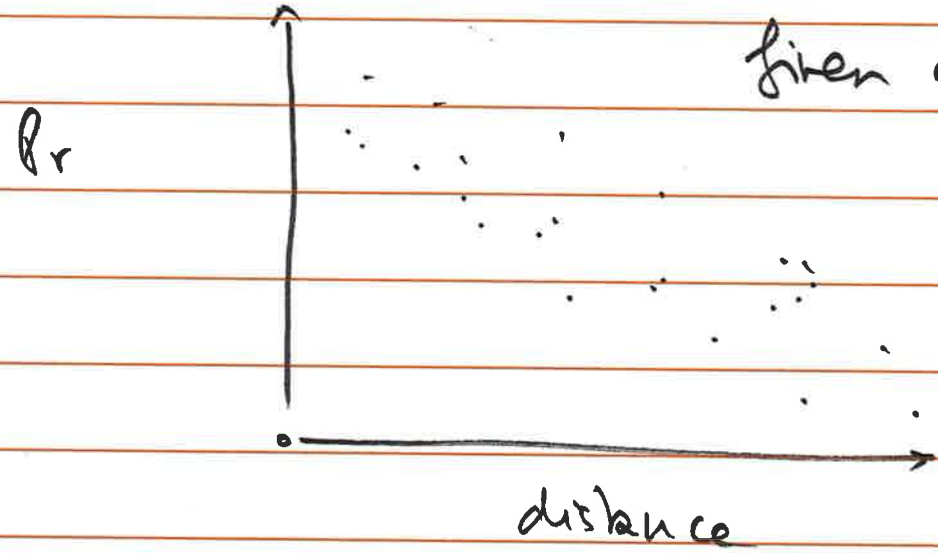


e.g. $d_0 \sim 1$ m

This model has a number of parameters:

- d_0, K ← typically hand-chosen
- η, σ^2_{dB} ← These specify the radio environment; these are empirically determined

given a data set:



P_T - given.

what should be the model parameters, i.e. d_0, K, η, σ^2_{dB}.

Performance of digital modulation under fading.

w/o fading:

$$P_b = f(\gamma)$$

w/ fading - how to describe the performance?

Approach 1: compute $E[P_b]$

$$= E_\gamma[f(\gamma)]$$

(Because SNR (γ) is itself a random variable, so is $f(\gamma)$, i.e. P_b)

if we know the pdf of the fading (SNR) is $g(\gamma)$, if $\gamma \sim g(\gamma)$

$E_{\gamma}[f(\gamma)] = ?$

f ← deterministic function

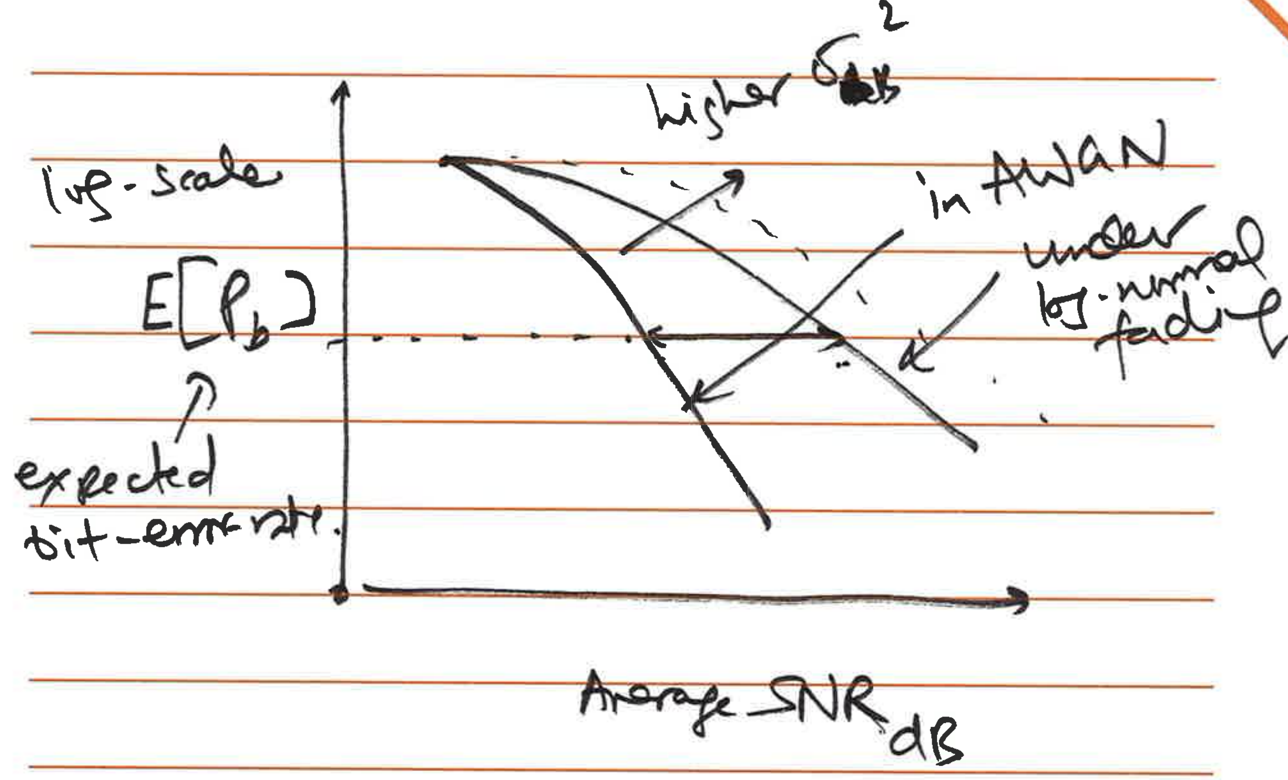
$$E[\gamma] = \int_{-\infty}^{\infty} \gamma \cdot g(\gamma)\, d\gamma$$

$$E[f(\gamma)] \neq f(E[\gamma])$$

$$E[f(\gamma)] = \int_{-\infty}^{\infty} f(\gamma) \cdot g(\gamma)\, d\gamma$$

$f(\gamma)$ ← function; $g(\gamma)$ ← pdf

Avg. bit error rate.



Approach 2 :

A different way of viewing performance under fading :

OUTAGE probability.

if SNR < Θ_{out} ← outage threshold

then the link is said to be in outage

outage probability $= \Pr(SNR < \Theta_{th})$

$Pr_{out} = \Pr(\gamma < \Theta_{th})$.

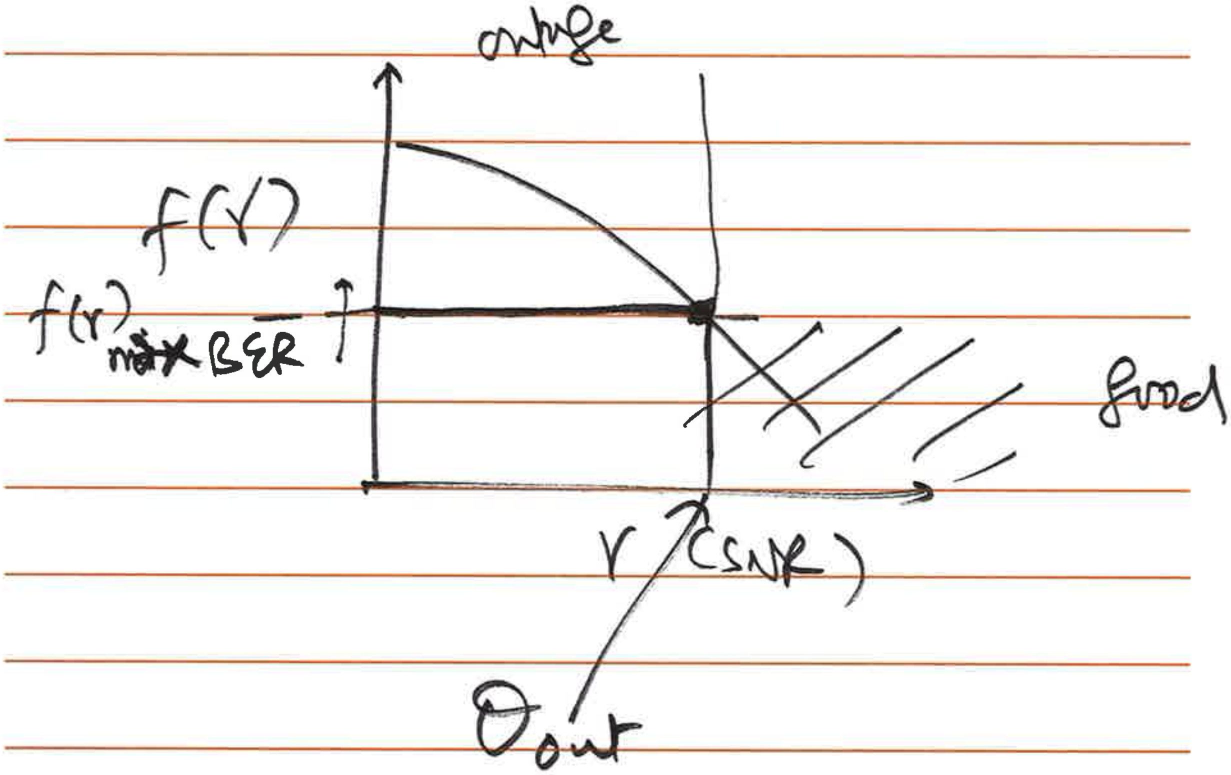
γ is a random variable due to fading

$g(\gamma)$ is its pdf.

$Pr_{out} = ?$ in terms of $g(\gamma)$

$$= \int_{-\infty}^{\Theta_{out}} g(\gamma)\, d\gamma$$

$\Theta_{out} \leftarrow$ outage threshold SNR



$$Pr_{out} = \int_{-\infty}^{\Theta_{out}} g(\gamma)\, d\gamma .$$

$g(\bar{\gamma})$ ← depends on the wireless environment

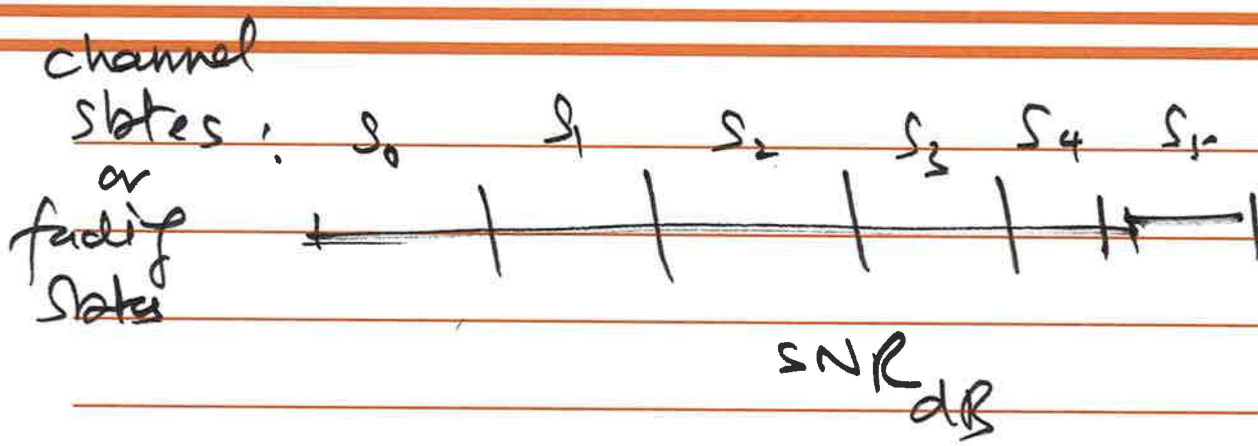
$f(\gamma)$ ← depends on the radio communication parameters — modulation scheme, coding, etc.

together they determine link performance

A different model for fading:

discrete-time Markov Chain model

divide the SNR range into discrete ranges.



By assigning $P_{s_i s_j}$ of going from state s_i to s_j in unit time (i.e. assume some time-step that is fixed), we get a markov-chain describing fading as a discrete random process.