EE 597 Lecture 3 5/25/2016

Recap.

Phy Layer

- Digital communications
  - modulation schemes
  - performance under AWGN.

- Radio propagation modeling
  - Simple path loss model w/ lognormal fading
  - Markov channel model
  - Performance of modulation schemes under fading.

monotonically decreasing

AWGN

SNR

$$\text{BER} \rightarrow P_b = f(\gamma)$$

characterizes the radio

fading channel

γ is a random variable with some pdf $g(\gamma)$

fading distribution

characterizes the environment-

2 approaches to characterizing performance.

Approach 1: $E_\gamma[P_b] = \int_{-\infty}^{\infty} f(\gamma)\, g(\gamma)\, d\gamma$

expected or average bit error rate

Approach 2: start w/ a BER threshold P_b^{th} s.t. want $P_b < P_{b}th$.

given $f(\gamma)$ can derive an SNR threshold θ_{out} s.t. we want $\gamma > \theta_{out}$

$$P_{outage} = P_r[\gamma < \theta_{out}]$$
$$= \int_{-\infty}^{\theta_{out}} f(\gamma)\, d\gamma$$

what can we do to control the $E[P_b]$ or P_{outage}?

- increase signal power
- change modulation scheme (lower rate)

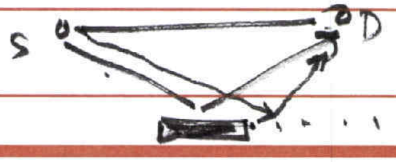
power - throughput - error rate

Frequency-Time view of fading

Delay Spread : due to multipath, some of the reflections arrive "late", causing inter-symbol interference.

Doppler Spread : due to mobility of the sender / receiver or even objects in the environment the carrier frequency gets shifted up/down!

Time		Frequency
Delay Spread T_s	$\Longleftrightarrow$	Coherence BW W_{coh}
Coherence time T_{coh}	$\Longleftrightarrow$	Doppler Spread f_s



$$T_{coherence} \propto \frac{1}{f_s}$$ → relative velocity

if $T_{coherence}$ is low (due to scattering/reflecting dynamics of nodes / objects in environment) then we say the env. is experiencing a fast fading.

Else, we call it slow fading.

$$T_s \propto \frac{1}{W_{coh}}$$

if W_{coh} is small, i.e. T_s (delay spread) is large, we say the env. shows frequency selective fading, else if W_{coh} is large ⇒ flat fading

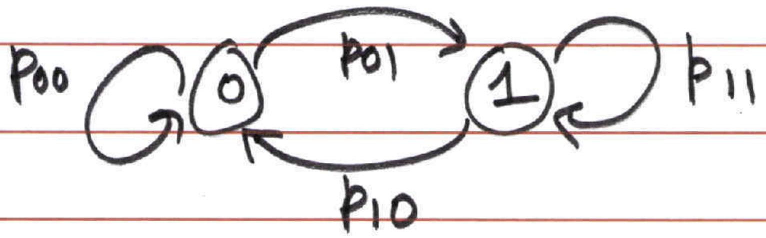
when we have inter-symbol interference

Discrete time Channel model.

Markovian

Simplest model : only 2 states.

Gilbert - Elliot model



0 - bad (outage : $\gamma < \theta_{OUT}$)

1 - good $\gamma \geq \theta_{OUT}$

constraints :

$$p_{00} + p_{01} = 1$$

$$p_{01} = 1 - p_{00}$$

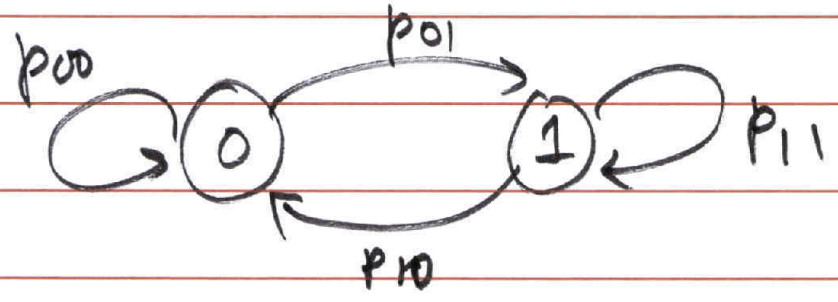
&

$$p_{11} + p_{10} = 1$$

$$p_{10} = 1 - p_{11}$$

to specify the model, enough to give 2 numbers : (p_{01}, p_{11})

Intuitively, say we keep p_{00} fixed, and vary p_{11} what should happen to $T_{coherence}$?



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p_{11} ↑ T_{coh} ↑

p_{00} ↑ T_{coh} ↑

If at state 1, how long to switch?

Expected value of a ~~[illegible]~~ Geometric r.v.

expected time to first success, w/ succ. probability p.

$$= \frac{1}{p}$$

If at state 1, takes $\frac{1}{p_{10}} = \frac{1}{1-p_{11}}$ time steps in expectation to switch. $= T_{coh}^{(1)}$

If at state 0, $\frac{1}{p_{01}}$ is the exp. time to switch $= T_{coh}^{(0)}$

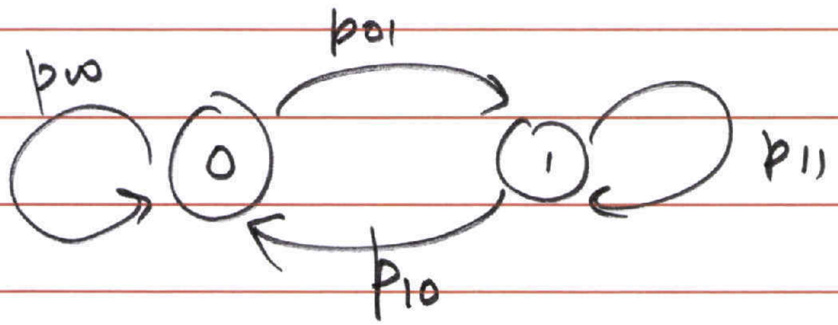
possible def. of Tcoherence:

$$T_{coherence} = \frac{1}{2}\left(T_{coh}^{(1)} + T_{coh}^{(0)}\right)$$

better definition:

$$\pi_0 \cdot T_{coh}^{(0)} + \pi_1 T_{coh}^{(1)}$$

steady state prob. of being in state 0



Global balance equations.
(recommend working this out on your own!)

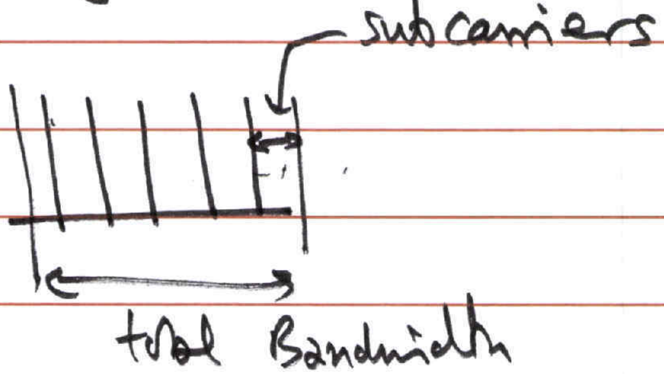
$$\pi_1 = \frac{p_{01}}{p_{01}+p_{10}} \qquad \pi_0 = \frac{p_{10}}{p_{01}+p_{10}}$$

$$\boxed{\begin{aligned} \pi_0 \cdot p_{01} &= \pi_1 \cdot p_{10} \\ \pi_0 + \pi_1 &= 1 \end{aligned}}$$

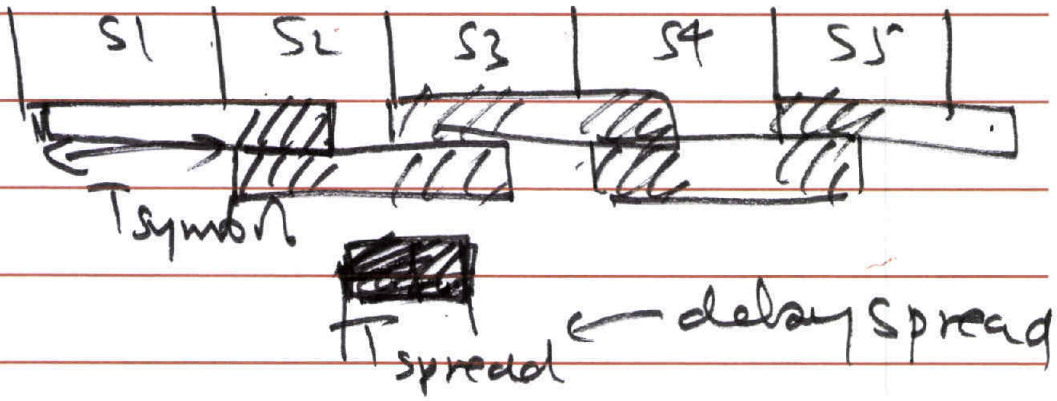
$$T_{coh} = \pi_0 \cdot \frac{1}{p_{01}} + \pi_1 \cdot \frac{1}{1-p_{11}}$$

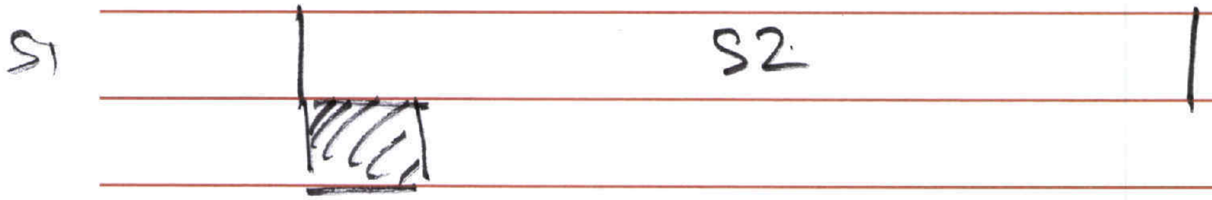
$$= \frac{p_{10}}{p_{01}+p_{10}} \cdot \frac{1}{p_{01}} + \frac{p_{01}}{p_{01}+p_{10}} \cdot \frac{1}{1-p_{11}}$$

OFDM - orthogonal frequency division multiplexing:



Total throughput = throughput / subcarrier × # subcarriers.

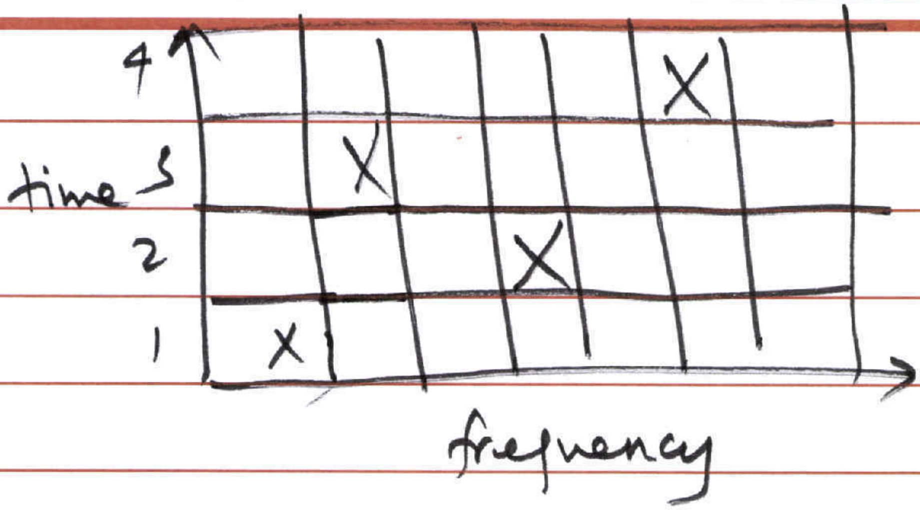




increasing symbol duration reduces interference from prev. symbol

Spread Spectrum
- Frequency hopping (FHSS)
- Direct sequence (DSSS)

FHSS initially developed to protect radio links from jamming

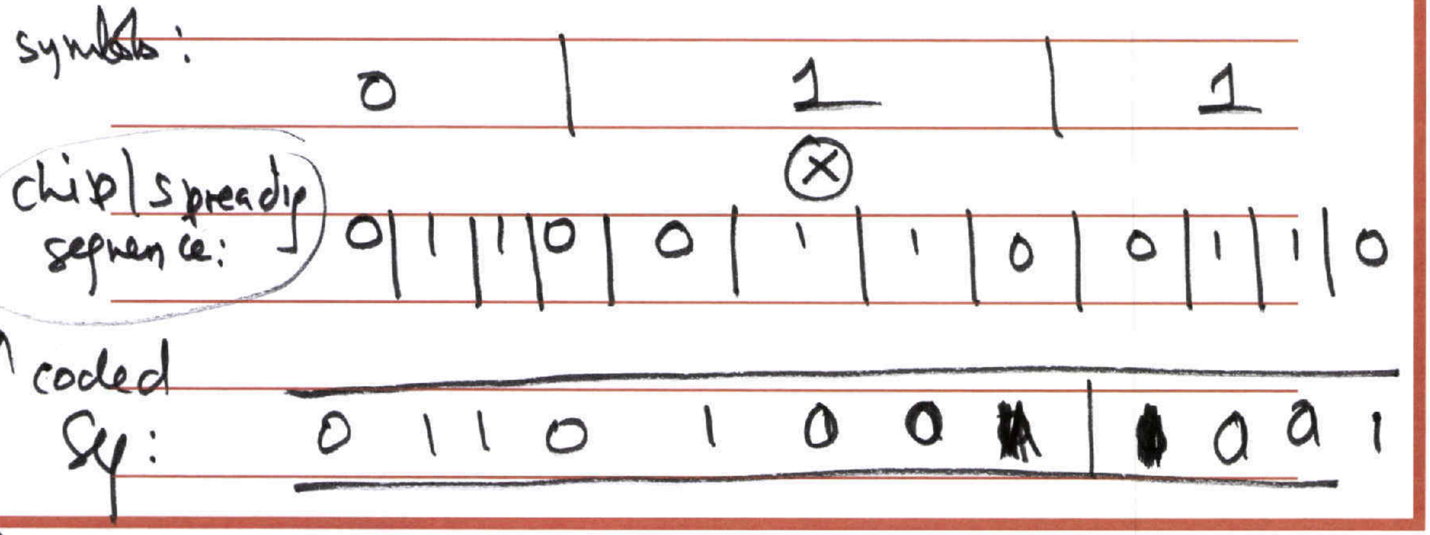


(used in Bluetooth)

a form of diversity improvement.

Chip

A spreading sequence that changes at a higher rate than the symbols:



designed to have certain properties:

e.g.: high autocorrelation

. low cross correlation w/ other sequences

Receiver: decode using the same seq. as at the transmitter.

received

0 1 1 0 1 · 0 $\otimes$ 0 1 1 0 0 1

decod seq: 0 1 1 0 0 1 1 0 0 1 1 0

$$\underset{0}{\underline{0\ 0\ 0\ 0}}\ \Big|\ \underset{1}{\underline{1\ 1\ 1\ 1}}\ \Big|\ \underset{1}{\underline{1\ 1\ 1\ 1}}$$

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the reflected / delayed versions of the signal will (for a properly chosen sequence with ~~large auto~~ peaked autocorrelation property) be filtered out.

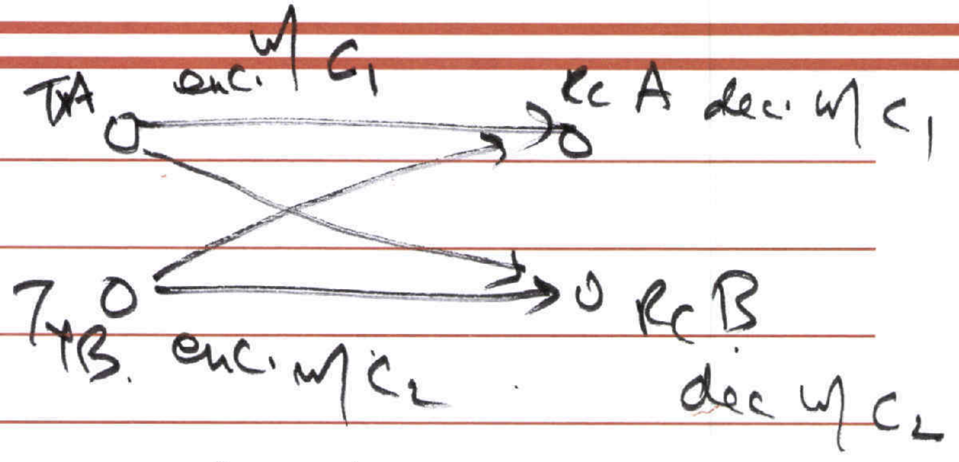
$$(S_t \cdot C_t) \cdot C_t = S_t$$

$$+$$

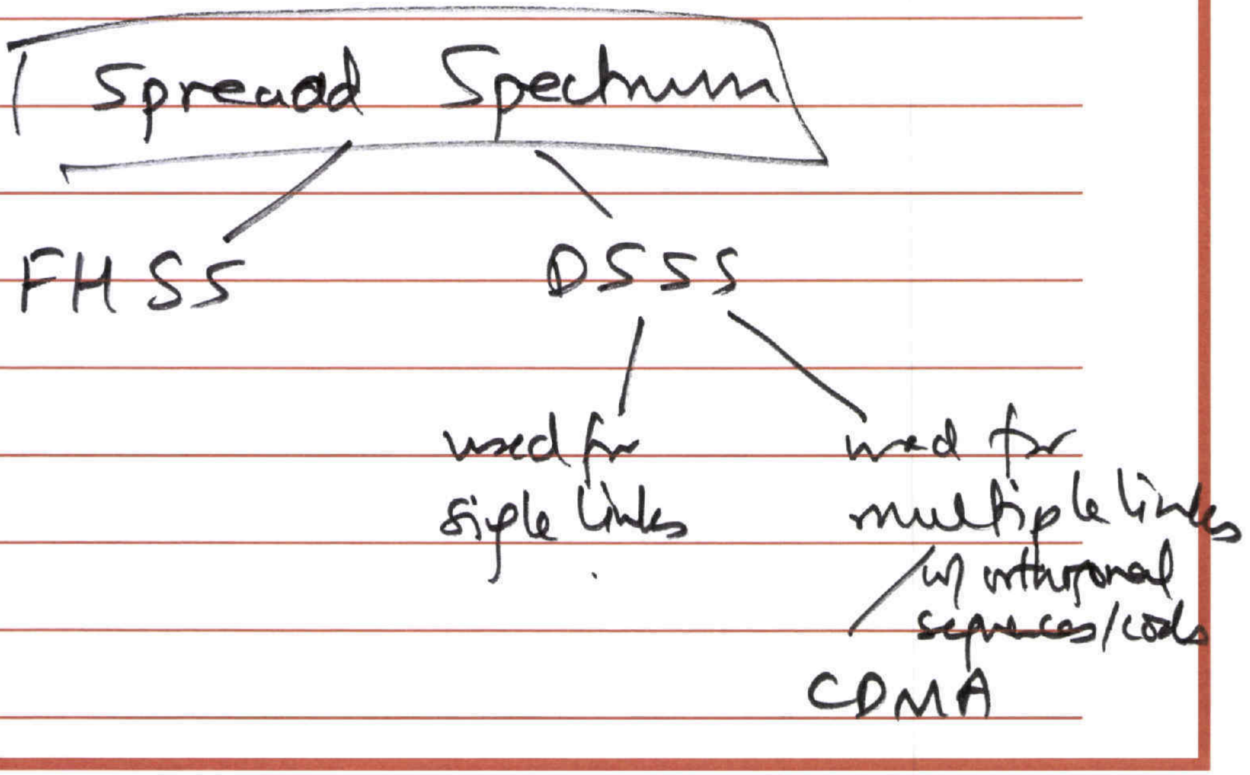
$$(S_{t-\tau}\, C_{t-\tau}) \cdot C_t = \sim 0$$

helps reduce ISI

if 2 different transmitters are sending data encoded with different chip/spreading sequences (carefully selected to be orthogonal or have low cross-correlation) then at the receiver can filter out the interfering signal.



Using orthogal spreading sequences to allow multiple links to access the same channel is known as CDMA.



Error-Control Coding

Forward Error Correction

<u>Basic idea:</u>
adding redundant bits
in order to reduce the
error rate.

Block Codes **Convolutional Codes**

Block codes: take a block of k bits &

output $\underbrace{n > k \text{ bits}}_{\text{codeword.}}$

additional $n-k$ bits are used to provide robustness to bit-errors.

most naive solution: replication

101 → 101101

111 → 111111

10⓪10①

. . z . . z

$\frac{k}{n}$ coding rate.

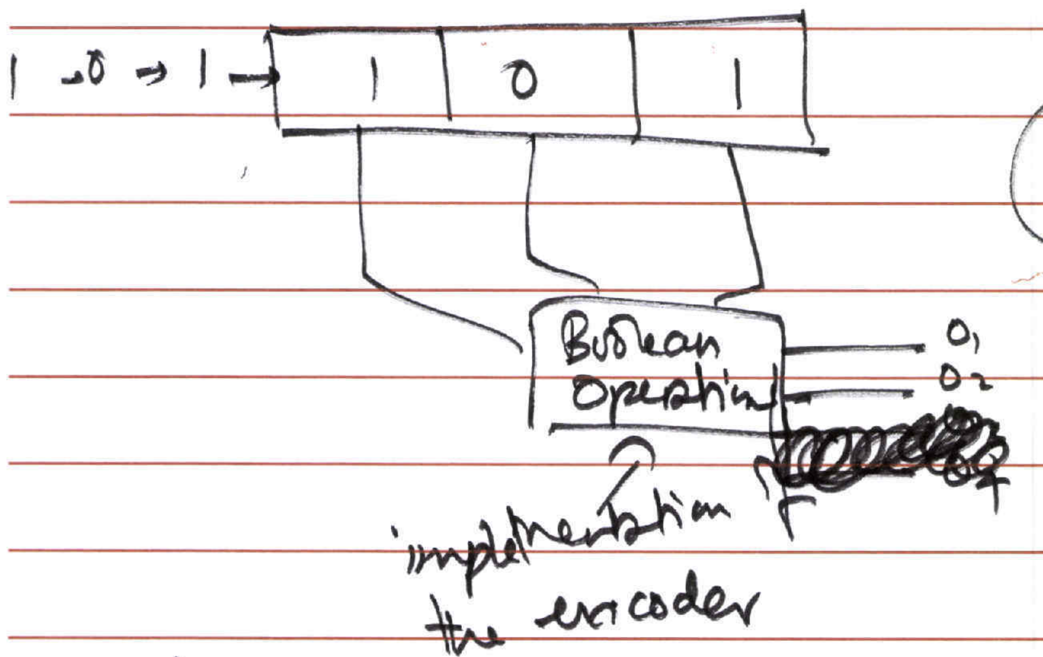
lower the coding rate, the better the error performance

Convolutional codes: still

described by a coding rate $\frac{k}{n}$

but the encoding is continuous/stream based.

Typically implemented using shift registers



($\frac{1}{2}$ encoder example)

On the decoder end, can use algorithms such as Viterbi algorithm advanced convolution encoding/decoding techniques : Turbo codes/ LDPC codes etc.

next time: MIMO & diversity.