Lecture 5

EE 597 Lecture Notes, Monday June 6, 2016.

Announcements:

- No class on Monday, June 13th
- On Wednesday, June 15th will have a special lecture focused on doing simulations of Wireless Networks on NS3 by the TA.

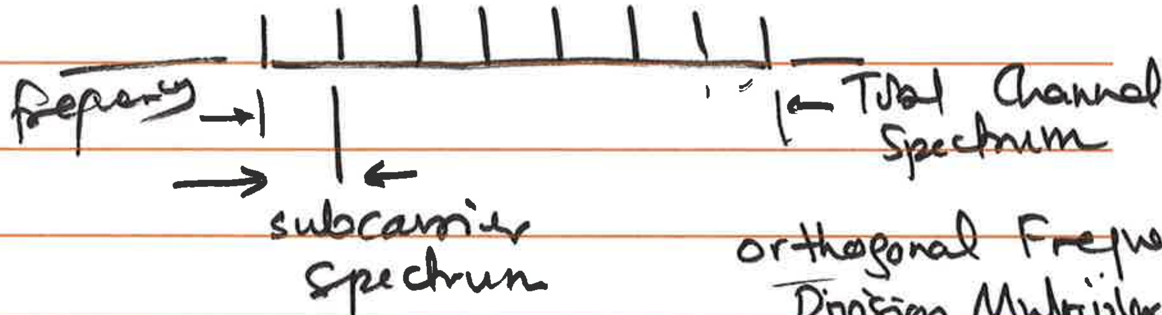
Phy

└ Always involves a tradeoff

Power - Error Rate - Throughput

Power Control & Power Allocation

OFDM subcarriers could be used to carry data for one or more users



orthogonal Frequency Division Multiplexing (OFDM)

Maximizing Sum-Throughput over Parallel Channels.

$$\max \sum_i R_i$$

→ throughput of ith channel.

Shannon's capacity (AWGN)

$$R_i = \underline{W_i} \cdot \log(1 + \underline{SNR_i})$$

W_i: Bandwidth

SNR_i: can be controlled at transmitter by setting power

$$\max \sum R_i$$

$$R_i = W_i \log(1 + SNR_i) = \log\left(1 + \frac{P_{T_i} g_i}{n_i}\right)$$

assume constant for all channels

normalize it to be 1.

where $P_{T_i} \leftarrow$ transmitter power on channel i

$g_i \leftarrow$ gain on channel i

i.e. $\boxed{P_r = P_T \cdot g}$

$n_i \leftarrow$ noise power on channel i

$$\text{s.t.} \sum_{i=1}^{N} P_{t_i} \leq P_{Total}.$$

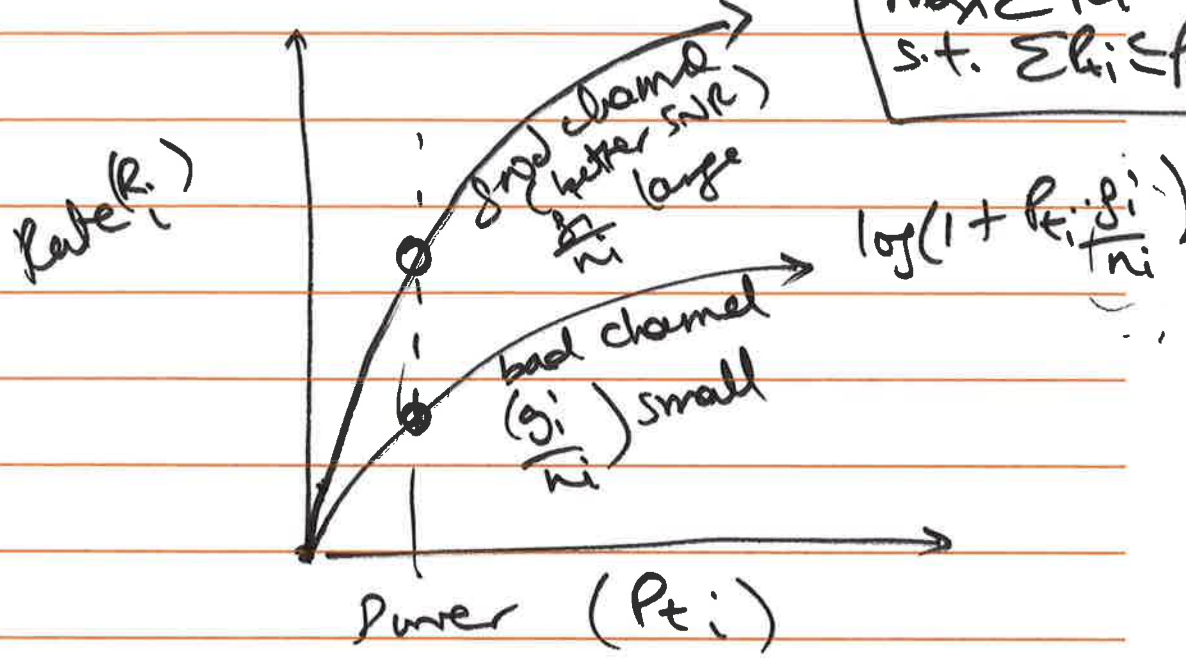
a possible solution: $P_{t_i} = P_t \; \forall i$

i.e. $P_{t,i} = \frac{P_{Total}}{N}$

i.e. uniform power allocation

Consider 2-channels

$\boxed{\begin{array}{l} \max \sum R_i \\ \text{s.t. } \sum P_{t_i} \le P_{total} \end{array}}$



g_i/n_i ⟵ Gain to Noise Ratio

(Law of diminishing returns)

Method of Lagrange Multipliers

$$\max \sum_{i=1}^{N} l_i = \max \sum_{i=1}^{N} \log\left(1 + \frac{P_{Ti}\, g_i}{n_i}\right)$$

$$\text{s.t} \sum_{i=1}^{N} P_{ti} \le P_{TOTAL}$$

$$P_{ti} \ge 0 \quad \forall i \in (1 \cdots N)$$

unknown design variable

multivariable optimization problem with constraints

$$\max \; f(x_1, x_2 \cdots x_N)$$

$$\text{s.t} \quad g_i(x_1, x_2 \cdots x_N) \le c_i \quad i = 1 \text{ to } M$$

Lagrangian $\rightarrow \mathcal{L}(\vec{x}, \vec{\lambda}) = f(\vec{x}) - \sum_{i=1}^{M} \lambda_i (g_i(\vec{x}) - c_i)$

original variables — Lagrange multiplier variables

$$\max \mathcal{L}(\vec{x}, \vec{\lambda})$$

KKT conditions: $(\vec{x}^*, \vec{\lambda}^*)$

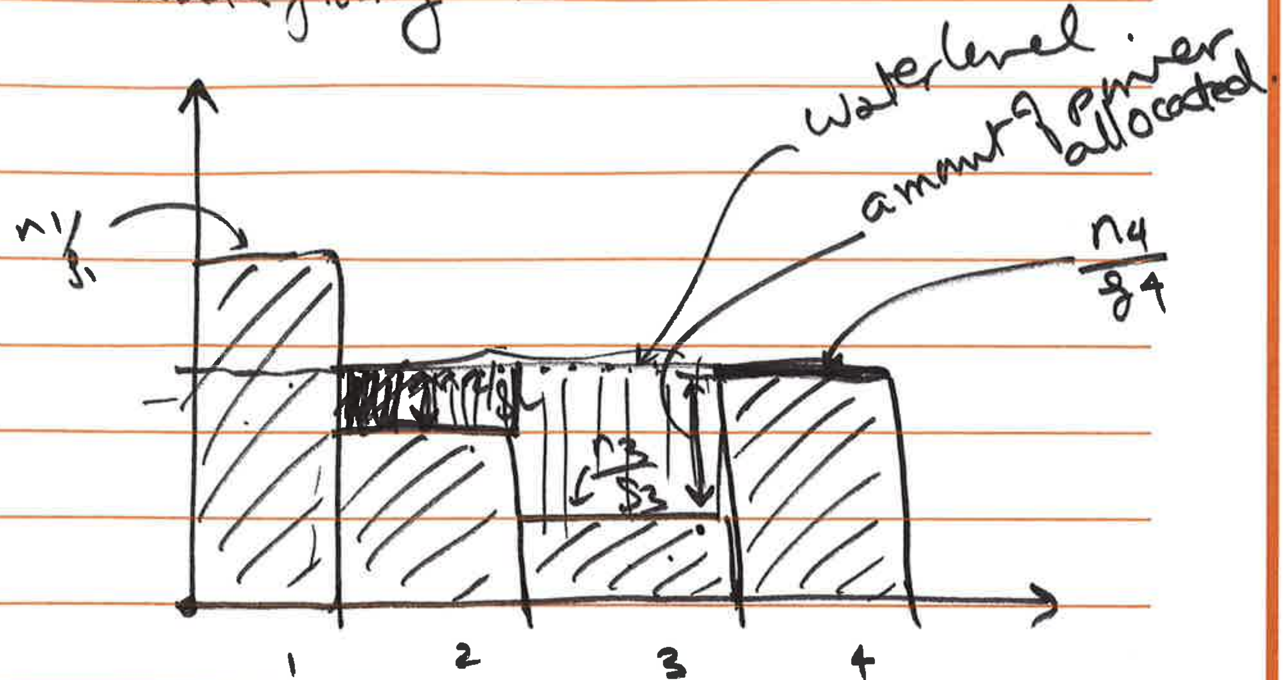
$$\left.\frac{\partial \mathcal{L}}{\partial x_i}\right|_{x^*, \lambda^*} = 0$$

$$\left.\frac{\partial \mathcal{L}}{\partial \lambda_i}\right|_{x^*, \lambda^*} = 0 \quad g_i(\vec{x}^*) \le c_i$$

complementary slackness conditions $\rightarrow$ either $\lambda_i^* = 0$ or $g_i(\vec{x}^*) = c_i$

Solution of the problem:

Waterfilling:



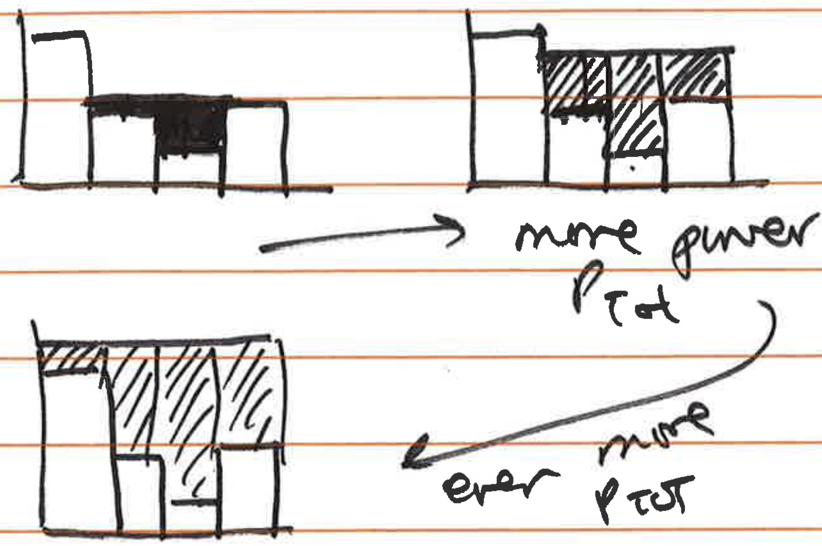
Water level is defined by the total power available : P_{Tot}.

Visually : 1. Channels that are really bad (high $\frac{n_i}{g_i}$ ratio) may not get any power at all

2. multiple channels may get allocated power (unequal in general).

3. The least noise channel gets the highest power

4. Increasing the total power available is equivalent to raising the water level.



algorithmically: this can be implemented by a greedy allocation strategy: say power allocation at each step is discretized to be some ΔP.

↳ allocate the next ΔP to the channel which yields the highest additional rate ΔR_i

Waterfilling equation

water level

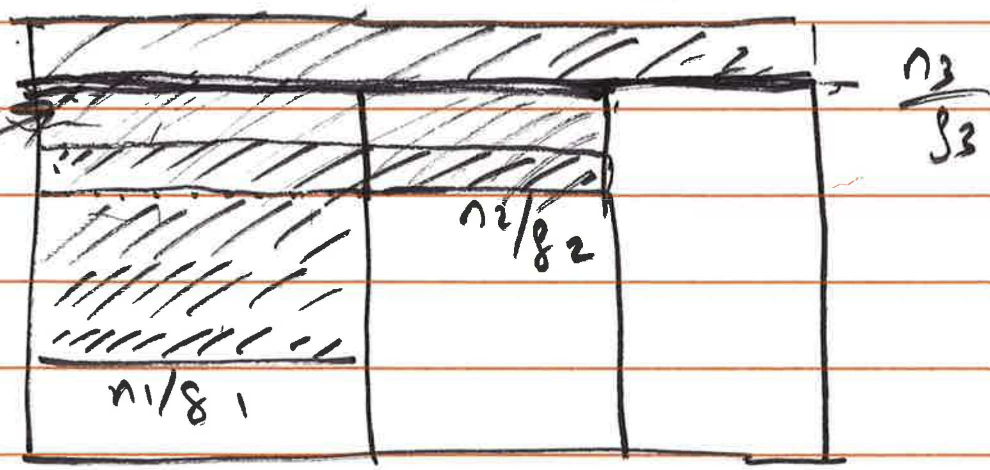
$$P_{t_i}^*(P_{tot}) = \left(l(P_{tot}) - \frac{n_i}{g_i}\right)^+$$

$$\sum_{i=1}^{N} \left(l(P_{tot}) - \frac{n_i}{g_i}\right)^+ = P_{tot}$$

total power

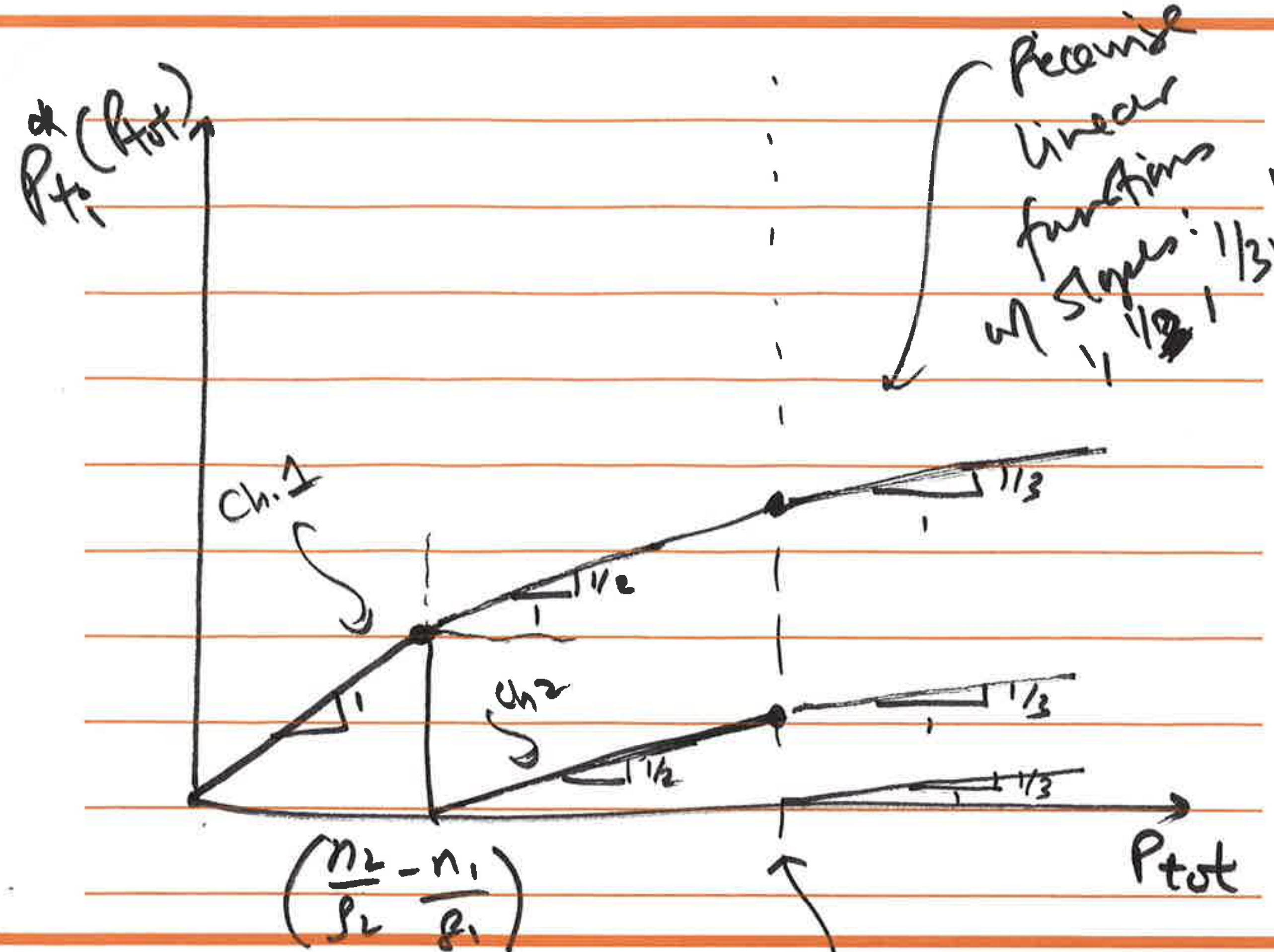
solve this equation to figure out $l(P_{tot})$

& then we know $P_{t_i}^*(P_{tot})$.



total power at this pt

$$\rightarrow \left(\frac{n_3}{g_3} - \frac{n_1}{g_1}\right) + \left(\frac{n_3}{g_3} - \frac{n_2}{g_2}\right)$$



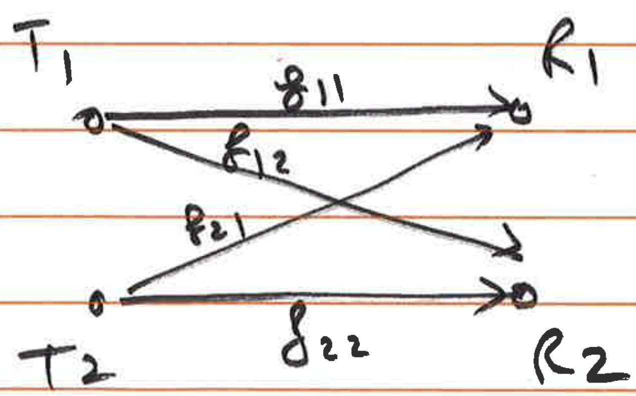
$$\left(\frac{n_3}{g_3}-\frac{n_1}{g_1}\right)+\left(\frac{n_3}{g_3}-\frac{n_2}{g_2}\right)$$

Power Resource allocation.

In waterfilling
more power = more rate

—

power control for independent links.



fixed-rate radios

desired threshold

$$\mathrm{SINR}_1 : \quad \frac{P_{T_1} \cdot g_{11}}{P_{T_2} \cdot g_{21} + N_1} > \theta$$

assume g_{ii}, g_{ij} are constant
(e.g. slow fading)

$$\mathrm{SINR}_2 : \quad \frac{P_{T_2} \cdot g_{22}}{P_{T_1} \cdot g_{12} + N_2} > \theta$$

for now, say $N_1 = N_2 = N$

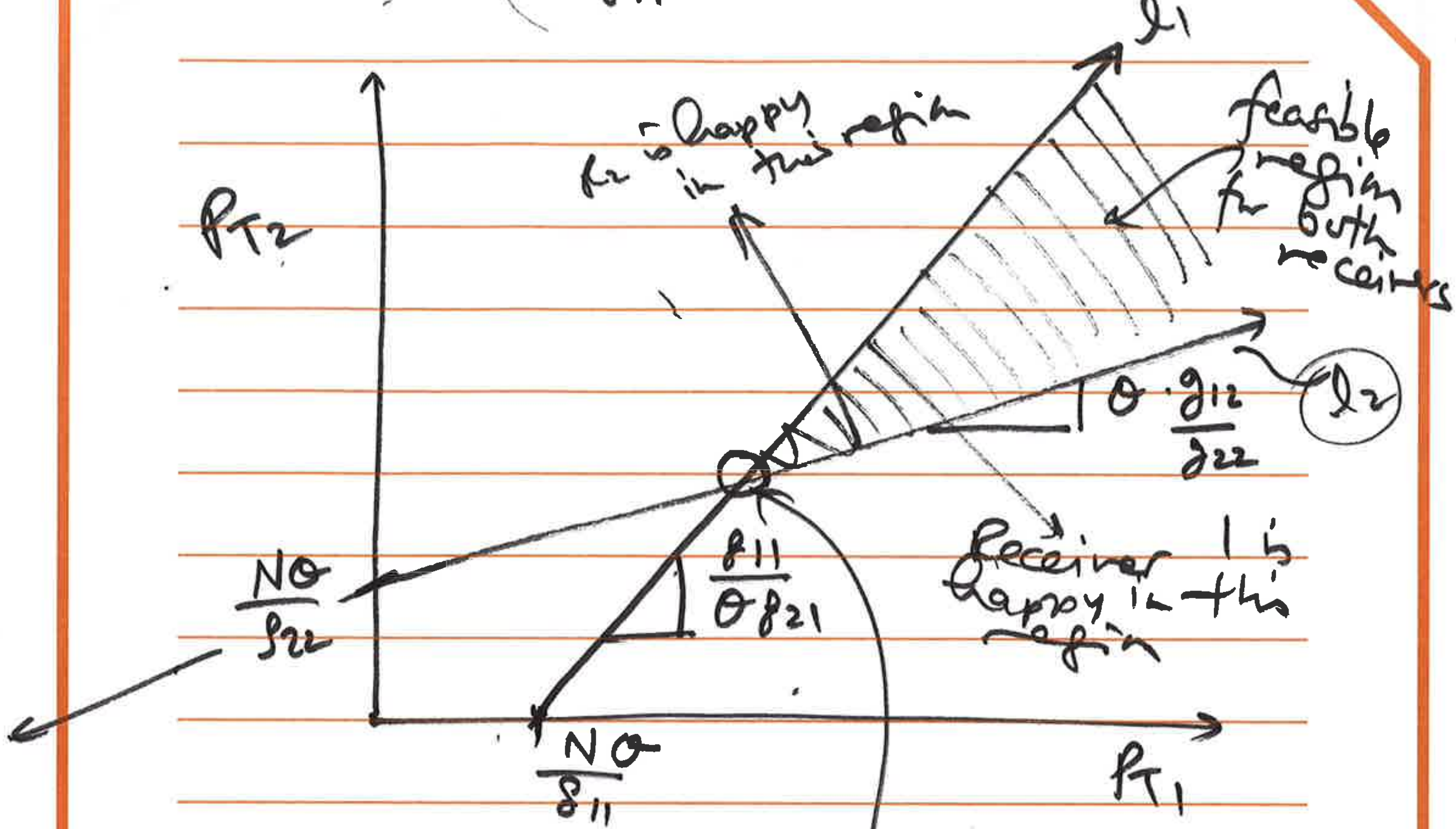
$$P_{T_1} \cdot g_{11} \geq \theta (P_{T_2} \cdot g_{21} + N) \quad \text{--- } 1$$

$$P_{T_2} \cdot g_{22} \geq \theta (P_{T_1} \cdot g_{12} + N) \quad \text{--- } 2$$

want both inequalities to hold for both links to be satisfied.

$$\underbrace{P_{T_2}}_{y} \geqslant \underbrace{\frac{\theta \cdot g_{12}}{g_{22}}}_{a} \cdot \underbrace{P_{T_1}}_{x} + \underbrace{\frac{\theta N}{g_{22}}}_{b}$$

$$P_{t_1} \geq \frac{\theta \cdot g_{21}}{g_{11}} \cdot P_{t_2} + \frac{N\theta}{g_{11}}$$



Close to here is good to operate (why?) saves power!

If these lines intersect, can easily compute the operating point exactly.
(2 equations in 2 unknowns)

There exists an intersection (i.e. solution where both receivers are happy) if and only if slope for l_2 < slope for l_1.

i.e. $\frac{\theta \cdot g_{12}}{g_{22}} < \frac{g_{11}}{\theta \cdot g_{21}}$

$$\Rightarrow \underbrace{g_{11} \cdot g_{22}}_{\text{forward signal gains}} > \theta^2 \cdot \underbrace{g_{12} \cdot g_{21}}_{\text{cross interference gains}}$$

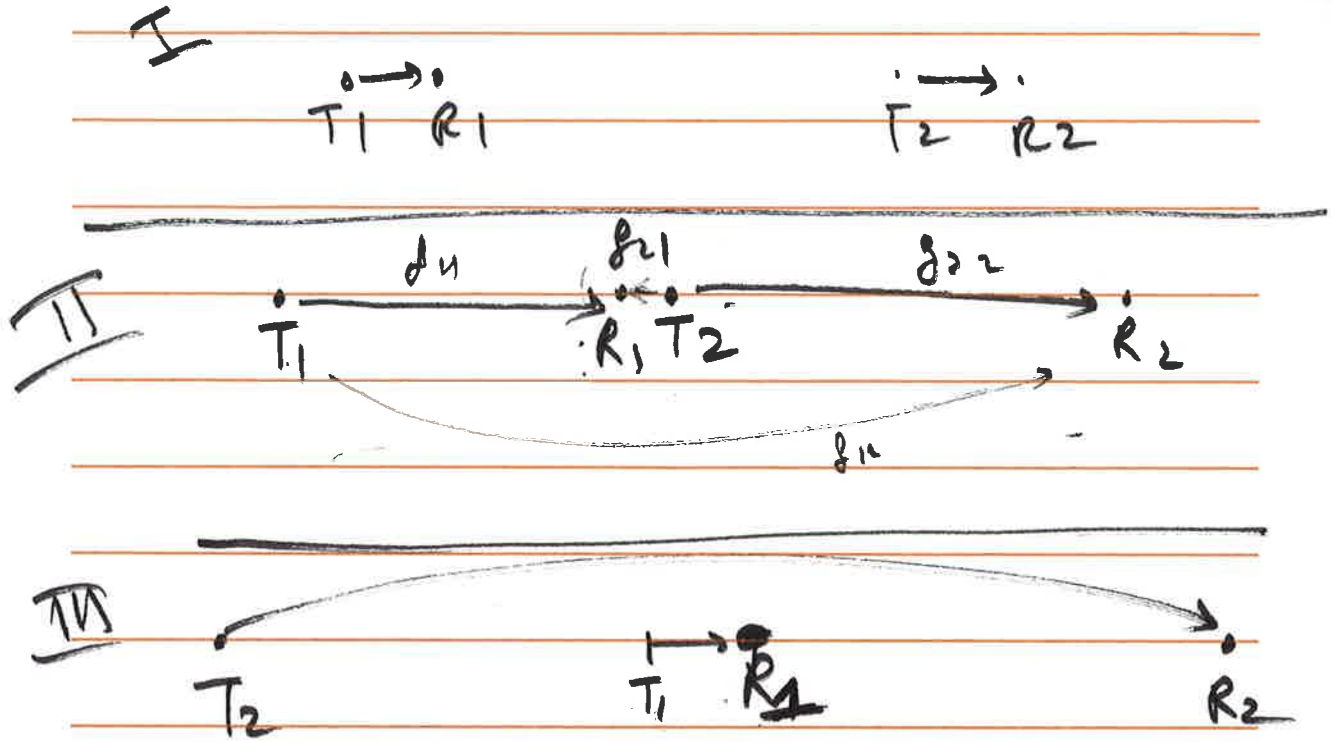
SINR threshold.

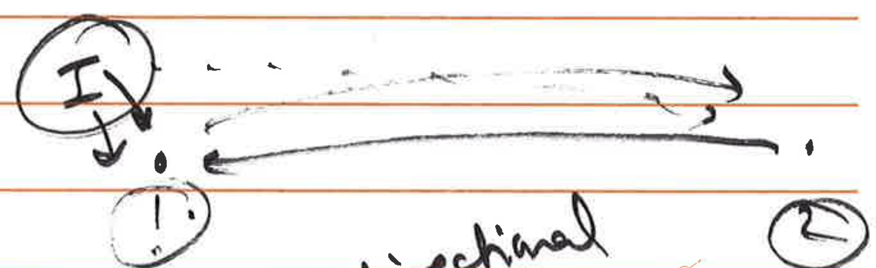
intuitively, harder to find a solution, when:

a) SINR threshold is high

b) cross-interference is high

c) signal gain is low.





unidirectional link

- Power allocation over parallel channels - waterfilling.
- Power control over (2) indep. channels