EE597 Lecture 7 June 20, 2016

Recap.

Medium Access Protocols.

Randomized access

slotted ALOHA.

n backlogged transmitters

each sends at each slot

independently, with probability p.

Throughput: expected # of pkts received per slot

# of pkts received is a Bernoulli r.v. i.e 0 | 1

$\therefore$ Expected value = Pr(1 pkt is received)

= Pr(only one transmitter sends)

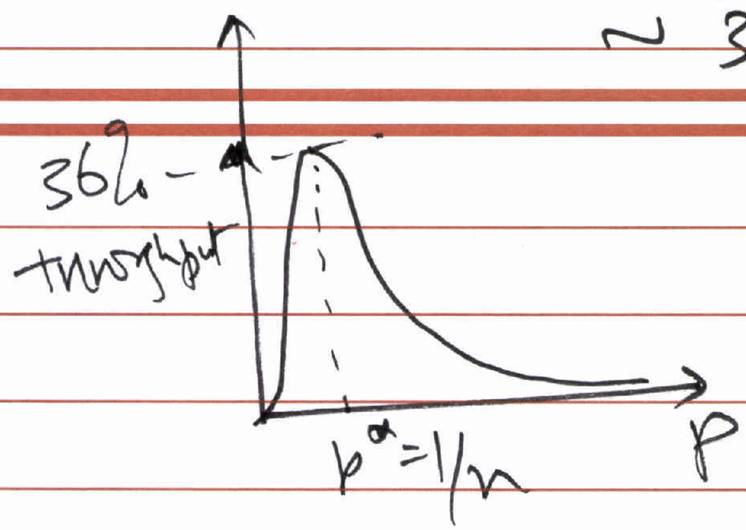
$\therefore$ Expected throughput : $n\,p\,(1-p)^{n-1}$

maximized when $p^* = \frac{1}{n}$

$\therefore$ max total throughput $= \left(1 - \frac{1}{n}\right)^{n-1}$

as $n \to \infty$ Total Throughput $\to \frac{1}{e}$

$\sim 36\%$



per node throughput $= \frac{1}{n} \cdot$ total Throughput

What if different users want different throughputs?

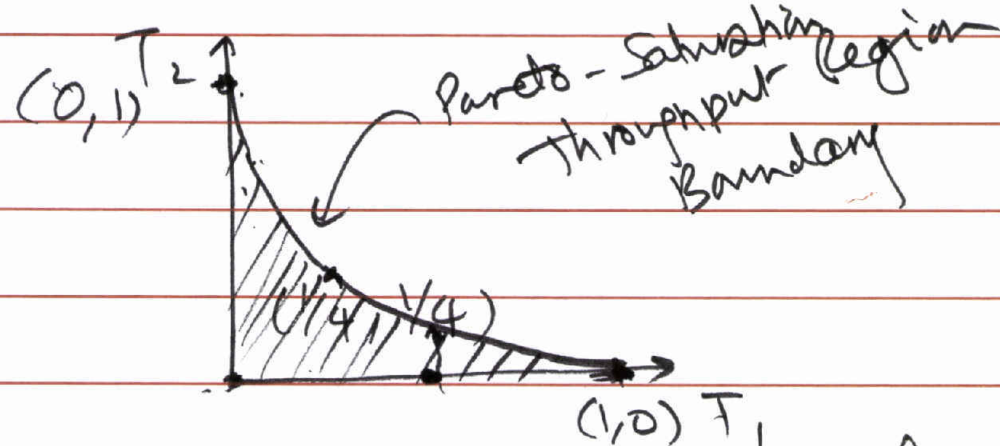
p_i - Slotted Aloha

e.g. 2 users: p_1, p_2

throughput for user 1

$T_1 = p_1 (1 - p_2)$

$T_2 = p_2 (1 - p_1)$



at the boundary $\sqrt{p_1} + \sqrt{p_2} = 1$ why?

finding the boundary:

idea $\max T_2$ s.t. $T_1 = c$ (fixed)

$$\max \; p_2(1-p_1)$$
$$\text{s.t.} \; p_1(1-p_2) = T_1 = c$$
$$\Rightarrow p_1 = \frac{c}{1-p_2}$$

Approach 1

$$\max \; p_2 \cdot \left(1 - \frac{c}{1-p_2}\right)$$

$$p_2 \cdot \left(\frac{1-p_2-c}{1-p_2}\right)$$

$$\max_{p_2} \left(p_2 - p_2^2 - p_2 c\right)(1-p_2)^{-1}$$

$$(1-p_2) = \frac{c}{p_1}$$

Approach 2

$$p_2 = 1 - \frac{c}{p_1}$$

$$\max_{p_1} \left(1 - \frac{c}{p_1}\right)(1 - p_1)$$

$$\max_{p_1} 1 - p_1 - \frac{c}{p_1} + c$$

$$\max_{p_1} \frac{p_1 - p_1^2 - c + cp_1}{p_1}$$

$$\max_{p_1} \underbrace{\left((p_1 - p_1^2 - c + cp_1)\cdot(p_1)^{-1}\right)}_{f(p_1)}$$

$$\frac{df(p_1)}{dp_1} = 0 \Rightarrow (p_1 - p_1^2 - c + cp_1)\cdot -1(p_1)^{-2}$$
$$+ (1 - 2p_1 + c)\cdot(p_1)^{-1} = 0$$

$$\Rightarrow -\frac{p_1 - p_1^2 - c + cp_1}{p_1^2} + \frac{(1 - 2p_1 + c)}{p_1} = 0$$

$$\Rightarrow \frac{-(\cancel{p_1} - p_1^2 - c + \cancel{cp_1}) + \cancel{p_1} - 2p_1^2 + \cancel{cp_1}}{p_1^2} = 0$$

$$c - p_1^2 = 0$$

$$p_1^2 = c$$

$$p_1^* = \sqrt{c}$$

$$p_2^* = 1 - \frac{c}{p_1^*}$$

$$= 1 - \frac{c}{\sqrt{c}} = 1 - \sqrt{c}$$

$$p_1^* + p_2^* = \sqrt{c} + 1 - \sqrt{c} = 1$$

regardless of $c = T_1$

$$T_2 = p_2^* \cdot (1 - p_1^*)$$

$$= (1-\sqrt{c})(1-\sqrt{c}) = (1-\sqrt{c})^2$$

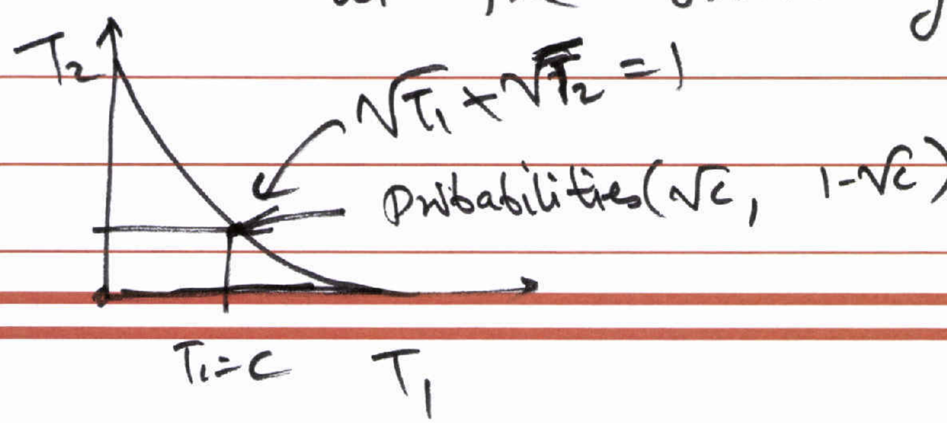
$$T_2 = (1 - \sqrt{T_1})^2$$

$$T_2 = (1 - \sqrt{T_1})^2$$

$$\sqrt{T_2} = 1 - \sqrt{T_1}$$

$$\Rightarrow \sqrt{T_1} + \sqrt{T_2} = 1$$

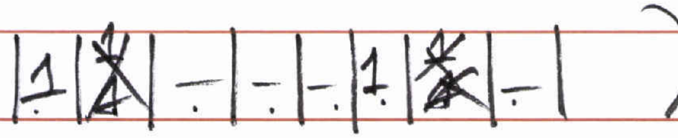
at the boundary!



The result that $\sum_{i=1}^{N} p_i^* = 1$ generalizes to any number of users.

CSMA

└ Carrier Sense Medium Access



typical Aloha timeline

the nodes contend over a small slot of size δ.

(I) If no one transmits in this small slot, go to the next contention slot (δ) & repeat.

(We are looking at a simplified version of CSMA called p-CSMA)

(II) If 2 or more nodes start to transmit in that slot $\rightarrow$ collision, & the transmission lasts for T_c time.

T_c ← collision time.

during time T_c there is no more contention from other senders, because when they sense the carrier they will find it busy.

IV If only 1 node transmits, it transmits for time T_s.

No one else will contend / try to transmit during this time. (because they will sense the carrier to be busy).

successful transmission time

each contention time / epoch ε

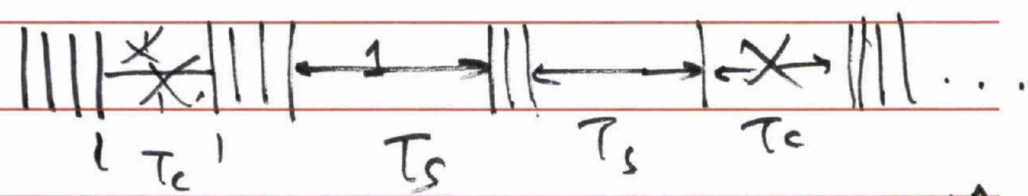
$\delta \leftarrow$ no transmitter (idle)

$T_c \leftarrow$ more than 1 transmitter (collision)

$T_s \leftarrow$ exactly 1 transmitter (success)

$\delta \le T_c \le T_s$

Expected Throughput for p-CSMA = ?



typical CSMA timeline

intuitively if $\delta << T_s$, then the throughput for CSMA should be better than slotted ALOHA.

in fact, one can even view slotted ALOHA as a special case of p-CSMA when $\delta = T_c = T_s$.

$$\boxed{\text{Expected throughput} = \frac{\text{prob. of success} \cdot T_s}{\text{Expected Epoch duration}}}$$

from Key Renewal Theorem

epoch duration — time between contention events.

↳ a random variable

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pmf of the epoch duration:

$$p_E = \begin{cases} \delta & \text{w/ prob: } (1-p)^n \\ T_s & n p(1-p)^{n-1} \\ T_c & 1-(1-p)^n - n p(1-p)^{n-1} \end{cases}$$

Expected Throughput:

$$\frac{n p (1-p)^{n-1} T_s}{(1-p)^n \delta + n p (1-p)^{n-1} T_s + \left(1-(1-p)^n - n p (1-p)^{n-1}\right) \cdot T_c}$$

as $T_s \uparrow$ compared to δ, T_c, the throughput increases.

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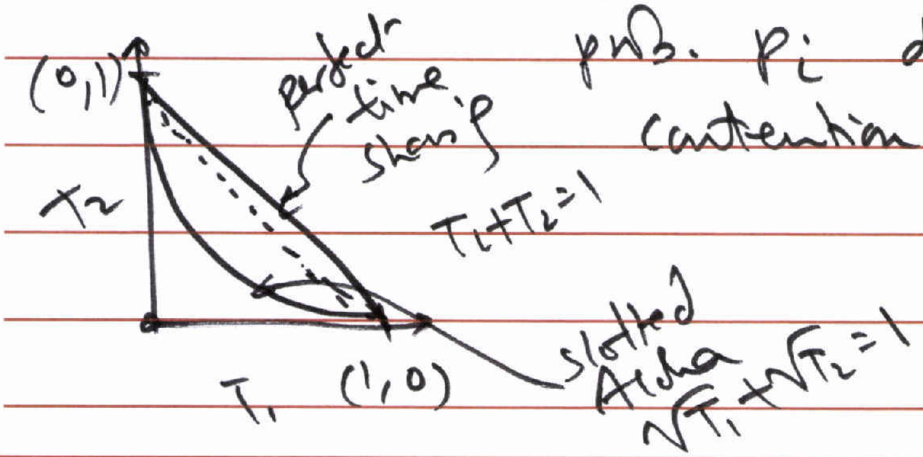
p-CSMA $\rightarrow$ p_i - CSMA

$\uparrow$ transmitter i chooses to contend w/ prob. p_i during contention slots



At the pareto-Boundary of the saturation throughput region of p_i-CSMA, the following condition holds:

$$1 - \prod_i (1-p_i) + \frac{T}{\delta}\left(\sum p_i + \prod(1-p_i) - 1\right) = 1$$

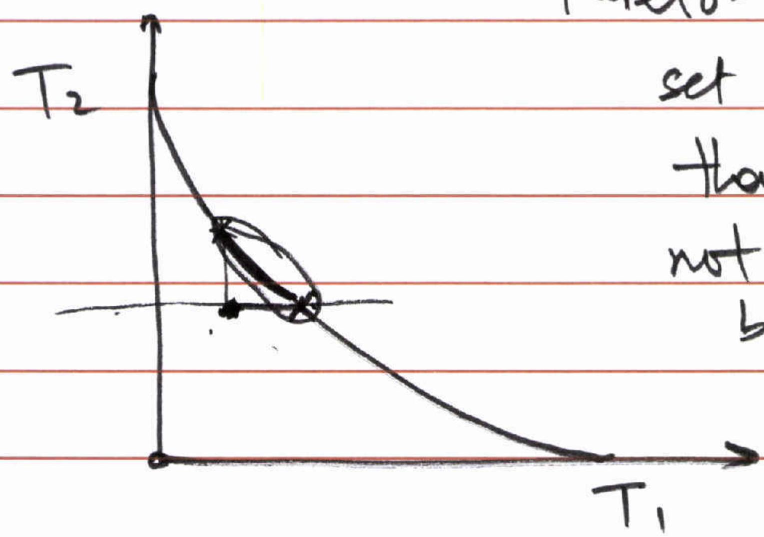
(assuming $T_s = T_c = T$)

$$\Leftrightarrow \quad \frac{T}{\delta}\left(\sum' p_i + \prod(1-p_i) - 1\right) = \prod_i (1-p_i)$$

when $\frac{T}{\delta} = 1$, this is just slotted Aloha

Note in this case the above simplifies to

$$\sum' p_i = 1.$$



Pareto-boundary:
set of points
that are
not "dominated"
by any other
throughput
vector.

IEEE 802.11a, b, g, n use the same access prob. for all sources

IEEE 802.11e

↳ wireless LANs w/ QoS.

different applications
can use different
access "probabilities"

CSMA implemented in IEEE 802.11 standards:

① Instead of choosing to access w/ prob p at each slot ($\Rightarrow$ Geometrically distributed access time) as in p-CSMA, in 802.11, the access time is chosen to be a uniform random variable from $[0, W-1]$

$\uparrow$ backoff window.

Binary-exponential Backoff

② The backoff period (~~n~~ size of the window) is increased each time there is a collision - doubled - up to a maximum window size.

stages of Backoff:

$W_0 \quad W_1 \quad \ldots \ldots \quad W_m$

$W_1 = 2 W_0$ initial backoff window size.

$W_2 = 2 W_1 = 2^2 W_0$

$\vdots$

$W_m = 2^m W_0$.

maximum backoff window size.

Can model the backoff counter's value along with the backoff stage as a Markov chain.

Given W_0, m, n, T_s, δ, T_c

can calculate the saturation throughput for IEEE 802.11.

G. Bianchi's analysis