Efficient Mechanism Design for Competitive Carrier Selection and Rate Allocation

Yanting Wu, Bhaskar Krishnamachari, George Rabanca, and Amotz Bar-Noy

***Abstract*—This paper investigates a problem where multiple operators (carriers) compete to carry data from a customer (transmitter). To optimally utilize its power and allocate rates, a transmitter needs truthful information about the link performance. However, in a scenario where selection and switching of carriers happens dynamically, it is challenging to regulate carriers' behavior using traditional payment design such as based on byte counting. We propose a payment mechanism based on a convex piecewise linear function, and prove this simple mechanism provides incentives for carriers to provide truthful information about link performance. We also show that as the number of bits per bid is increased, more accurate information about the link performance can be encoded in the bids, consequently, the transmitter's power and rate allocation approaches the optimal with perfect information about channel statistics. To validate the performance of the model, we conduct simulations using real base station locations in London, and show that not only the customers benefit by having higher throughput, this model is also profitable to the operators due to more potential customers and more efficient use of the channels.**

I. INTRODUCTION

Wireless networks are on the verge of a third phase of growth, in which the traffic is dominated by videos. According to industry reports, the average mobile user uses multiple gigabytes of data per month, including significant video and audio traffic. The massive growth of wireless mobile traffic has led to an accelerating pace of research and development in wireless area. With technologies such as OFDM, MIMO, a high rate data stream can be split into multiple lower rate streams and transmitted simultaneously over multiple subcarriers. This greatly boosts the link speed. However, it still barely keeps pace with the fast growing demands from mobile traffic. To solve the bandwidth thirst problem, another trend is to increase the number of base stations (BSs) with smaller cells. It is predicted that in 10 years, there is likely to be more BSs than mobile devices [1], with one mobile device parallel connecting to multiple BSs. Compared to the system capacity which a mobile device must be connected to a single contracted operator's BS, system capacity almost grows quadruply when a mobile device is allowed to connect to any nearby BSs even they are from different operators [2].

Though in current wireless communication networks, each operator uses certain licensed frequency bands exclusively, due to high cost and spectrum scarcity it can be expected that efficient use of spectrum in 5G networks will require innovative ways of sharing rather than exclusive licenses. Ideas about spectrum sharing and data splitting among mobile network operators have attracted researchers' attention and related ideas can be found in [3]–[6]. Such an idea is also supported by current industry trend [7]. It is likely that in the future, the end users will be allowed to move from long-term single operator agreements to more opportunistic service models, and that standards may be enhanced accordingly.

Motivated by these trends, we consider a futuristic scenario in which a mobile device (transmitter) is able to split its data stream across multiple independent channels from different operators (carriers). The transmitter pays the carriers to use the channels. We assume that the carriers are self-interested entities and compete with each other to get payments from the transmitter. We model the problem as an auction: carriers send bids indicating their channels' link performance, and the transmitter selects channels based on the bids and its traffic.

The main focus of this study is to design a payment mechanism to provide incentives for truthful biddings from the carriers, and as a result, the transmitter can allocate power and rate efficiently to meet its traffic requirement. We propose a payment mechanism using a convex piecewise linear function of the link performance, and prove that bidding truthfully is always a preferable action for a carrier. Since carriers are bidding truthfully, unlike existing literature on competitive rate allocation, which typically requires multiple iterations to converge, our proposed mechanism is one-shot and does not require iterative convergence. We also prove that the throughput obtained by the transmitter approaches the optimal as the number of bits per bid increases. Our proposed mechanism is win-win for both customers and operators since the customers have better service (and/or lower payment) and the operators may potentially obtain larger revenues due to more customers and more efficient use of the channels, and the bandwidth burden is amortized among multiple operators.

In this work, we consider the scenario where competition happens between different operators. We use this scenario to show that the tuning of the payment mechanism can help prevent untruthful bidding. This idea can also be extended to a wider application, such as selecting eNBs in LTE dual connectivity assuming that the they are from different providers, selecting services offered by different parties and so on. These scenarios share one thing in common: the buyer relies on the sellers' information to make decisions, the sellers offer similar

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services, and there exists competition among sellers.

Related Work

One well known methodology of doing dynamic rate and power allocation across multiple channels is to use water-filling under a power constraint [8], [9]. In our work, the transmitter also does water-filling across channels. However, many of prior works assume truthful feedback regarding the channel statistics, in our problem, with autonomous selfish carriers, truthfulness is no longer a trivial thing.

Similar to the dual connectivity [10] in LTE network, our idea is also about simultaneously transmitting data over multiple channels to improve the throughput. Unlike dual connectivity, the cellular network is heterogeneous, carriers in our scenario are treated more or less the same. Our proposed pricing mechanism can be easily adapted to regulate competing small cells behavior in LTE dual connectivity.

Our idea is also about spectrum sharing and data splitting among different operators. Different from [5], [6], which focus on the architecture or low level implementations about spectrum sharing, our work builds upon the assumption that such a sharing and data splitting is doable and we focus on providing mechanisms to ensure performance even if competition exists among operators. In [3], the bids dynamically converge to the optimal; in [4], the users evaluations are known. The bids in our model are about the link performance, and it stays the same if the link performance does not change. Also, the traffic requirement of the mobile user is assume to be private information and not revealed publicly.

Pricing and auction mechanisms in dynamic spectrum access are also related. In such systems, primary users are the channel sellers and the secondary users are the buyers. In most of these works, the bids are about the prices and dynamically changes, and the main focus is to prove the existence [11], uniqueness [12] or convergence [13] of equilibrium. Unlike these previous studies, our pricing mechanism is predefined, and the bids are about link performance, rather than price. Since our mechanism ensures the truthfulness from the carriers, it requires no iterative converging process.

Our work provides incentive schemes to make the self interested entities play strategies that are aligned with the mechanism designer's goals. This has something in common with the mechanism design using the intervention framework [14] or Smart Data Pricing (SDP) [15]: the former uses intervention to manipulate the users actions, and the latter uses price to manage the end user's behaviors and control congestion. Our work has a fixed and predefined intervention rule, thus unlike [14], it requires no iterations to converge to the optimal rule, and different from SDP, we use price to adjust operators' behavior rather than the end users' behavior.

This work significantly extends our previous work [16], in which we consider a single transmitter two carriers case: the bids are binary and the transmitter either allocates full power to the higher bidder or splits power equally between two bidders. The transmitter rewards successful transmissions (proportional to the amount of data transmitted) and penalizes failure. The key idea of [16] is to set the "right" penalties to guarantee a throughput bound, while in this correspondence, we extend the binary bid to be a multiple-bit bid, and the key aim is to ensure truthfulness. We also show that the optimal is achieved when the length of quantization interval approaches 0. Moreover, this work is more general: the number of carriers can be arbitrary, and an arbitrary scalar link performance metric can be used.

The correspondence is organized as follows: section II introduces how the system works; section III proposes a payment mechanism design which ensures the truthfulness; section IV conducts simulations and evaluates the performance; and section V concludes the correspondence.

II. SYSTEM MODEL

We consider a practical scenario that there exist multiple users/transmitters and competing carriers. When the transmitter has some data to sent, it initializes an auction by broadcasting a message requesting channel resources. The nearby carriers who have available channels reply to the request with a bid indicating the link performance [1] of their respective channels. Higher bids indicate better links, and consequently, they are more expensive. The transmitter ranks the bids, calculates best power and data rate allocation according to its traffic requirement, selects a set of channels and replies on such channels with data rate information. The selected channels confirm the transmission, and the transmitter starts transmitting on those channels. For those channels which are not selected, they will not receive any reply from the transmitter. After a time out, such channels can assume that they are not selected, and can participate other auctions initialized by other transmitters. The transmitter stays with the same set of channels until it finishes or announces another auction. We call this one request-reply cycle an auction cycle. The usage (such as how long to use which channel) is recorded, and the determined payment is paid over a longer time period, such as monthly billing.

The duration of the auction cycle can be made dynamic, and it could depend on various issues such as overhead, protocol/standard constraints on signaling and control frequency, and also performance. For example, in a stable environment, we can use coherence time as the time length of the auction cycle. In a highly varying environment, the auction cycle can made longer to trade-off overhead reduction for performance reduction, and the bids are about the average link performance.

The transmitter's allocation strategy depends on the bids and the transmitter's traffic requirement. When the traffic is heavy, the transmitter may want to rent multiple high quality channels to maximize the total data rate under a power constraint. However, when the traffic is light, the transmitter may only need a few or even a single channel.

To emulate the real system, we have the following assumptions: **A1:** A transmitter can transmit data simultaneously over K channels depending on its available antennas. In a system with multiple transmitters, K can be different for different transmitters; **A2:** The number of competing carriers N are different with respect to different time and location; **A3:** link

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[1] The bid can be about the SNR, SINR, link throughput, packet success rate, etc. Our framework is general enough to cover any bidding content so long as it offers a scalar indication of link performance which can be measured by the UE.

performance dynamically changes over time due to many factors, however, in one auction period, we assume that the channel statistics stay the same; **A4:** In one auction cycle, the transmitter stay with the same set of carriers. However, in different auction cycle, the selection of carriers can change; **A5:** It is difficult for a carrier to monitor other carriers' channels' link performance, or estimate the transmitter's traffic requirement; **A6:** The carriers are risk averse, which in our context means the carriers tend to choose actions which may give a possibly lower, but a more quantifiable expected payoff rather than choose actions which give unquantifiable payoffs. Although the latter actions may sometimes give high returns, there also exists the risk to get a lower or even negative expected reward; **A7**: In a system with multiple transmitters, a carrier who is not selected by one transmitter can participate the auction initialized by other transmitters.

III. MECHANISM DESIGN

We assume that given the full knowledge about the channel link performance, the transmitter is able to allocate power and rate optimally. We assume that the link performance is quantized into one of the $n = 2^l$ smaller intervals, denoted as $[0, \alpha_1], [\alpha_1, \alpha_2], ..., [\alpha_{n-1}, \alpha_n]$. The transmitter relies on the carriers' bids (which are intended to represent their link qualities) to make decisions on power and rate allocation [2].

In the auction, the players are the carriers. Each player's strategy is an l-bit bid, to represent one of the 2^l link quality intervals. As the true link quality is private information known only to each carrier, the bid need not be *a priori* truthful. For instance, if the transmitter were to naively trust the carriers and offer a payoff that increases monotonically with the claimed link performance, the carriers would have an incentive to lie by always claiming to have the highest quality link. Thus we have to carefully design the utility function to be dependent on both the private value and the bid.

For different bids i, we design the expected payoff to be based on a different linear function (corresponding to line L_i, as shown in Fig. 1). Specifically, assuming the link performance is q, the expected payoff for bidding i is $k_i q + m_i$ where k_i is the slope of the line L_i and m_i in the y-intercept.

By construction the expected payoffs must satisfy two conditions:

- **A:** k_i (the slope of the line L_i) is monotonically increasing in i. In other words, $j < i \implies k_j < k_i$.
- **B:** Lines L_{i-1} and L_i intersect at α_i.

Note that in our design, carriers' payoff is not a simple function of private information (i.e. channel quality), rather, it is a set of linear functions that depend on its bid and channel quality.

Theorem 1. *If the expected payoff of truthful bidding is a convex piecewise linear function with respect to link performance, the carriers will bid truthfully.*

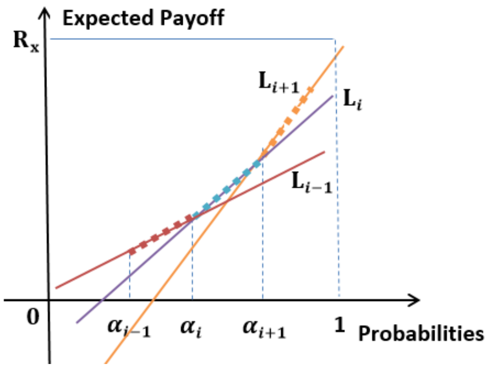

Figure 1: Payoff vs Probabilities

Proof: Assuming that a channel's link performance is in the interval $[\alpha_i, \alpha_{i+1}]$, the expected payoff for truthful bidding is based on line L_i. From conditions A and B, it can be deduced that for all $q < \alpha_{i+1}$, the payoff for line L_i is greater than or equal to the payoff for line L_{i+1}. Further, by straightforward induction on i starting from $i + 1$, it can be shown that for all $q < \alpha_{i+1}$ the payoff for line L_i is greater than or equal to L_j for all $j > i$. Similarly, it can be deduced that for all $q > \alpha_i$, the payoff for line L_i is greater than or equal to the payoff for line L_{i-1}. By reverse induction starting from $i - 1$ down to 0, it can be shown that for all $q > \alpha_i$, the payoff for L_i is greater than or equal to the payoff for line L_j for $j < i$. Thus, we get that for all $q \in [\alpha_i, \alpha_{i+1}]$, the payoff for line L_i is greater than or equal to the payoff for any other line L_j where j not equal to i.

For a channel which is selected, the above analysis shows that the corresponding carrier has no incentive to lie. For a channel which is not selected, under the assumptions A4 $\sim$ A7 introduced in section II, overbidding is still not a good idea for the following reasons: First, transmitter selects channels based on its traffic requirement. In our model, since the payment is convex, to deliver the same amount of traffic, it is more expensive to rent channels which have better link performance, though they provide less delay and higher throughput. If the traffic is light, a transmitter may need only a few channels and they are not necessarily the best channels. Second, without the knowledge of the traffic and other channels' link performance, a channel does not know how much it should overbid; Third, if a channel overbids and is selected, the corresponding carrier does not know whether this is due to overbidding or other competitors' bad link performance; if it is the latter, overbidding yields less expected reward; Fourth, in a real system with multiple transmitters, a channel which is not selected by one transmitter may be selected by another, and a channel which is bad to one transmitter may turn out to be good to another transmitter. Thus, as a risk averse carrier, bidding truthfully is always a more preferable action. ■

Note that no matter how many competitors there are, we have shown that the best action for a carrier is to bid truthfully since it yields the highest expected payoff. Under truthful bidding the carrier's utility will be the piecewise linear function obtained by taking the maximum over all these linear

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[2] These α_is are predefined in the contract depending on system requirements or conditions. How to optimally select α_is is out of the scope of this correspondence.

functions. Thus the set of the expected payoff for truthful bidding composes a convex piecewise linear function, shown as dotted line in Fig. 1.

We define the data rate efficiency, denoted by η, as the ratio of the throughput obtained from the auction to the optimal throughput given perfect information about the channel link performance.

Theorem 2. *When the granularity of the link performance interval approaches 0, η approaches 1.*

Proof: Since the expected received data rate is a continuous function of link performance, there exists a grid length ϵ which makes $V_{opt} - V_{min} < \delta$, where V_{min} is the minimum expected data rate among all possible allocation strategies in that grid. When $\epsilon \to 0, \delta \to 0$, $\eta = \frac{V_{opt}-\delta}{V_{opt}} \to 1$. ■

Remark: *The key part of this incentive mechanism design is the convexity of the function. In general, any convex piecewise linear function can provide incentives for truthful bidding.*

Commonly used payment based on byte counting (payment is a linear function with respect to transmitted data) will not ensure truthfulness. The rational action for a carrier is to always send the highest bid since lying is not penalized, and overbidding may increase the possibility to be selected. For the commonly used Single Operator Contract Model (SOCM), it may be easier to detect the degradation of the performance. However, in our scenario that a transmitter can dynamically select and switch carriers, monitoring the carriers' performance and recording at the customer side is much more challenging. Our proposed mechanism tunes the payment model slightly, and it provides incentives for the carriers to consciously regulate their behavior. Moreover, our mechanism is consistent with the current service charge model. The better the quality of service is, the more expensive it is. To deliver the same amount of traffic, using channels with better link performance costs more, but the service is better, such as shorter delays, less errors, and so on.

IV. SIMULATIONS

In our simulations, we consider the uplink channels. We use the probability that a channel in state good (i.e. the channel gain to interference plus noise ratio $GINR$, which is the SINR normalized by transmit power, is above a predefined threshold $GINR_0$), denoted as $p = Pr(GINR \geq GINR_0)$, as the link performance indicator. We assume that the transmitter can select one of the following two coding and modulation schemes: a conservative one, denoted as f_l and an aggressive one, denoted as f_h. When f_l is used, no matter what the state the channel is in, the transmission is always successful. When f_h is used, the transmission succeeds with probability p (We assume that nothing gets sent if the transmission fails). Note that while the performance metric and communication scheme used here is somewhat simplified for illustrative purposes, it can be extended in real systems to incorporate more complicated link adaptation schemes and performance metrics.

We evaluate the performance of our proposed Auction Model (AM) by comparing it with the commonly used Single Operator Contract Model (SOCM). We use the data set from Ofcom's Sitefinder [17], and obtain precise coordinates of BSs from two major operators over an 2km × 2 km area in London as shown in Fig. 2. There are 158 BSs (marked as blue) from Operator 1 and 128 BSs from Operator 2 (marked as red). We sample 5000 customers from Operator 1 and 5000 customers from Operator 2 located uniformly at random in this area. We evaluate the mobile devices' throughput as well as the carriers' net profit.

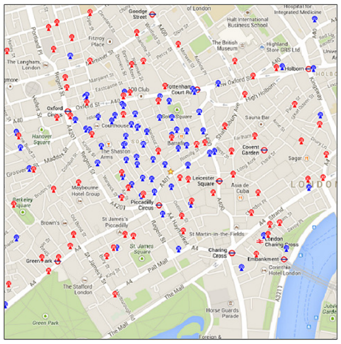

Figure 2: 2km × 2km view of the BS deployment by two major cellular operators over an area in London

We use the simple path loss model with log-normal shadowing and interference, where the gain to interference plus noise ratio in dB is given as $GINR = L - 10\gamma \log_{10}(\frac{d}{d_0}) + \psi - 10\log_{10}(U_i \cdot C) - N$, where $\psi \sim N(0, \sigma^2)$ is the log-normal shadowing term with mean 0 and variance σ^2. We set the loss $L = -34$dB at reference distance $d_0 = 1$m, path loss exponent $\gamma = 3.5$, log-normal shadowing standard deviation $\sigma = 10$, noise power $N = -120$dB. And we choose the threshold of $GINR$ to be $GINR_0 = 6$dB in our simulation. U_i is the number of users within a range such that $p(GINR > GINR_0) > 0.3$; we use U_i to model the interference from other users, C is a scaling term that captures both a scaling up of the number of users (the 5000 users in simulation are only a sample) and a scaling down of the amount of interference they may be causing (since many of them are likely to be in other channels). We normalize the total power to be 1, and use $f_l = 10\log(1+2P)$ and $f_h = 10\log(1+100P)$ to represent low data rate function and high data rate function, where P is the amount of the power allocated.

In SOCM model, each transmitter contracts with a single carrier and is only allowed to connect to a single BS from the carrier it is bound to. Since the transmitter allocates all power to a single channel, substituting $P = 1$ in f_l and f_h, we get rates $R_l = f_l(1) = 11$, and $R_h = f_h(1) = 46$ in the unit of kb/slot [3]. When $p < \frac{11}{46}$, the transmitter transmits with rate R_l, otherwise, with rate R_h. The payment to the carriers is proportional to the amount of the transmitted data. We assume that it is 10^{-4} in the unit of \$/kb (\$10 for 1 Gb data). Consequently, the payment is $u = 10^{-4}R_x$ in the unit of \$/slot. AM is our model. In this evaluation, we consider $K = 2$ case. We sample 10^4 customers placed at the same random

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[3] The transmissions are slot-based in our simulation. We use it as an example to show the effectiveness of our proposed mechanism.

locations as in the single carrier contract model and use 8-bit bids: the whole probability range are evenly divided into 2^8 smaller intervals. We use convex function $g(p) = 1.1^{p-1} \times 10^{-4} R_x\,(p \in [0,1])$ to design the payment: $(\alpha_i, g(\alpha_i))$ and $(\alpha_{i+1}, g(\alpha_{i+1}))$ determines line L_i, and payment is based on line L_i for bidding i in the unit of \$/slot.

Fig. 3 compares the GINR of SOCM model and AM model. The dark blue and light blue bars represent the distribution of GINR using SOCM model, and the yellow bar and brown bar represent the distribution of the selected better channels' GINR using AM model. We can see that GINR are improved significantly using our proposed model as it allows the customers' mobile devices connecting with base stations that are nearby, offering a better communication quality to the user.

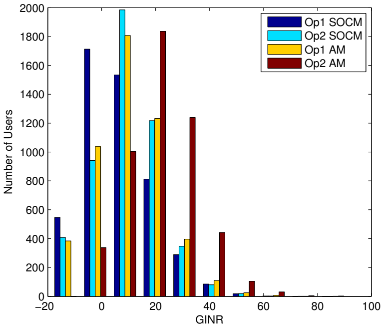

Figure 3: Distribution of GINR: SOCM versus AM

The simulation results are shown in Table I. We can see that our proposed model provides a win-win strategy to solve the wireless bandwidth thirst problem. The average throughput of a transmitter almost doubles while paying less per byte, and the operators make more revenue due to more potential customers and more efficient use of the spectrum by mainly serving the nearby mobile devices.

Table I: SOCM vs AM

	SOCM		AM	
	Op1	Op2	Op1	Op2
Transmitter's average throughput(kb/slot)	23.96	28.16	51.45	
Transmitter's average payment per kb (\$)	10^{-4}		7.96×10^{-5}($<10^{-4}$)	
Carrier's average profit per slot (\$/slot)	11.98	14.08	17.08	23.86

Note that the actual contract adopted in practice may depend on other market factors, but these examples show the overall benefit of carrier flexibility to both users (in terms of increased throughput and possibly reduced marginal cost) and operators (in terms of increased profit). We believe that the gain can be even more significant in future wireless system with greater carrier diversity and higher traffic.

V. CONCLUSION

We have investigated a competitive rate allocation in which multiple selfish carriers compete to carry data from a transmitter in exchange for a payment. We have shown that even if the transmitter is unaware of the stochastic parameters of the channels, it can set the payment in such a way that the carriers' strategic bids yield an expected throughput that is close to the optimal. The payment is designed according to a convex piecewise linear function. With this design, a carrier will get lower expected payment if their claimed performance is different their actual performance, thus, it gives the incentive for the carriers to bid truthfully. With the number of bits per bid increases, the throughput obtained by the transmitter approaches the optimal. Through simulations, we have shown that our proposed model could be beneficial to both the mobile users as well as the operators. Further work is needed to bring our proposal to practice. In particular, implementation of our auction requires different operators to agree to coordinate on a common channel, requiring new standardization efforts; however, our results do indicate such cooperation would be in their self-interest.

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