EE597 Lecture 8 June 22, 2016

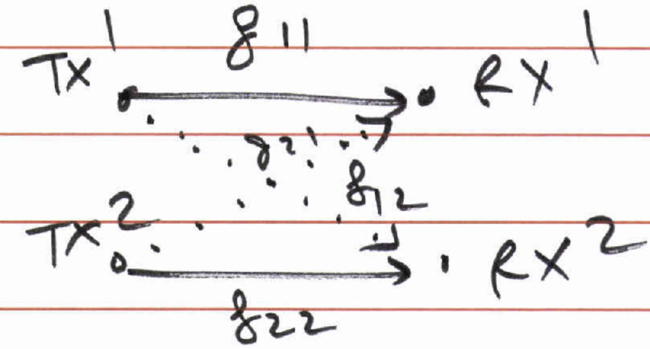
Link Layer

- Power Allocation over parallel channels (sum rate maximization with constrained power)
  - Waterfilling

- Power Control over independent links
  - talked about how this works for 2 links
  - today: generalization to any # of links

randomized channel access
- slotted Aloha
- p-CSMA
- IEEE 802.11 CSMA

reminder of the 2 links problem:

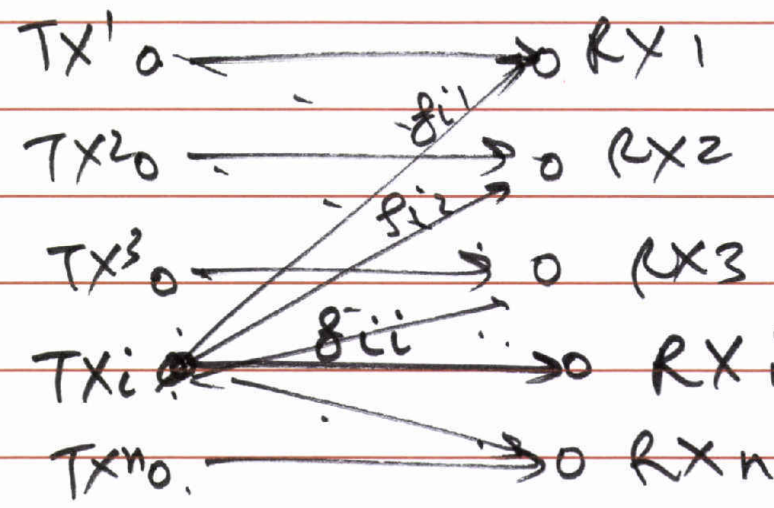


gains are given/known, and not varying — slow fading

$$SINR \geq \theta$$

$$\frac{P_1 \cdot g_{11}}{P_2 \cdot g_{21} + N} \geq \theta$$

$$\frac{P_2 \cdot g_{22}}{P_1 \cdot g_{12} + N} \geq \theta$$



$$SINR_i \geq \Theta \quad \forall i$$

$$\frac{P_i \cdot g_{ii}}{\sum_{j \neq i} P_j \cdot g_{ji} + N} \geq \Theta \qquad i = 1 \ldots N$$

$$P_i g_{ii} \geq \Theta \sum_{j \neq i} g_{ji} \cdot P_j + \Theta N$$

$$P_i g_{ii} - \sum_{j \neq i} \Theta \cdot g_{ji} \cdot P_j \geq \Theta N \quad \forall i$$

$1 \times N$ $\quad$ i^{th} term $\quad$ $N \times 1$

$$\begin{bmatrix} \cdot & , & \cdots & \boxed{g_{ii}} & \cdot & , \end{bmatrix} \begin{bmatrix} P_1 \\ \vdots \\ P_i \\ \vdots \\ P_N \end{bmatrix} \geq \theta N$$

j^{th} term

$-\theta \cdot g_{ji}$

$$\begin{bmatrix} -\theta g_{1i} & -\theta g_{2i} & \cdots & g_{ii} & \cdots & -\theta g_{ni} \end{bmatrix}$$

$$\boxed{A \vec{P} \geq \vec{b}}$$

where A is a matrix $(N \times N)$

$\vec{P}$ is the vector $\begin{bmatrix} P_1 \\ \vdots \\ P_N \end{bmatrix}$

$\vec{b}$ is the vector $\begin{bmatrix} \theta N \\ \vdots \\ \theta N \end{bmatrix}$

$$A_{ij} = \begin{cases} \delta_{ii} & \text{for } j = i \\ -\delta_{ji} \cdot \theta & \text{for } j \neq i \end{cases}$$

$$\underbrace{\underbrace{A \vec{p}}_{N \times N \; \& \; N \times 1}}_{N \times 1} \geq \vec{b}$$

N×1 vector

elementwise

Linear Matrix Inequality

When can we solve: $\underset{n \times n}{A} \; \underset{n \times 1}{\vec{p}} = \underset{n \times 1}{\vec{b}}$

if A is invertible, then

$$A^{-1} A \vec{p} = A^{-1} \vec{b}$$

$$\mathbb{I} \vec{p} = A^{-1} \vec{b}$$

$$\boxed{\vec{p} = A^{-1} \vec{b}}$$

for 2x2 case :

$$A = \begin{bmatrix} g_{11} & -g_{21}\theta \\ -g_{12}\theta & g_{22} \end{bmatrix}$$

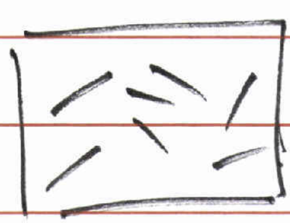
$$|A| = g_{11} \cdot g_{22} - (g_{12} g_{21} \cdot \theta^2) \geq 0$$

$$g_{11} g_{22} \geq g_{12} g_{21} \theta^2$$

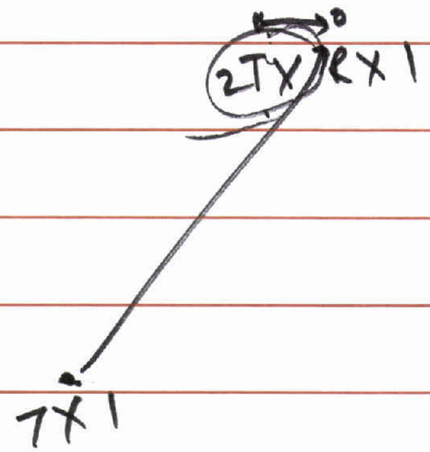
This gives us

① A test for if all N links can be scheduled together

② If so, a power vector that guarantees all SNR values are met w/ equality



Near-far problem in CDMA



Back to random access.

- slotted Aloha
  - n nodes, each transmits at each slot w/ prob. p.

~~carrier sense~~ medium access

- p-CSMA / p-persistant CSMA

- contention slots of small duration δ
- successful pkts take T_s time
- collisions last T_c

$\delta \leq T_c \leq T_s$.

(Special case when $\delta = T_c = T_s$, $\underline{\underline{\text{is}}}$ slotted Aloha)

Expected throughput: $\frac{\text{Exp. Success Duration/epoch}}{\text{Expected Epoch duration}}$

$$\frac{n p (1-p)^{n-1} \cdot T_s}{\delta \cdot (1-p)^n + n p (1-p)^{n-1} \cdot T_s + \left(1 - (1-p)^n - n p (1-p)^{n-1}\right) \cdot T_c}$$

(Pi-CSMA ← Throughput region)

W_0 W_1 W_2 W_3 W_4 W_5 - W_m

16 → 32 → 64 → 128 → 256 → 512 → 1024

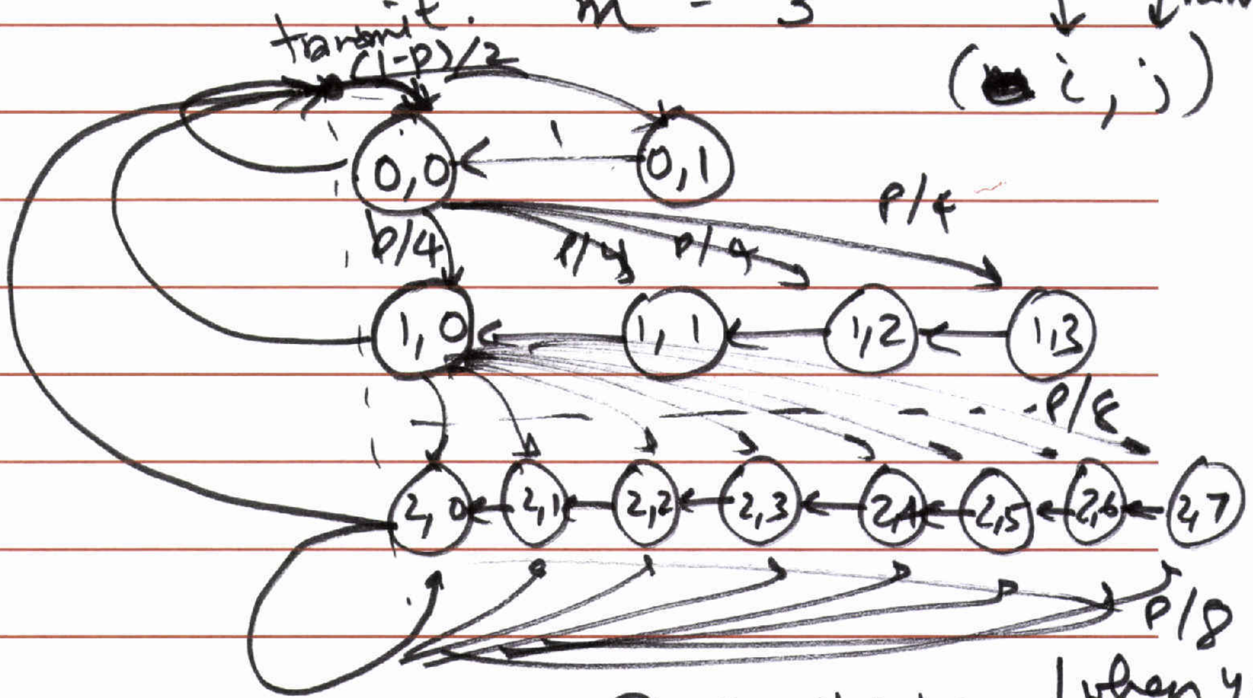
1st stage ... mth stage

The combination of Backoff counter & stage number can be viewed as a state for the CSMA protocol in IEEE 802.11

<u>Toy example:</u> $W_0 = 2$ stage #   backoff value

$m = 3$

(i, j)



p ← prob. of collision | when you transmit

$1-p$ ← " of success

state : $(s(t), b(t))$

$b(t)$: 0 to $W_s - 1$

$s(t)$: stage: 0 to m

$b_{i,k} \leftarrow$ steady state probability of being in state $(s = i, b = k)$.

$$\vec{\pi} \mathbb{P} = \vec{\pi}$$
$$\Sigma \pi_i = 1$$

Can show, for any m & initial window size W, & collision prob. p,

$$b_{0,0} = \frac{2(1-2p)(1-p)}{(1-2p)(W+1) + pW(1-(2p)^m)}$$

if $p = 0$ $\quad b_{0,0} = \frac{2}{W+1} = \tau$

$$b(i,0) = p^i \cdot b_{0,0}$$

$$\sum_{i=0}^{m} b(i,0)$$ = steady state probability of being in a transmission state

$$\tau = \sum_{i=0}^{m} p^i \cdot b_{0,0} \approx \frac{b_{00}}{1-p}$$

$$\tau = \frac{2(1-2p)}{(1-2p)(W+1) + pW(1-(2p)^m)} \quad \text{——— (1)}$$

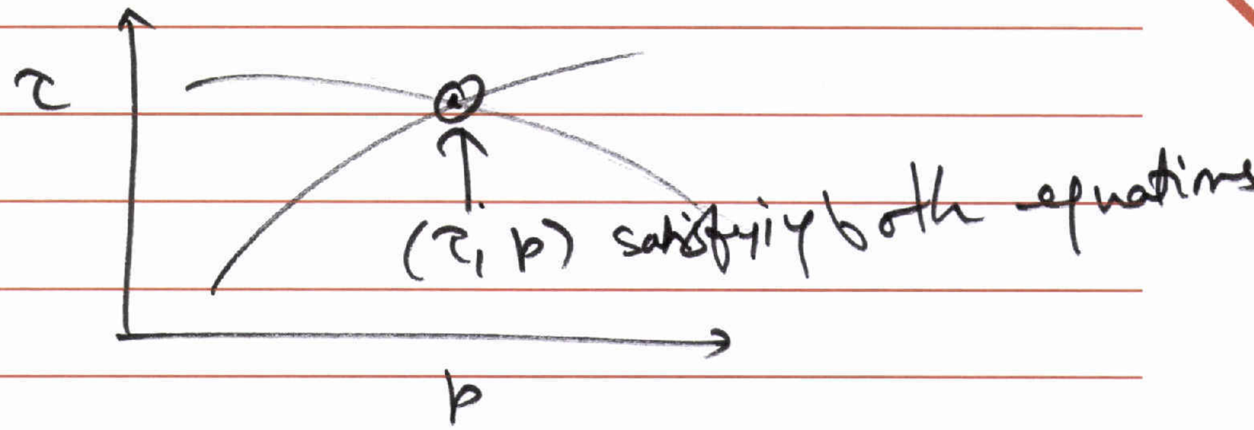
prob. that a node transmits in a given slot

use a "p-CSMA model" to determine relation between τ & p.

$$1-p = (1-\tau)^{n-1} \quad \text{——— (2)}$$

$$p = 1-(1-\tau)^{n-1}$$

# of nodes contending.



then we know p, τ

Substitute τ into the throughput expression for p-CSMA & you get the (approximate) saturation throughput for IEEE 802.11 CSMA.

Overall, this gives a procedure to calculate the throughput given $\underbrace{W, m}_{\text{protocol parameters}}, \underbrace{n}_{\text{\# of contending nodes}}$.