Computing Inter-Encounter Time Distributions for Multiple Random Walkers on Graphs

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***Abstract*—For intermittently connected mobile networks such as sparsely-deployed vehicular networks, it is of great interest to characterize the distribution of encounter times. We consider a very general mobility model in which each device is assumed to be moving through a given graph following a general random walk with arbitrary transition probabilities. We consider first the pairwise inter-encounter time distribution for a pair of random walkers and present a recursive polynomial-time computation that yields the exact solution. We then consider the individual-to-any inter-encounter time (i.e., the time between contacts of a particular walker with any of the other walkers in the population). For this harder problem, we give an approximate computation that is also polynomial time. We validate the accuracy of the presented solutions using numerical simulations. We discuss how the model can be generalized to consider multiple populations.**

***Index Terms*—Random walk ; Encounter Distributions ; Mobile Networks**

I. Introduction

Intermittently connected mobile networks (ICMN) are networks in which communication links and contacts among mobile nodes are not fixed but created highly dynamically and intermittently. There may be no instantaneous end-to-end paths for any pairs of source and destination in such a network. Such networks are relevant for instance for peer to peer mobile device applications and for sparsely-deployed vehicular networks with short range vehicle to vehicle communication links.

Modeling of ICMN aims at prediction of different parameters and features of the mobility networks in order for proposing corresponding algorithms, enhancing performance of routing as well as reducing total delay of data dissemination in network of vehicles. Due to unstable links among objects and devices, store and forward schemes are typically used in an ICMN, in which a node keeps the data, travels and transmits the data whenever there is a contact. If two nodes meet each other more often, they may have a higher probability to transmit data to each other. And if a node meets any other node quickly after an encounter in which it received some data itself, it can help disseminate that data faster through the network. Therefore, among other metrics, inter encounter times are a crucial key metric characterizing information dissemination and routing in such opportunistic networks.

This work was supported in part by NSF through grants CNS-1217260, and CCF-1423624.

We consider in this work a general model of multiple random walkers (representing the mobile devices) travelling with a random movement pattern in a network represented by a graph. All of the walkers make movement decision every time slot; each walker follows a transition probability matrix staying at its original location or moving from one vertex to another. We consider two important random variables relate to the encounter process: the Pairwise Encounter Time (PET), the inter-encounter time between any pairs of walkers, and the Inter-Any Encounter Time (IAET), the inter-encounter time between a particular walker and any of the other walkers in the network.

The following are our key contributions:

- We present the first exact computations for the PET distribution that are polynomial in the number of walkers, the size of the graph, and the support of the distribution.
- We present the first approximate-computation for the IAET distribution on a general connected graph and show through numerical simulations that this approximation is quite accurate.
- For greater generality, we further extend our results from a single community case (all walkers follow the same movement pattern) to multiple communities case (multiple groups of walkers in the network, wherein the walkers within each group follow the same movement pattern but different groups have different patterns.) For this generalization also the complexity of our computations remains polynomial. In addition, we validate this extension through numerical simulations.

The rest of this paper is organized as follows: section II lists related work; section III describes the problem formulation and its corresponding complexity; section IV presents our approach to computing encounter distributions; V extends the model to consider multiple communities. And finally, we present a concluding discussion in section VI.

II. Related Work

Inter-contact time has been of interest to scientists and researchers working on intermittently connected mobile networks because of its critical importance in developing store-and-forwarding algorithms and their practical validity. Numerous empirical studies have tried to reveal characteristics of inter-contact time in various categories of wireless networks.

Investigations into wireless local area network (WLAN) users encounters based on USC WLAN traces have shown BiPareto distribution, and connectivity richness enables potential in information flooding without infra-structure [1]. Passarella *et al.* have provided a statistical analysis of pairwise inter-contact patterns in 3 different Delay Tolerant Network data set (Dartmouth, iMote and MIT) and have proven the well-fitness of log-normal distribution and exponential curves to the inter-contact time distribution [2] while the researchers have investigated the characteristics of inter-contact time of Mobile Ad-hoc Networks through empirical observation of taxis and buses traces in Shanghai [3]. They have concluded that the inter-contact time has an exponential tail which contrasts with prior results based on other forms of mobility indicating power law distribution [4].

Beyond works focused on statistical analysis of real/realistic traces, there have also been theoretical studies that have tried to mathematically characterize mobility characteristics of such opportunistic networks. Because of its tractable mathematical analysis, random walk and its various types have become widely used for modeling random movement simulation of mobile nodes in Wireless Ad-hoc Sensor Network [5], in Vehicular Ad hoc Networks [6] and in Delay Tolerant Network [7]. Kalay in [8] studied the statistics (including mean and variance) of the first passage time (the time such that a specific node encounters its target) in a finite 1D lattice partitioned into domains and a 2D lattice for a single random walker and an immobile target. Furthermore, first passage time distribution for the prior case was presented. Moreover, Colin Cooper *et al.* gave precise results of cover time (the time to broadcast a piece of information to all of the particles given that they can communicate with each other when meeting at a vertex) by using multiple random walkers on a random regular graphs under different scenarios [9]. Furthermore, James *et al.* investigated the encounter probability for individual pair of random walkers in 1D, 2D and 3D lattices [10] while another analytical model looked into aggregated inter-encounter time given that distribution of individual pair inter-contact time is known in advance for both unified and general heterogeneous network (when not all pairs contact patterns are the same) [11]. Finally, Sanders in [12] provided the exact mean time of a given particle in a system of multiple particles undergoing random walks and indicated that:

> "the full probability distribution of encounter times, and the effect of different network structures on those results are subjects for future studies"

While all of previous works presented statistical results including mean and variance of first hitting time, mean of cover time, individual pair's encounter probability, mean of inter-contact time and distribution of aggregated pair inter-contact time with prior knowledge of individual pair inter-contact time, our work presents for the first-time a method to numerically calculate aggregated pairwise inter-encounter time and approximate the inter-any encounter time distribution for a particular walker with other walkers in a general graph given their movement patterns.

III. Problem Formulation

Assume that we have a road network scenario as described in Figure 1. There are total 4 areas $A1, A2.A3, A4$ identified by the 4 squares in the map. There are also 3 car paths color-coded as red, blue and purple along with timestamp specifying their current corresponding position (current corresponding area A1, A2, A3 or A4) at the time. From the real world road network scenario, we can consider each area as a node, and each car as a random walker (meaning that any two cars at the same node at a given time slot are considered encountering each other). Every car can make decision to move to another area or continue staying at the same area in the next time slot. Transition probability matrix is used to capture the movements of the vehicles in the system. Having all their positions at all the time slot, we could proceed to calculate the corresponding Pairwise Encounter time (PET), and Inter-any encounter time (IAET) in Figure 2 for the given scenarios. We collect all PET for any pairs of cars and all IAET for any car in the network to estimate the corresponding distribution.

We formulate the problem as a random walk model, in which vehicles are random walkers walking on a general connected graph. There are total N walkers walking on the connected graph characterized by V vertexes and S edges. given the connected graph, a walker starting from a vertex can choose any of its neighbour vertexes or choose to stay at that vertex following a transition probability matrix $\mathbb{P}$. The available location set of one single walker is $A = \{1, 2, 3, .., |V|\}$. The available location set of all pairs is $B = \{(1, 1), (1, 2), (1, 3), ..., (|V|, |V|)\}$.

We want to study PET and IAET in this network. Concretely, in the following subsections, we will show how to exactly compute the PET distribution and approximately calculate the IAET distribution.

Fig. 1: Real world scenario

A. Computation of distribution of PET

PET is the inter-encounter time for aggregated pair of walkers [1]. We use DTMC to analyze the PET distribution, in which the states are pairs of [location of walker 1,location

[1]Since all walkers follow the same transition probability matrix $\mathbb{P}$, inter-encounter time for aggregated pairs are the same as for any individual pairs.

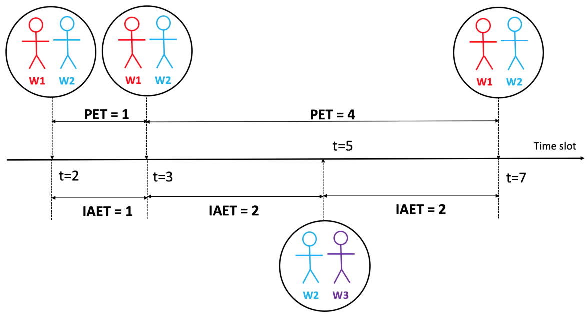

Fig. 2: Corresponding encounters

of walker 2]. Let's illustrate $M_{(x,y)(x',y')}$ as the probability for walker 1 to move from x to x', and walker 2 to move from y to y' in one time step. Therefore, we can always derive the corresponding transition matrix M for the above defined DTMC. Let $P(x,y,t)$ be the probability given that the walker 1 initially stays at vertex x, walker 2 initially stays at vertex y, they can first meet after t time steps. We can generate a recursive set of equations:

$$\begin{aligned}
&P(x,x,0) = 1, \forall x \in A \\
&P(x,y,0) = 0, \forall x,y \in A, x \neq y \\
&P(x,y,1) = \sum_{z \in A} M_{(x,y)(z,z)}, (x,y) \in B \\
&P(x,y,t) = \sum_{\substack{(x,y)\in B \\ (x',y')\in B \\ x' \neq y'}} P(x',y',t-1) \cdot M_{(x,y)(x',y')} \forall t \geq 2
\end{aligned} \tag{1}$$

From the constructed DTMC, π_x is the steady distribution for a walker to be at location x; π_{xy} is the steady distribution such that a pair of walkers stay at vertex $[x,y]$ $(x,y \in A)$: $\pi_{xy} = \pi_x \cdot \pi_y$. Therefore, π_{zz} is the steady distribution of a pair meeting at location $z, z \in A$. The probability that the pair meet at location $z, z \in A$ is denoted as $\widetilde{\pi}_z$:

$$\widetilde{\pi}_z = \frac{\pi_{zz}}{\sum_y \pi_{yy}} \tag{2}$$

Therefore, the PET distribution is calculated in the following equation:

$$P_{PET}(t) = \sum_{z \in A} P(z,z,t) \cdot \widetilde{\pi}_z \tag{3}$$

For the time horizon T, the time complexity of the above procedure is $O(T|V|^2)$. Moreover, there will be V^2 available positions for a pair of walkers. Therefore, it takes $O(V^4)$ to compute transition probability matrix M.

B. Approximation of IAET distribution

Based on the PET modelling, we can proceed on computationally approximating IAET, the inter-encounter time between a particular walker and any other walkers on the graph. To approximate IAET, we assume that in the beginning, the particular walker meets only one of the remaining walkers, and all of the others are distributed following their steady state distribution. We already compute $P(x,y,t)$, the probability any two walkers meet after t time slots given initial location at x, y respectively, following the equation set (1). Based on $P(x,y,t)$, we can compute $\overline{P}(x,y,t)$, the probability that with the same initial location profile, the pair hasn't met for t time slots:

$$\overline{P}(x,y,t) = \overline{P}(x,y,t-1) - P(x,y,t), x \in A, y \in A \tag{4}$$

Initially, N walkers start with an initial location profile $L = (l_1, l_2, ..., l_N)$, $l_i \in A$. Without loss of generality, we denote the particular walker as the walker at location $z, z \in A$. All walkers move independently on the graph. Therefore, meeting any other walkers is independent for the particular walker.The probability, denoted as $\overline{P}(L_z, t)$, that the particular walker hasn't met any other walkers since then up to time slot t starting with such an initial profile $L_z = (z, l_2, ..., l_N)$:

$$\overline{P}(L_z, t) = \prod_{i=2}^{N} \overline{P}(z, l_i, t) \tag{5}$$

The distribution of such an initial profile, $l_i \in A$ for $i = 2, 3.., N$, is in fact very difficult to obtain; $\hat{P}(z,t)$, the probability that the particular walker meet one of remaining walkers at location z and hasn't met any other walkers since then up to time slot t can be approximated as [2]:

$$\hat{P}(z,t) = \frac{\sum_{L_z} \overline{P}(L_z, t)}{|V|^{N-2}} \tag{6}$$

However, we can rewrite $\hat{P}(z,t)$ according to the next equation:

$$\hat{P}(z,t) = \overline{P}(z,z,t) \cdot \frac{(\sum_{i=1}^{|V|} \overline{P}(z, l_i, t))^{|N|-2}}{|V|^{N-2}} \tag{7}$$

Because the initial meeting location can be any of $|V|$ vertexes, and the meeting probability is calculated following equation 2, $\overline{P}_{IAET}(t)$,the probability that the particular walker meet one of remaining walkers and hasn't met any other walkers since then up to time slot t:

$$\overline{P}_{IAET}(t) = \sum_{z \in A} \hat{P}(z,t) \cdot \pi_z \tag{8}$$

Therefore, the IAET distribution is calculated in the following equation:

$$P_{IAET}(t) = \overline{P}_{IAET}(t-1) - \overline{P}_{IAET}(t) \tag{9}$$

For the time horizon T, the total complexity of the above procedure is $O(TN|V|^2 + |V|^4)$

[2]We will show numerically that this approximation still yields accurate results

IV. SIMULATION RESULTS

A. Walkers on a circular strip:

We consider N walkers walking through the circular strip having M cells. In every time step, one walker can move right or move left or stay in the same cell with probability 1/3. Figure 3 demonstrates the distribution of PET and IAET for M=24 and N=7:

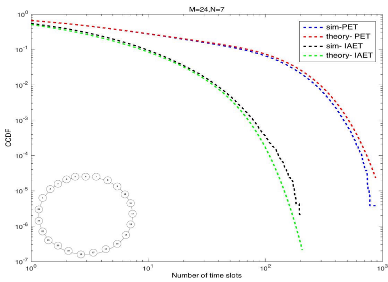

Fig. 3: Circular strip : M=24, N=7

B. Walkers on a general connected graph:

Given number of vertexes V and number of walkers N, we vary number of edges S and generate a random connected graph (characterized by V and S) as well as a random transition probability matrix P on the corresponding graph. We follow the proposed computation for PET and IAET. Figure 4 reports PET and IAET distributions when the walkers follow a random transition probability matrix.

V. PET-IAET COMPUTATION OF MULTIPLE COMMUNITIES ON GENERAL CONNECTED GRAPHS

So far, we only considered one single transition probability matrix P, which means all of the walkers in the system move in the same manner. To extend our results, let's say the system has total C communities indexed by $1, 2, ..., C$, and walkers for each communities follow its own transition probability matrix. Moreover, there are C transition probability matrix $P_1, P_2, ..., P_C$ for C communities correspondingly. Moreover, there area N_i walkers in communities i and total N walkers. We can say that there are total $C \cdot (C-1)/2$ types of encountering pair indexed by $11, 12, ..., CC$. Using the same approach in III-A, we compute $P_{PET}(11,t), P_{PET}(12,t), ..., P_{PET}(CC,t)$ for all types of encountering pair. Moreover, proportion for a pair coming from the same community i denoted as α_{ii} and is calculated following the equation:

$$\alpha_{ii} = \frac{\binom{N_i}{2}}{\binom{N}{2}} \tag{10}$$

Similarly, the proportion for a pair coming from two different communities i and j is:

$$\alpha_{ij} = \frac{N_i \cdot N_j}{\binom{N}{2}} \tag{11}$$

Finally, the PET distribution is calculated:

$$P_{PET}(t) = \sum_{i=1,j=1}^{i=C,j=C} P_{PET}(ij,t) \cdot \alpha_{ij} \tag{12}$$

To analyze IAET for multiple communities, without loss of generality, let's consider the particular walker is the first person coming from community s, and he meets the first walker from community d at location z initially ($s = 1, 2, ..., C$,$d = 1, 2, ..., C$). Additionally, we denote li_j is the location of jth person from community i. All walkers move independently on the graph. Therefore, meeting any other walkers is independent for the particular walker. The probability, denoted as $\overline{P}(L_{s,d,z}, t)$, that the particular walker hasn't met any other walkers from all of the communities since then up to time slot t starting with such an initial profile:

$$L_{s,d,z} = (z, l1_1, ..., l1_{N1}, ls_2, ..., ls_{Ns}, ld_2, ..., ld_{Nd}, ..., lC_{N_C}) \tag{13}$$

$$\begin{aligned} \overline{P_{sd}}(L_z, t) = \prod_{i=1}^{N1} \overline{P}(z, l1_i, t) ... \prod_{i=2}^{Ns} \overline{P}(z, ls_i, t) ... \\ \cdot \prod_{i=2}^{Nd} \overline{P}(z, ld_i, t) ... \prod_{i=1}^{N_C} \overline{P}(z, lC_i, t) \end{aligned} \tag{14}$$

Because we assume again a uniform distribution of such an initial profile, $li_j \in A$ for $i = 1, 2, 3.., C$, $j = 1, 2, ..., N_i$ accordingly, $\hat{P}(s,d,t)$ is the approximated probability that the particular walker from community s meet one of walkers from community d at location z and hasn't met any other walkers since then up to time slot t:

$$\hat{P}(s,d,t) = \frac{\sum_{L_{s,d,z}} \overline{P}(L_{s,d,z}, t)}{|V|^{N-2}} \tag{15}$$

However, we have to consider the proportion of communities from which the initially meeting pair is coming. Furthermore, proportion for the particular walker and another walker coming from the same community i denoted as β_{ii}:

$$\beta_{ss} = \frac{N_s \cdot (N_s - 1)}{N \cdot (N-1)} \tag{16}$$

Proportion for the particular walker and another walker coming from two different communities s and d:

$$\beta_{sd} = \frac{N_s \cdot N_d}{N \cdot (N-1)} \tag{17}$$

Finally, the approximated IAET distribution is calculated as:

$$P_{IAET}(t) = \sum_{s=1,d=1}^{s=C,d=C} P_{IAET}(s,d,t) \cdot \beta_{sd} \tag{18}$$

Figure 5 shows the result for a 3-community case of large input case with $N1 = 12, N2 = 12, N3 = 12, V = 36$.

In general,we will assume a community of N walkers waking on a general connected graph with V vertexes, and each of them follows his own transition pattern, the complexity of the above procedure is $O(TN^2|V|^2)$. The time complexity for C communities is $O(TCN^2|V|^2)$.

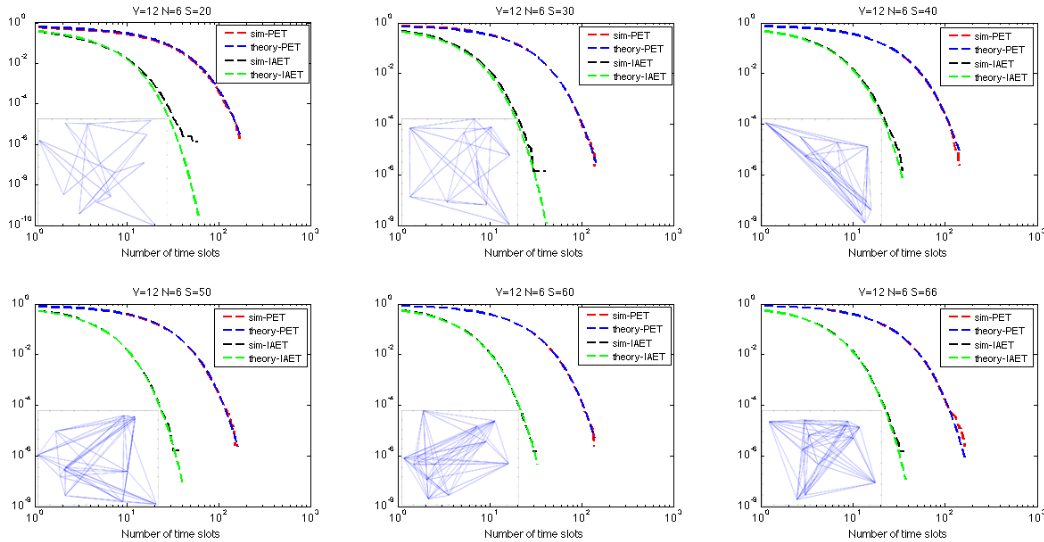

Fig. 4: PET and IAET analysis of a general connected graph, V=12, N=6

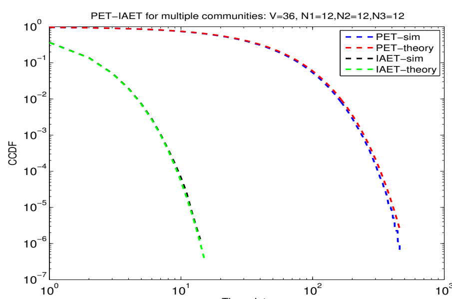

Fig. 5: PET and IAET for a 3-communities network (N1=12,N2=12,N3=12)

VI. Conclusion

Modeling of opportunistic networks in general and ICMN specifically aims at characterizing patterns of movement and predicting future encounters to reduce data dissemination delay as well as improve routing algorithms performance in network of vehicles. To study the crucial inter encounter times in such networks, we considered an encounter mobility model utilizing simple random walks to numerically calculate Pairwise Encounter Time (PET) as well as approximate inter-any encounter time (IAET) distributions for all the nodes in the network in polynomial time. In the future, we would like to validate the model using real world data traces. Modeling of a network protocol or application utilizing the computed encounter distributions is also of interest in future work.

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