EE 597 Lecture 4 6/1/16

Recap.

Phy Layer

- Digital comm in AWGN
- fading & RF propagation
  - Simple Path Loss +
  - LogNormal fading
  - Discrete Markov Chain model

Quantify performance under wireless fading channels:

$\int_{-\infty}^{\infty} f(\gamma) g(\gamma) d\gamma \rightarrow$ Expected probability of error

$\int_{-\infty}^{\theta_{out}} g(\gamma) d\gamma \leftarrow$ Outage Probability.

depend on the stochastic environment $g(\gamma)$ (pdf)

& the underlying communication scheme $f(\gamma)$

- modulation
- error control coding

Models of fading

- iid fading: over time, the received power or SNR is being drawn in iid fashion from $\underline{\underline{f(\gamma)}}$

$SNR = \frac{P_r}{P_{noise}}$ e.g.: log-normal fading

Other distributions are used:
- Rayleigh distribution
- Ricean distribution
- Nakagami-m distribution

HW2 – problem asking for outage prob. w/ a lognormal fading: $P_r[\gamma < \theta_{out}]$

$$= \underbrace{P_r[\gamma_{dB} < \theta_{out\,dB}]}_{\text{can be expressed in terms of Q-function}}$$

quick comment:

if $X \sim N(\mu, \sigma^2)$

$$Y = \underbrace{\frac{X-\mu}{\sigma}}_{\substack{\text{scaling a Gaussian r.v.} \\ \text{to make it a standard normal (Gaussian) r.v.}}} \sim N(0,1)$$

fading models w/ memory:

- let's assume we are only interested in discrete time.

$$\underbrace{\{X_i\}_{i=0,1,\ldots}}_{\text{discrete time random process}}$$

r.v. for each time

e.g. Markov model: $P[X_i = x_i \mid X_{i-1} = x_{i-1} \ldots X_0 = x_0)$

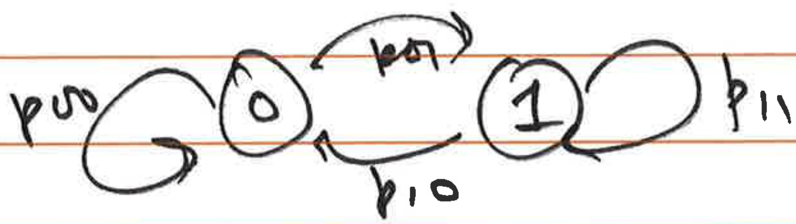
$= P[X_i = x_i \mid X_{i-1} = x_{i-1}]$

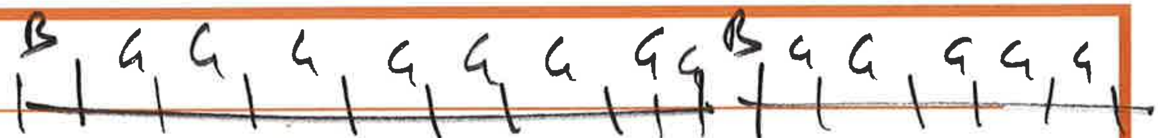
A further simplification is:

$X_i \ \forall i \in S$

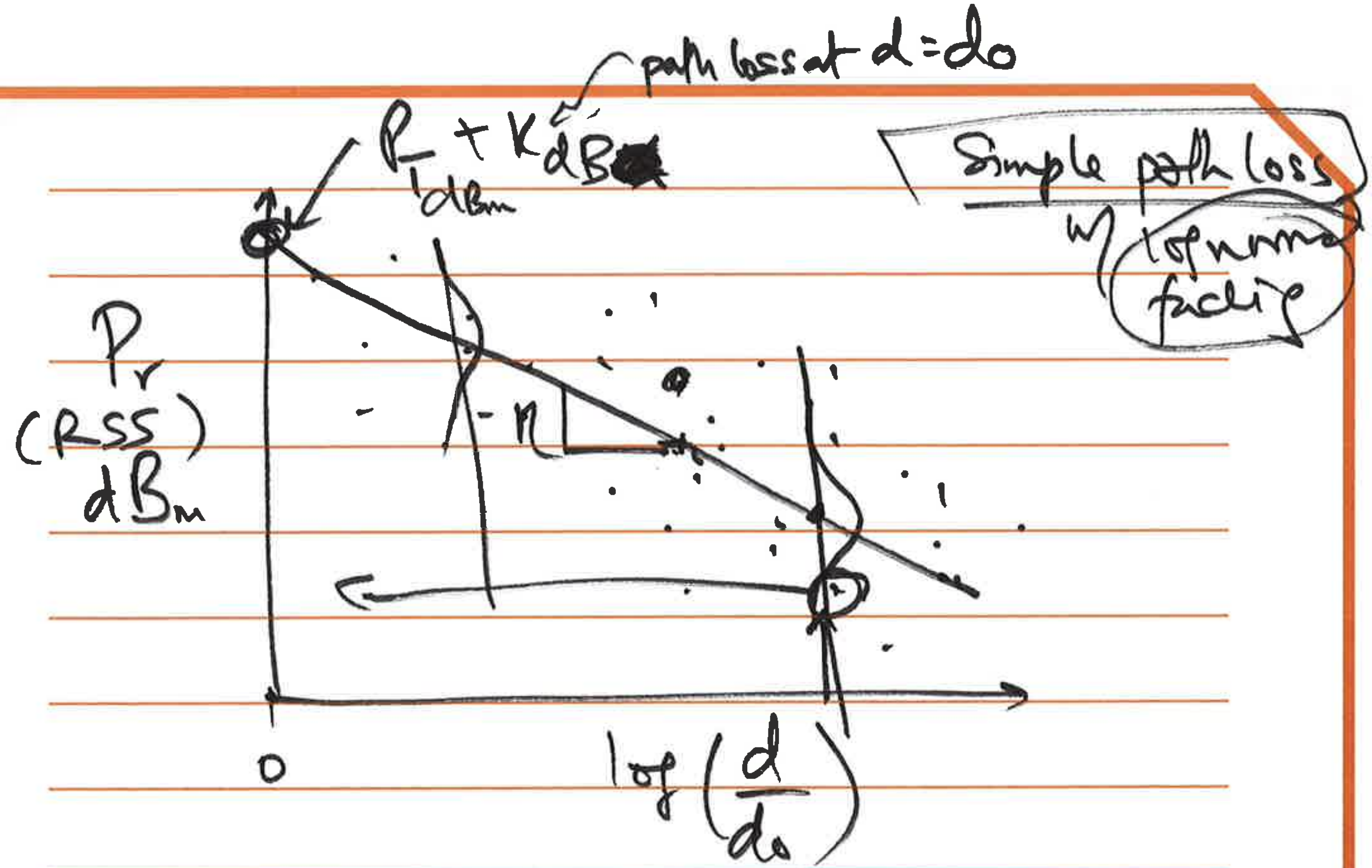
where S is a finite set.

This gives a discrete time discrete state M.C. model





<u>HW2</u>: problems of fitting model parameters based on real / simulated data



1. use linear regression to compute the best fit line

2. slope $\rightarrow \eta$ $d_0 = 1m$
   y-intercept $\rightarrow K_{dB}$

3. subtract mean path loss from each sample (corr. to the right distance) → yields random samples that should be Normally distributed w/ mean 0.
their variance → estimate of σ^2_{dB}

why do model fitting?

- extrapolate beyond measurement
- characterize the general environment

Real World Environment

uncertain | noisy

samples measurements

STATISTICAL PARAMETER FITTING (EE517)

model of reality

503

Probabilistic model (r.v., random process)

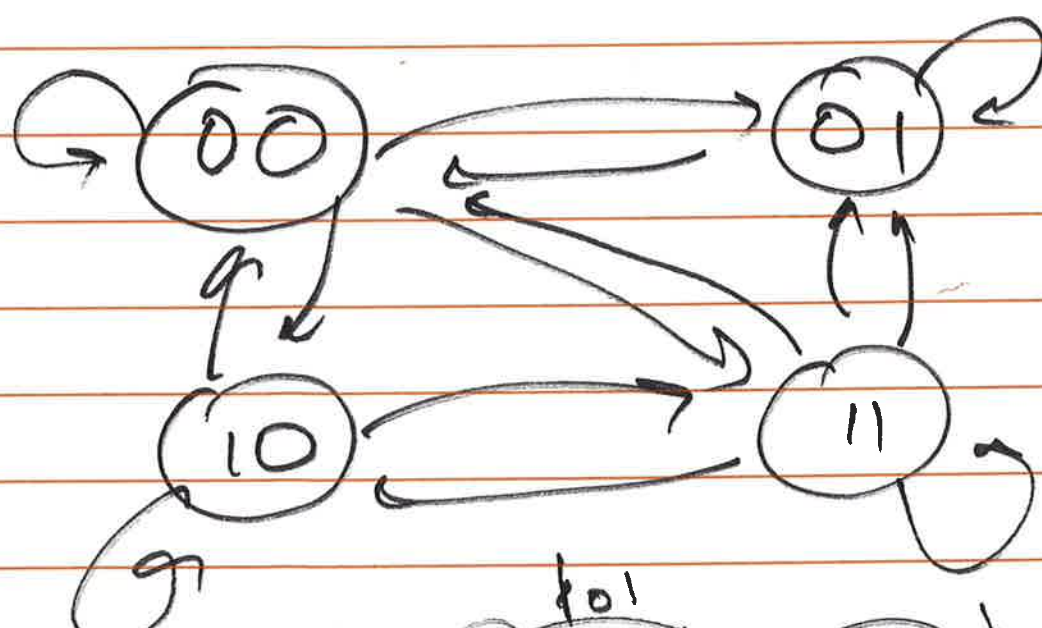
use for performance analysis, design, evaluation

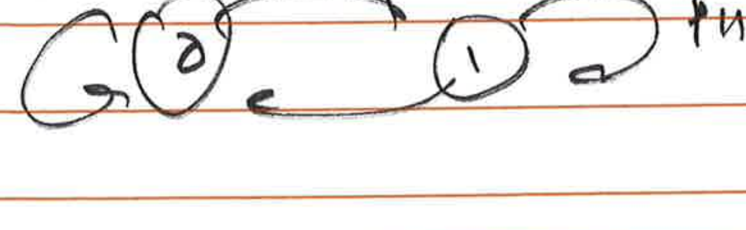
Art of modelling:

- simple: easy to work w/ & compute, less realistic
- complex: more realistic, hard to compute w/.

e.g. consider a markov model w/ memory of 2 steps (2nd order Markov model)

$$P(X_{i+1} = x_{i+1} \mid X_i = x_i, X_{i-1} = x_{i-1} \ldots X_0 = x_0)$$
$$= P(X_{i+1} = x_{i+1} \mid X_i = x_i, X_{i-1} = x_{i-1})$$





mobility ∝ | fast – slow fading
coherence time / Doppler spread

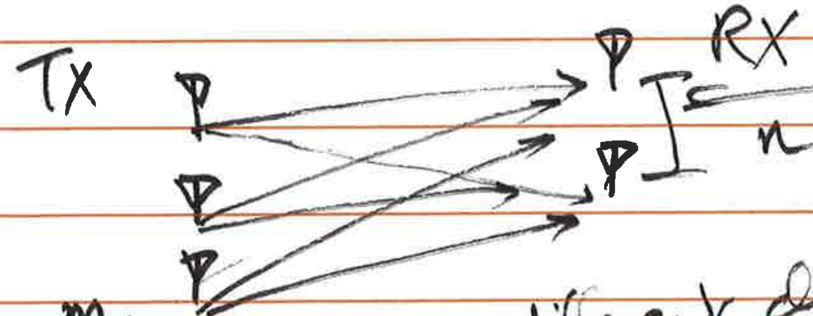
Environment "echoes" ∝ | flat – frequency selective fading
Delay Spread | Coherence BW

↳ Inter-Symbol Interference (problem at high data rates)

~~CDMA~~ DSSS OFDM

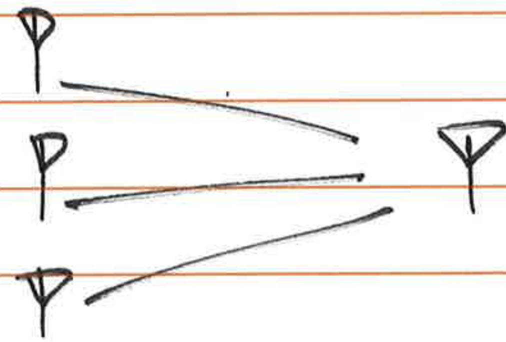
CDMA, Spread Spectrum – DSSS – FHSS
OFDM, Error Control coding

Diversity & MIMO
using multi-antenna systems.

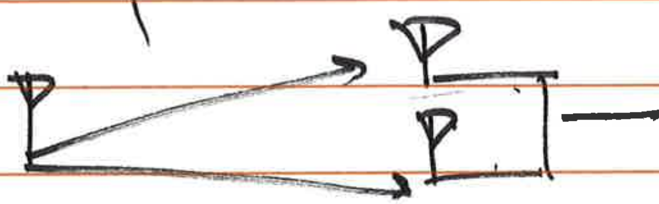


spaced apart on the order of ½λ ...

m·n different channels



Receive diversity

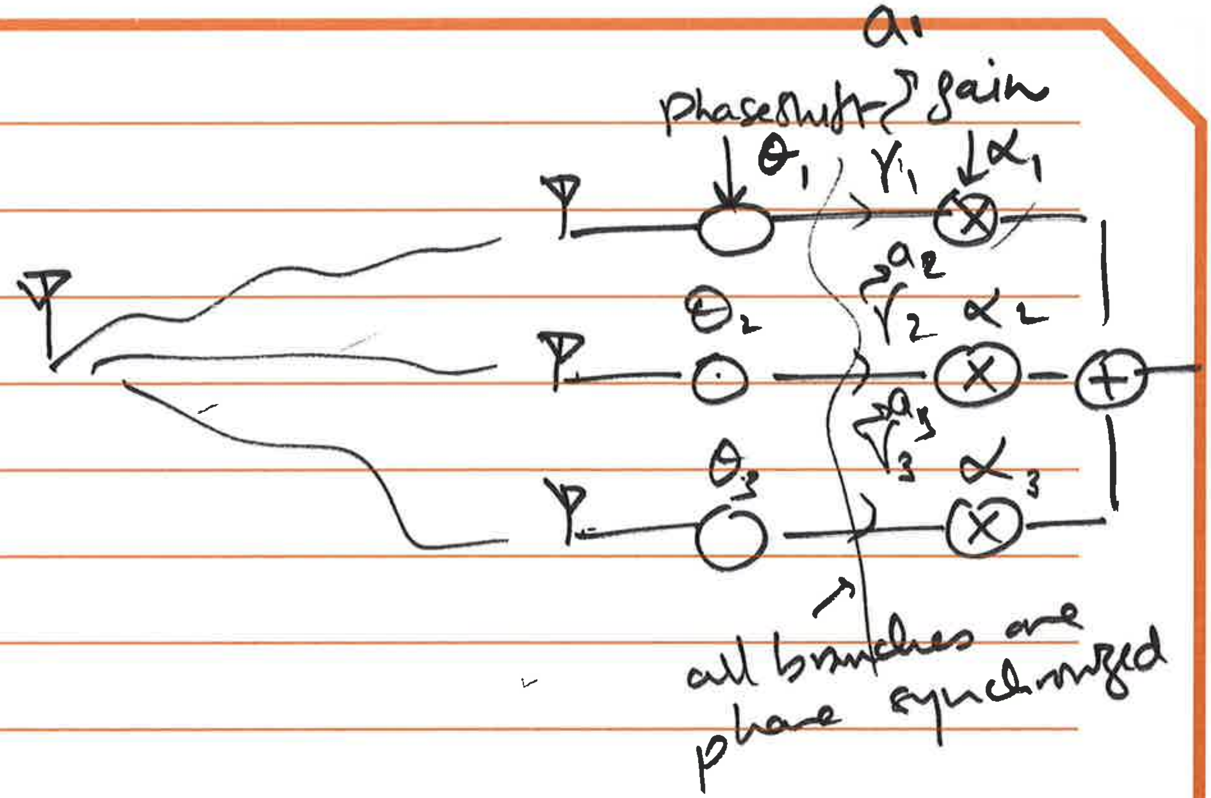


Selection diversity : pick the stronger signal.

Combination diversity

Equal combining.

maximum ratio combiner (MRC)



$$\frac{a_1^2}{n^2}, \quad \frac{a_2^2}{n^2}, \quad \frac{a_3^2}{n^2}$$

$$\frac{(\alpha_1 a_1)^2}{n^2}, \quad \frac{(\alpha_2 a_2)^2}{n^2}, \quad \frac{(\alpha_3 a_3)^2}{n^2}$$

total SNR : $\displaystyle\sum_{i=1}^{L} \frac{(\alpha_i a_i)^2}{n^2}$ what α_i ~~are~~ gives the best choice?

MRC

$\boxed{\alpha_i \propto a_i}$ maximizes the output SNR

gain

signal amplitude

$$E[SNR_{out}] = \underbrace{L}_{\text{\# of antennas}} \cdot \underbrace{E[SNR]}_{\text{for each branch.}}$$

$$\text{error rate} \underset{\sim}{\propto} \frac{1}{SNR^{L}}$$ ← diversity order

(at high SNR values)

In general,
max diversity order $L = m \cdot n$.

Diversity benefit of MIMO systems
↳ more channels ⇒ higher effective SNR & lower error rate.

Multiplexing benefit of a MIMO system:
more antennas can be translated to throughput improvements as well.

Capacity of an $m \times n$ MIMO system (at high SNR, fading)

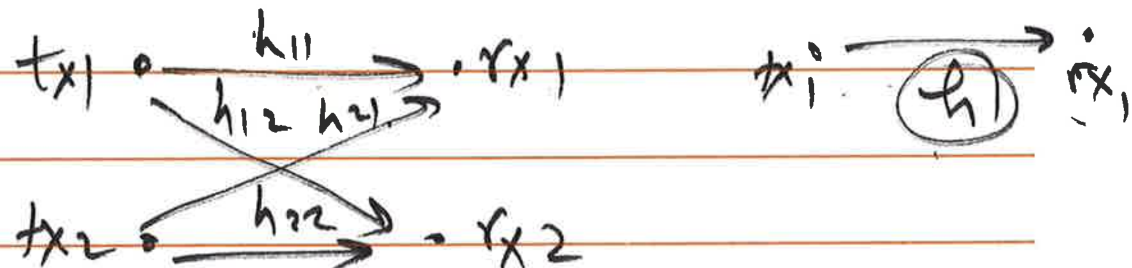
$$\approx \underbrace{\min(m, n)} \cdot \log(\mathrm{SNR})$$

e.g. 3 × 3 MIMO system

$\min(3,3) = 3.$

3 × 2

$\min(3,2) = 2$



$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \leftarrow$$ MIMO channel matrix

$$\begin{bmatrix} \ \ \end{bmatrix} \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \begin{bmatrix} \ \ \end{bmatrix}$$

SVD

MIMO systems show a diversity-multiplexing tradeoff:

max-diversity $\sim m \cdot n$
(error rates)

max-multiplexing $\sim \min(m, n)$

if we have a multiplexing gain of r

diversity gain $\sim (m-r) \cdot (n-r)$

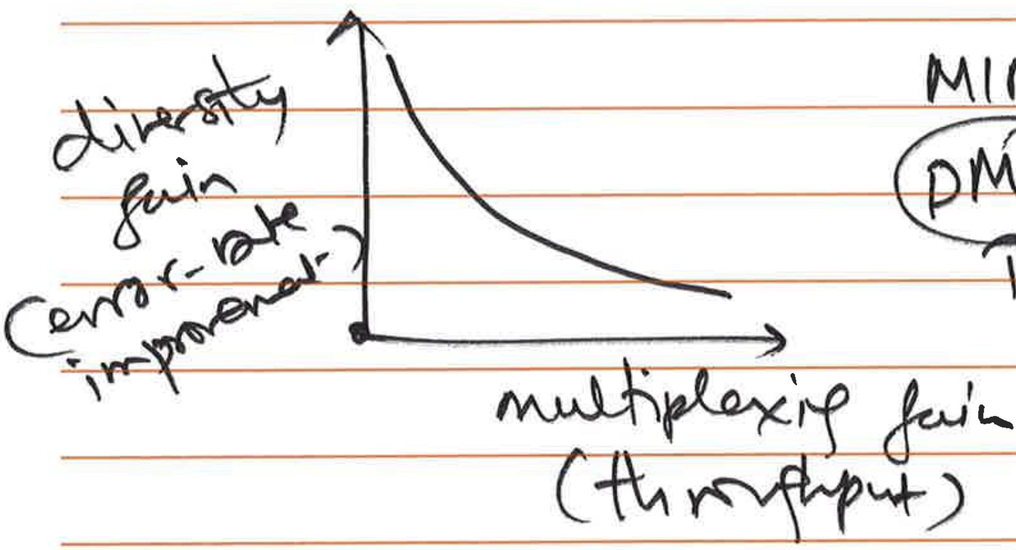
e.g. 4x4:

$g_d = 16$ $\quad g_m = 0$ → single channel

or $g_d = 4$ $\quad g_m = 2\times$

$g_d = 0$ $\quad g_m = 4\times$

← single channel.



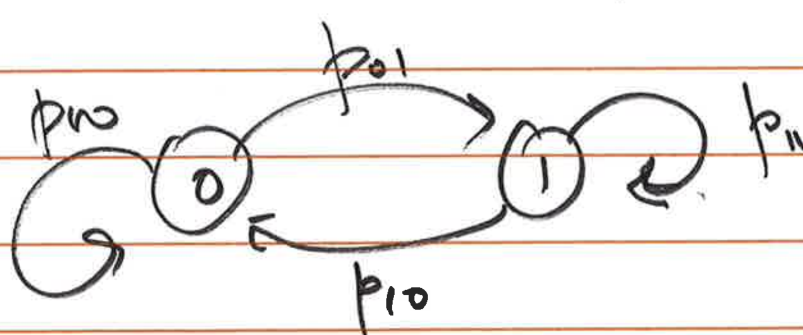
MIMO

DMT

diversity-multiplexing tradeoff.

HW 2: estimating parameters of a two-state MC from data.

0 1 1 0 0 0 1 1 1 0 0 ...



maximum likelihood estimate:

for what value of p_{01}, p_{10} sep. is the observed most likely?

$$p_{01} = \frac{\# 0 \to 1}{\# 0 \text{ is observed}}$$

$$p_{00} = \frac{\# 0 \to 0}{\# 0 \text{ is obse}}$$

$$p_{01} = 1 - p_{00}.$$

sim. for p_{10} & p_{11}

no class on June 13^{th}
(we will prepare a lecture)

on June 15^{th} TA will teach
how to do NS3 simulations
of Wireless Networks