EE 597 July 6, 2016

<u>Graph Coloring</u>

↳ A tool for allocating channels (time, frequency, codes) to potentially interfering cells or links.

Announcement: Mid-term exam is on monday, July 11

The exam will be in-class, for the usual class duration (12.25 - 2 pm)

Open notes, books, print-outs (anything on paper you want to bring in)

<u>BUT</u> no electronics: cell-phones, calculators, computers, tablets etc.

We will do a brief review of topics to date in class

TA will post sample midterm exams and problems on the course site. But NOT the solutions.

exam format (likely):
A series of short questions/definitions/fill in the blanks to test your understanding of basic concepts.
followed by $n=3$ or $4$ calculation problems, from which the best $n-1$ count.

Recommend working on what seems to you to be easiest first.

Another recommendation for exam prep:
go over the sample problems
& see if you identify
"classes" of problems.

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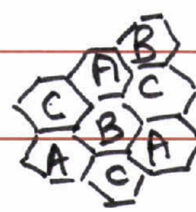
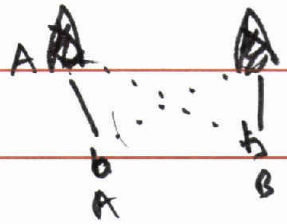
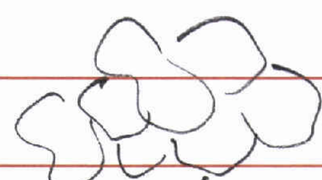
<u>Recap:</u> Cellular wireless networks



← a particular "reuse pattern" of channels that ensures that nearby / neighboring cells do not have the same channels.




↑
Ideal



← Reality: irregular cell shapes: due to propagation effects as well as cell/BS placement constraints as well as transmit power levels and also to match user traffic.

The Graph coloring approach can be used if we first abstract cells & their interference relationships in the form of a graph $G = (V, E)$

cells

interference relationships.

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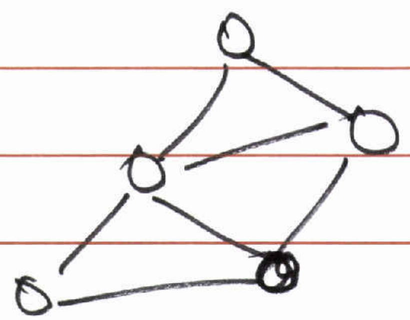

<u>Graph coloring</u>:

Given G, find the smallest no. of colors, s.t. each vertex gets a color different from the color of its neighbors.
(Vertex coloring)

<u>NP-hard problem</u>

In general, no exact alg. that is tractable/efficient exists.

An Example heuristic that is often used: Greedy vertex degree ordering heuristic

Not guaranteed to be always minimal but works well in practice.

Other algorithms & heuristics also exist for this problems, e.g. Simulated Annealing, Genetic Algorithms, etc.

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Graph coloring is applicable to other channel allocation problems as well. E.g. is channel allocation for links in multihop wireless networks.

$$\omega(G) \le \chi(G) \le \Delta_{max}(G) + 1$$

maximum clique size

chromatic # of a graph

max degree of the graph

optimal soln: min. number of colors that can be assigned feasibly.

GVDO

clique: fully connected subgraph

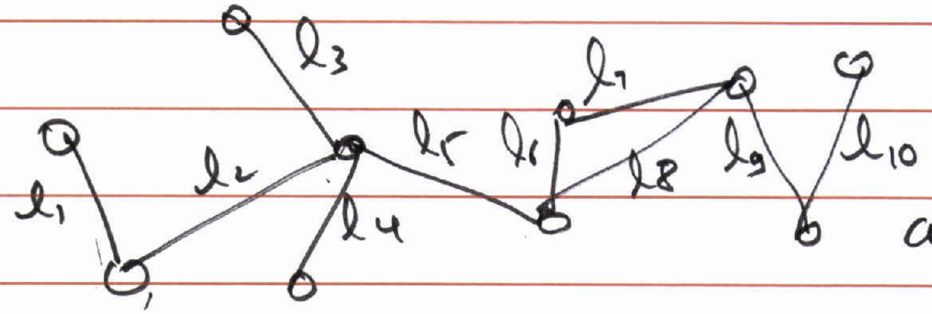
E.g. IEEE 802.15.4e / TSCH

Time Slotted Channel Hopping

A standard for Industrial wireless sensor / embedded networks where the links are synchronized and allocated specific time slots & frequency

channels in order to be more robust, high throughput & low latency.

multi-hop wireless network




communication graph

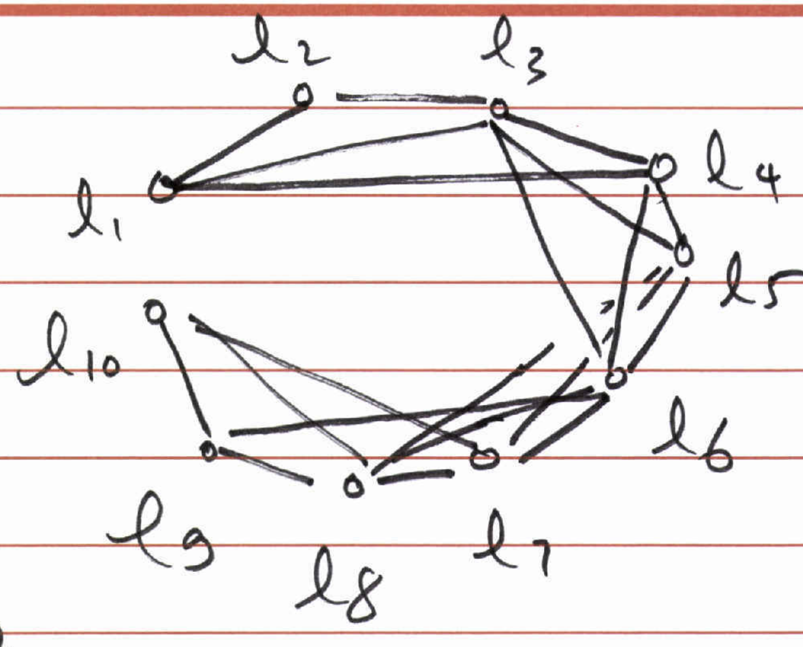
e.g.: mesh networks, wireless sensor networks / low power wireless networks (IEEE 802.15.4),

mobile ad-hoc networks (MANETs)

e.g. 802.15.4 std. provides for ~16 channels in the 2.4 GHz range

step 1: identify pairs of interfering links

Step 2 : construct a conflict graph that captures these interference patterns.
In this graph the vertices correspond to links in the original graph.



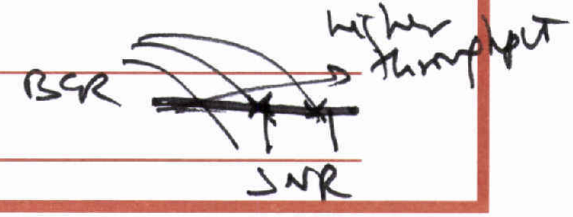

Conflict graph H

Step 3
Color the conflict graph: min. # of freq. that we can allocate w/o interference.

# Review of the Class so far

Phy Layer & MAC/Link Layer.

Phy Layer: · Digital modulation schemes
- constellation diagrams
- power (SNR) - throughput - error tradeoff




Rate $\propto \log(1+SNR)$

- Radio Propagation
  - simple path loss

$$P_R = P_T \cdot K \left(\frac{d}{d_0}\right)^{-\eta}$$

$\eta$ ← path loss exponent ($\sim 1$ to $5$)

$d_0$ → ref. distance

$d \geq d_0$

fading: happens due to multipath reflections, absorption, material properties of env. etc.

└ iid random variable

fading r.v.

$$P_R = P_T K \left(\frac{d}{d_0}\right)^{-\eta} \cdot \psi$$

Typical model for $\psi$ is log-normal distribution

$$P_{R_{dBm}} = P_{T_{dBm}} + K_{ref,dB} - 10\eta \log_{10}\left(\frac{d}{d_0}\right) + \psi_{dB} \sim N(0, \sigma_{dB}^2)$$

fading variance

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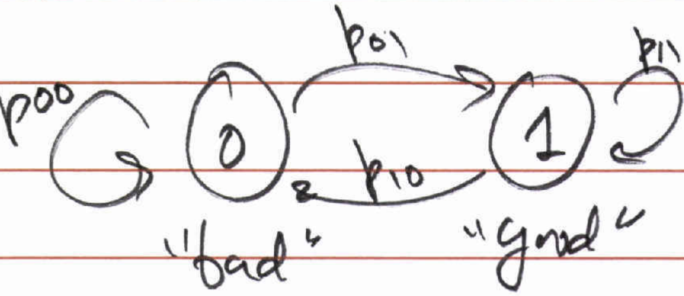
Another set of typical models that capture temporal correlations are discrete markov models




simple 2-state Gilbert-Elliot model.

AWGN channel: bit error rate $P_b(\gamma)$

Two metrics for fading channels:

1. Expected bit error rate

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$$

2. Probability of outage:

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Probability that $\gamma < \Theta$.

r.v.

SNR threshold (predefined)

$$\int_{-\infty}^{\Theta_{out}} f_X(x)\, dx$$

SNR fading distribution.

other physical layer concepts:

ways to characterize fading

- Doppler Spread & Coherence Time
- Delay Spread & Coherence BW

slow or fast fading → low $T_c$, high doppler spread

→ high $T_c$, low doppler spread

→ intersymbol interference results if symbol duration is not large compared to the delay spread.

flat or frequency-selective fading

→ OFDM is one solution: it divides a single high rate stream into multiple low-rate streams / subcarrier w/ large symbol duration used in CDMA

· Spread spectrum : · DSSS
· FHSS used in Bluetooth

· MIMO (multiple antennas at sender & receiver)
- maximum ratio combining - diversity
- Diversity - multiplexing tradeoff: improve throughput or error-rates (or some combination of these)

· Coding ___

MAC & Link layer:
- power control & allocation:

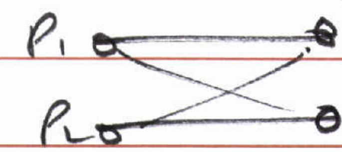
allocating constrained power to parallel channels to maximize the sum-rate (or other functions of rate)
— Waterfilling
(applying method of Lagrange multipliers to calculate opt. power allocation)

We assumed:

$$R_i \propto \log\left(1 + P_i \cdot \frac{g_i}{N_i}\right)$$

this ⇒ adaptive radio

Power control w/ indendent links.




$SINR_1 > \theta$

$SINR_2 > \theta$

fixed rate radio

Slotted Aloha - homogeneous case

(p)

heterogeneous case: $p_i$

↳ saturation throughput region

$\sum p_i^* = 1$

max Throughput

$\sim \frac{1}{e}$

p-CSMA, $p_i$-CSMA

$$\dot{\delta} \leq \dot{T_c} \leq \dot{T_s}$$

Throughput expressions for p-CSMA take these into account

max throughput $\rightarrow 1$ as $\frac{T_s}{\delta}$ gets larger

- 802.11 CSMA

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Time/frequency allocation using Graph coloring