Recap.

Phy Layer
- Digital comm in AWGN
  - Fading + RF propagation
    - Simple Path Loss
    - LogNormal fade
    - Discrete Markov Chain model

Quantify performance under wireless fading channels:

\[ f(y) \text{ (pdf)} \rightarrow \text{Expected probability of error} \]

\[ g(y) \text{ (pdf)} \rightarrow \text{Outage probability} \]

- Depend on the stochastic environment \( f(y) \)
- The underlying communication scheme \( f(y) \)
  - Modulation
  - Error control coding
Models of fading

- iid fading: over time, the received power or SNR is being drawn in iid fashion from \( g(r) \)

\[
\text{SNR} = \frac{P_r}{\text{Noise}} \quad \text{e.g.: log-normal fading}
\]

Other distributions are used:
- Rayleigh distribution
- Rician distribution
- Nakagami-M distribution

HW2 - problem asking for outage probability of a lognormal fading: \( P_r[\{ Y < \text{Out} \}] = P_r[\{ Y_{dB} < \text{Out}_{dB} \}] \)
can be expressed in term of & function.
Faster comment:

\[ X \sim \mathcal{N}(\mu, \sigma^2) \]

\[ y = \frac{x - \mu}{\sigma} \sim \mathcal{N}(0, 1) \]

scaling a Gaussian r.v. to make it a standard normal Gaussian r.v.

Fading models of memory:

Let's assume we are only interested in discrete time r.v. for each time:

\[ \mathbb{E}[X_i \mid 0] \]

describe a random process.

e.g.: Markov model: \[ P(X_i = x_i \mid X_{i-1} = x_{i-1}, \ldots, x_0 = x_0) = P(X_i = x_i \mid X_{i-1} = x_{i-1}) \]
A further simplification is:

\[ X_i + \tau_i \in S \]

where \( S \) is a finite set.

This gives a discrete time discrete state M.C. model.

\[ \text{p.00} \quad \text{p.10} \]\n
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HW3: problems of fitting model parameters based on real / simulated data
1. use linear regression to compute the best fit line
2. slope $\rightarrow \eta$  
   $\eta$-intercept $\rightarrow K dB$
3. subtract mean path loss from each sample (correct to the right distance) $\rightarrow$ yields
   random samples that should be normally distributed w/ mean 0.  
   their variance $\rightarrow$ estimate $\sigma^2$
Why do model fitting?

- Extrapolate beyond measurement
- Characterize the general environment

Real World Environment

Uncertain/Noisy

Statistical Parameter Fitting (Est.)

Model vs. Reality

Use in performance analysis, design, evaluation

Easy to understand?

Act 2: Modelling

Simple: Less realistic

Complex: More realistic

Hard to compute
e.g consider a markov model of memory of 2 steps (2nd order markov model)

\[ P(X_{t+1} = x_{t+1} \mid X_t = x_t, X_{t-1} = x_{t-1}, \ldots, X_0 = x_0) \]

\[ = P(X_{t+1} = x_{t+1} \mid X_t = x_t, X_{t-1} = x_{t-1}) \]
mobility (fast- slow fading) 
coherence time / Doppler spread

Environment "echoes" of flat - frequency selective fading
Delay Spread / Coherence BW

Inter-Symbol Interference (problem at high data rates)

DSSS, OFDM

CDMA, Spread Spectrum - FHSS
OFDM, Error Control Coding

Diversity & MIMO
using multi-antenna systems.

TX P  P  P  P  P  P  P  P
I n  spaced apart
RX n  m  m in different channels
Receive diversity

Selection diversity: pick the stronger signal.

Combination diversity

Equal combining

Maximum ratio combiner (MRC)
\[ \frac{a_1^2}{n^2}, \frac{a_2^2}{n^2}, \frac{a_3^2}{n^2} \]

\[ \frac{(\alpha_1, a_1)^2}{n^2}, \frac{(\alpha_2, a_2)^2}{n^2}, \frac{(\alpha_3, a_3)^2}{n^2} \]

\[ \text{SNR: } \sum_{i=1}^{L} \frac{(\alpha_i, a_i)^2}{n^2} \]

\[ \text{MRC} \]

phase shift, gain
\[ \theta_1, \theta_1, \theta_1 \]

\[ g \]

all branches phase synchronized

\[ g \]

signal amplitude
\[ E[\text{SNR}_{\text{out}}] = L \cdot E[\text{SNR}] \]

for each branch.

error rate \( \propto \frac{1}{\text{SNR}} \) for diversity order. (Low SNR values)

In general, max diversity order \( L = \text{MIN} \).

Diversity benefit of MIMO systems

- more channels / effective SNR & lower error rate.

Multiplexing benefit of a MIMO system:

more antennas can be translated to throughput improvements as well.
Capacity of an MxN MIMO system (at high SNR, fading)

\[ \approx \min(m, n) \cdot \log(\text{SNR}) \]

e.g. 2x3 MIMO system

\[ \min(3, 3) = 3 \]

\[ 3 \times 2 \]

\[ \min(3, 2) = 2 \]

\[ \begin{bmatrix}
    h_{11} & h_{12} \\
    h_{21} & h_{22}
\end{bmatrix} \quad \text{MIMO channel matrix} \]

\[ \begin{bmatrix}
    a_1 \\
    a_2
\end{bmatrix} \]
MIMO systems show a direct multiplexing tradeoff:

max - diversity ~ m \cdot n
(error rates)

max - multiplexing ~ \min(m, n)

if we have a multiplexing gain of r

\bullet \text{ diversity gain } \sim (m-r)(n-r)

\text{e.g., } 4 \times 4:

\text{if } d = 16 \quad \text{gm} = 0 \quad \text{single channel}

\text{or } \quad d = 4 \quad \text{gm} = 2 \times

\text{if } d > 0 \quad \text{gm} = 4 \times \quad \text{single channel}
Diversity gain (Clerckx) improves, multiplexing gain (throughput) decreases.

MIMO PDMT
directly multiplying tradeoff.

HW 2: estimating parameters of a two-state MC from data.

011000011100...

Maximum likelihood estimate:

for what value of \( p_0, p_1, p_{10} \) is the observed sequence most likely?
$p_{01} = \frac{\# 0 \rightarrow 1}{\# 0}$ is assessed.

$p_{00} = \frac{\# 0 \rightarrow 0}{\# 0 \rightarrow 0} \quad \# 0 \rightarrow 0$

$p_{01} = 1 - p_{00}$.

Sim. for $p_{10}$ & $p_{11}$

No class on June 13th
(we will prepare a lecture)

On June 15th, TA will teach how to do NS3 simulations of Wireless Networks