Joint Rate-Routing Control for Fair and Efficient Data Gathering in Wireless sensor Networks

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Abstract

In wireless sensor networks, fair and efficient rate allocation is an essential mechanism to avoid congestion collapse and system degradation. While most prior work in this context has focused on a static tree, we consider the joint optimization of routing and rate allocation in this work. We formulate LP problems to obtain max-min fairness and sum-rate efficiency. We show the tradeoff between fairness and efficiency in this setting, and develop distributed algorithms based on Lagrange duality to achieve these objectives.

1 Introduction

Due to the unpredictable nature of wireless communication, such as varying link quality and bandwidth, congestion is a common problem in wireless sensor networks. This is even true for small wireless sensor networks with known periodic traffic. Moreover, for contention-based MAC protocols, like CSMA, concurrent data transmissions over different links may interfere with each other and aggravate congestion, finally causing congestion collapse [9]. In order to avoid this, each sensor node must appropriately control its own rate to improve channel utilization and per-node end-to-end throughput, especially in a multi-hop network. There have been many papers on rate allocation in wireless sensor networks to alleviate these problems [5, 6, 7, 8, 9, 10].

Intuitively, it is important to improve the efficiency of data collection in terms of bandwidth and also energy and lifetime [11]. However, for many applications, such as earthquake monitoring, it is more important to collect data from all nodes in various geographical locations in a balanced manner rather than just gathering a large amount of data from one location. Therefore, fairness is key ([9], [7], [8]). Not only it might increase per-node based throughput, but also imply a longer lifetime of a wireless sensor network.

In this paper, we consider a general network with a single sink and multiple randomly distributed wireless sensors. A carrier sense multiple access (CSMA) MAC is used. Assuming no data compression, each sensor generates information data which should finally reach the sink, possibly in multi-hop. In order to study the tension between fairness and efficiency, we formulate two separate optimization problems. One of our objective is to achieve a network-wide optimal rate allocation in terms of fairness, while the other is for efficiency, measured by the amount of data extracted from the network with some minimal rate requirement.

There are different definitions of fairness [1, 12, 13] in the context of resource allocation. We identify max-min fairness as the most appropriate one for our setting. The efficiency is defined as the network throughput in terms of the sum rates of all nodes. In wired networks, additive increase and multiplicative decrease (AIMD) [17] rate adjustment strategy is widely used for fair and efficient data transmission. However, due to the broadcast nature of wireless medium, the bandwidth consumption consists both useful data and interference. Furthermore, the interference at each node highly depends on the topology. Due to these enormous differences between wired and wireless communication, the design of rate control mechanisms for wireless networks is not trivial. We need to solve the following problems.

First of all, we need to appropriately capture the interference model. In [1], Sridharan *et al.* propose a receiver capacity model, which has been experimentally proved accurate for a CSMA-based link layer. Therefore, we also apply this model in our optimization problem formulation. Also, in this problem domain (using IEEE 802.15.4), data rate is very low (less than 250 kbps). So, the radio communication bandwidth is the key constraint.

The majority of previous work on fair rate allocation in wireless sensor network is based on a predefined routing tree [5, 6, 9, 10]. By this means, rate allocation and routing are separated. However, we believe that the selection of paths for routing can impact the fairness or efficiency of rate allocation. Different from previous work, we model networks as directed graphs.

We assume that a sensor can dynamically adjust its data generation rate and data transmission rates on all its outgoing links. We follow a cross-layer design to explore how to determine routes (at the network layer) and set rates for sources (at the transport layer) to maximize network utilization in a fair manner.

The rest of the paper is organized as follows. In Section 2, we present the network model and formulate the optimization problems into linear programming (LP) problems. Then, in Section 3, the tradeoff between fairness and efficiency has been studied. In Section 4, we analyze the LP problems and propose distributed algorithms for them. Further, we report the simulation results of these algorithms. Finally in Section 5, we give our conclusion and future work.

2 Network Model and Problem Formulation

We model the topology of a wireless sensor network as a directed graph $G(V, E)$, where V is the set of all nodes (including the sink), and E is the set of links. An edge $(i, j) \in E$ represents a communication link from sensor i to j. Let r_{ij} represent the data transmission rate on link (i, j) . We assume G is a connected graph, *i.e.*, every sensor has at least one path to the sink. r^i_{src} is the data generation rate on sensor i. We further define O^i as the set of sensors having links outgoing from sensor i and I^i as the set of sensors having links incoming to i. N^i is the set of sensor i's neighbors. For example, in Figure 1 (a), $N^3 = \{0, 4, 5\}$, $I^3 = \{4, 5\}$ and $O^3 = \{0\}$. We consider a wireless sensor network used for environment monitoring. Each sensor node generates data and is able to relay data for other nodes. All sensed data by nodes are finally transmitted to the sink.

In many existing work, rate allocation and optimization are based on fixed routing trees, usually generated by the SPT (Shortest Path Tree) method. Here, by modeling the network as a graph, we can select paths for routing and explore fairness and efficiency in wireless networks. In Figure 1, we compare the tree-based and non-tree-based routing. As we can see, without the routing tree, a sensor may take different paths to transmit data to the sink. For a given network, different routing paths may lead different network utility and max-min fair rate. If we consider a network as a graph instead of a tree, it can reduce the impact of bottleneck and increase the fairness. However, the network utility may decrease. Thus, we believe that joint routing and rate allocation can improve the fairness of networks.

As we earlier mentioned, to model the interference we use the receiver capacity model [1]. In this

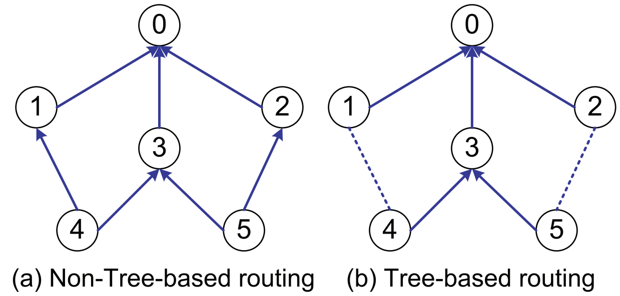

Figure 1: An example to compare the fairness and efficiency of networks with and without predefined routing tree. Sensor 4 and 5 can only send data to 3 in (b), while in (a), they have multiple paths.

model, every node is considered as a receiver. Due to the broadcast nature of the wireless medium, the bandwidth of a receiver is consumed by both useful data and "noise" (interference). Here, we use noise to refer to the data received but not targeted to the receiver. Let's use Figure 1 (a) to explain the model. This figure shows a 6-node sensor network. A solid line represents a communication channel, which is half-duplex and symmetric. For node 4, it has two communication links. r^4_{src} is the data generation rate at node 4. When node 4 sends its data, it takes either link 4 $\rightarrow$ 1 or 4 $\rightarrow$ 3 or both. r_{41} and r_{43} are respectively defined as the allocated rates on these two outgoing links of node 4. And there is $r_{41} + r_{43} = r^4_{src}$. Since node 4 does not relay data for other node, only r^4_{src} (or r_{41} and r_{43}) is its useful data. However, due to the symmetry of the link, the bandwidth of node 4 is also consumed by the "noise" from node 1 and 3. Therefore, we have the following inequality, where B^4 is the bandwidth of node 4.

$$r^4_{src} + r^1_{noise} + r^3_{noise} \leq B^4 \qquad (1)$$

In the above inequality (1), r^1_{noise} is the data sent by node 1, but not targeting to node 4. As we can see, the useful data of node 1, including its own sensed data r^1_{src} and the relayed data r_{41}, is sent out to node 0 (the sink). Therefore, $r^1_{noise} = r^1_{src} + r_{41}$. Similarly, we can obtain $r^3_{noise} = r^3_{src} + r_{43} + r_{53}$, where the data relayed by node 3 is $r_{43} + r_{53}$.

As we can see, the receiver capacity model successfully capture the feature of wireless communication. Also, it has been found empirically good in modeling the interference for a CSMA MAC. Receivers in the network are bandwidth constrained and have a finite receiver bandwidth capacity given by $\vec{B}$, which can be set as the saturation throughput of the CSMA MAC [21]. We can obtain a general form of receiver capacity constraint at node i as follows:

$$\sum_{j\in O^i} r_{ij} + \sum_{j\in N^i}\sum_{l\in O^j} r_{jl} \le B^i \qquad (2)$$

where $\sum_{j\in O^i} r_{ij}$ is all the data sent out by node i and considered as useful data. While $\sum_{j\in N^i}\sum_{l\in O^j} r_{jl}$ is all the data listened by node i, including both useful data and "noise".

In order to prevent the loss of data during transmission, flow conservation is needed. That is, the amount of data transmitted by a sensor is equal to the sum of all received data and new data generated by the sensor. Flow conservation is modeled as follows:

$$\sum_{j\in O^i} r_{ij} - \sum_{l\in I^i} r_{li} = r^i_{src} \qquad (3)$$

To study the fairness and efficiency, we formulate two optimization problems. Max-Min fairness has widely been accepted as a formulation of fairness in many settings. We also identify it as the most appropriate one in our problem domain. Thus, our first optimization problem is to obtain the max-min fairness in a wireless sensor network. Let 0 represent the sink and $V' = V - \{0\}$. The problem is formulated as follows:

$$\begin{aligned}
\textbf{(P1)} \quad \textbf{max:} \quad & r_{min} \\
s.t. \quad & \sum_{j\in O^i} r_{ij} + \sum_{j\in N^i}\sum_{l\in O^j} r_{jl} \le B^i && \forall i \in V \\
& \sum_{j\in O^i} r_{ij} - \sum_{l\in I^i} r_{li} = r^i_{src} && \forall i \in V' \\
& \sum_{i\in I^0} r_{i0} = \sum_{i\in V'} r^i_{src} \\
& r_{min} \le r^i_{src} \le B^i && \forall i \in V'
\end{aligned}$$

For the second optimization problem, our goal is to maximize the sum of rates generated by all nodes ($\sum_{i\in V'} r^i_{src}$), while maintaining a minimal required rate (r_{req}). The max-min rate can be used as the minimal required rate, but not necessarily. This problem is formulated as follows:

$$\begin{aligned}
\textbf{(P2)} \quad \textbf{max:} \quad & \sum_{i\in V'} r^i_{src} \\
s.t. \quad & \sum_{j\in O^i} r_{ij} + \sum_{j\in N^i}\sum_{l\in O^j} r_{jl} \le B^i && \forall i \in V \\
& \sum_{j\in O^i} r_{ij} - \sum_{l\in I^i} r_{li} = r^i_{src} && \forall i \in V' \\
& \sum_{i\in I^0} r_{i0} = \sum_{i\in V'} r^i_{src} \\
& r_{req} \le r^i_{src} \le B^i && \forall i \in V'
\end{aligned}$$

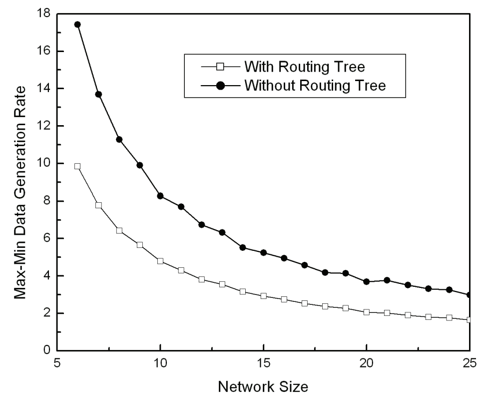

Figure 2: Average Max-Min rate of Tree-based routing and Graph-based routing with different network sizes.

For the above formulation, we should note that we ignore the overhead and quantization effects associated with packetization of data. We compare the fairness obtained by our proposed approach (as **P1**) and the traditional tree-based routing (with SPT). Networks with different sizes (from 6 nodes to 25 nodes) have been evaluated. Figure 2 shows the result. In order to avoid randomness, every point in the figure is the average of max-min rates of 100 randomly generated networks with same size. This figure proves that joint routing and rate allocation can significantly improve the fairness. In the following section, we discuss more on the relationship between fairness and efficiency in wireless sensor networks.

3 Tradeoff between Fairness and Efficiency

There are many different definitions of fairness. In the existing work, researchers use different flavors of fairness for rate allocation. Kun *et al.*[18] propose a congestion control algorithm for wireless sensor networks designed to obtain *proportional fairness* of flows in the network. In [16], Tassuilas *et al.* use a centralized algorithm to obtain a stronger sense of fairness, the *lexicographic max-min fairness*, in wireless ad hoc networks. Other forms of fairness are also used. In our paper, we aim to obtain the max-min fairness of rate allocation. That it, the minimum data generation rate allocated to any node is the maximum over all possible allocations.

Efficiency also has many different definitions. A widely used one is the network throughput measured by the sum rate ($\sum_{i\in V'} r^i_{src}$). Obviously, this kind of efficiency severely biases the rate allocation in large-scale, multi-hop sensor networks. It favors the nodes that can directly communicate with the sink and the nodes with less interference. Especially, by only considering network throughput, under heavy traffic load, it's impossible to successfully deliver packets that traverse many hops.

Fairness-efficiency tradeoffs are relatively well-understood in wired networks [2]. Due to the interference inherent in wireless networks, this tension between efficiency and fairness is even stronger. However, this problem is not well studied in the area. Let the Max-Min rate be an indicator of fairness. We define $\frac{\sum_{i \in V'} r^i_{src}}{N-1}$ as the *average data generation rate* in the network, which is used as the indicator of efficiency. We combine the fairness and efficiency into one objective as follows:

$$
\begin{aligned}
max: \quad & \alpha * r_{min} + (1-\alpha) * \frac{\sum_{i \in V'} r^i_{src}}{N-1} \\
s.t. \quad & \sum_{j \in O^i} r_{ij} + \sum_{j \in N^i} \sum_{l \in O^j} r_{jl} \le B^i \quad \forall i \in V \\
& \sum_{j \in O^i} r_{ij} - \sum_{l \in I^i} r_{li} = r^i_{src} \quad \forall i \in V' \\
& \sum_{i \in I^0} r_{i0} = \sum_{i \in V'} r^i_{src} \\
& r_{min} \le r^i_{src} \le B^i \quad \forall i \in V'
\end{aligned}
$$

By changing the value of α ($\alpha \in [0,1]$), we assign different weights to fairness and efficiency. We choose network sizes ranging from 6 to 45. For each network size, 100 instances of network deployment are randomly generated. Then each node is randomly set a bandwidth of either 100 or 200. Ten bandwidth distributions are generated for each network instance (for networks with size less than 10, it's possible to have some duplicated instances.). We solve the above LP problems and obtain the efficiency-fairness curve. Figure 3 and Figure 4, respectively, show the fairness and efficiency with different α for different networks. It is clear if we increase the weight to efficiency, fairness will decrease. When $\alpha = 0$, maximal efficiency is obtained at the cost Max-Min rate $r_{maxmin} = 0$. In Figure 3, the Max-Min rates are very close when $\alpha = 0.9$ and $\alpha = 1$. Figure 5 is the efficiency-fairness curve (for networks with 45 nodes). It shows the possible region of efficiency and fairness. The bold dot on the curve is obtained by maximizing the network utilization after obtaining the max-min rate. By this means, we can obtain the maximal fairness and efficient network utilization (over 83% of the maximal possible network throughput when $\alpha = 0$) at the same time.

Compared to lexicographic max-min, max-min is a weaker kind of fairness. However, due to the trade-off between fairness and efficiency, max-min fairness is more suitable to obtain fair and efficient rate allocation. In the next section, we analyze and propose some distributed algorithms to achieve our goals.

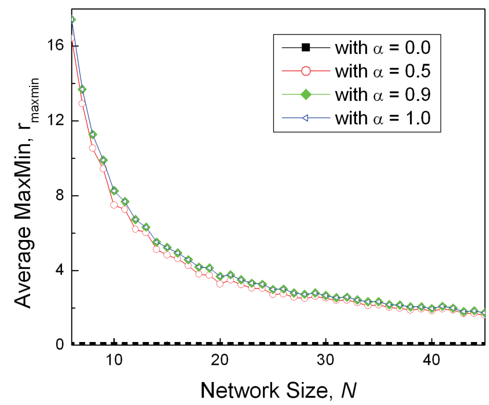

Figure 3: Average Max-Min rate (the fairness) vs. the network size.

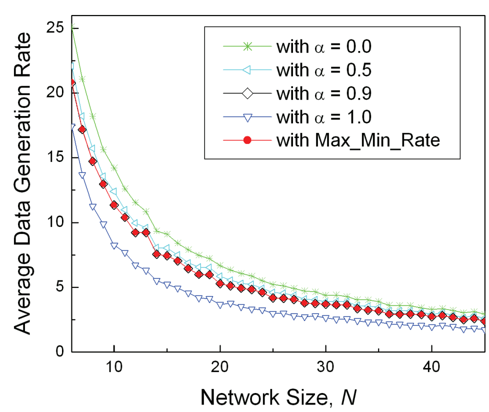

Figure 4: Average Data generation rate (the efficiency) vs. the network size.

4 Distributed Algorithms

In wireless sensor networks, after the deployment of sensors, usually it is not easy to access sensors again. Also, due to the autonomous property and unpredictable channel, distributed algorithms are much desirable in wireless environment. In this section, we propose several distributed algorithms using the shadow price interpretation to solve the optimization problems we described in section 2. Especially, we focus on solving the optimization problem of fairness in

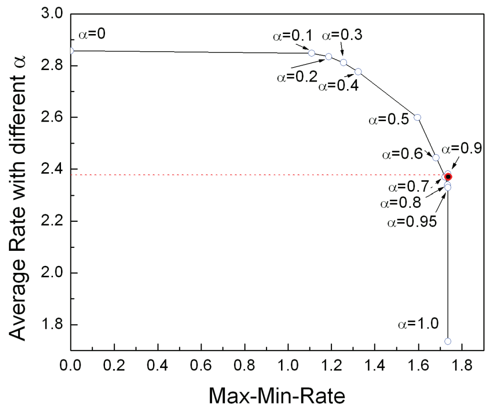

Figure 5: Efficiency vs. Fairness. This plot is for a network with 45 nodes.

a distributed manner. After obtaining the Max-Min rate, the sum-rate maximization problem will become easier. Since for the sum-rate optimization, it always favors the nodes close to the sink.

In [12], Kelly *et al.* first applied optimization theory to rate control algorithms. This method was quickly accepted by researchers. Later, more approaches of optimization have been introduced into this area, such as duality and sub-gradient methods. The dual-based method and sub-gradient methods are rapidly used to analyze and design distributed algorithms, especially for the emerging world of wireless sensor networks. Particularly, due to the dynamical and unpredictable feature of wireless networks, an optimization problem usually involves different elements and different stack layers. Therefore, Chiang *et al.* [19] and Johansson *et al.* [20] proposed cross-layer optimization and introduced the dual decomposition techniques. These works establish the basis of our following algorithms. In section 4.1 and 4.2, we elucidate these algorithms in detail.

4.1 Partially Distributed Algorithms for Fairness

Define $X = \{0 \le r^i_{src} \le B^i \quad \forall i \in V', 0 \le r_{ij} \le B^i \quad \forall (i,j) \in E\}$ as the domain and relax the flow conservation and bandwidth constraints. We applied the sub-gradient method to solve the dual problem and obtained the optimal dual variables. However, this solution may not be primal feasible. For example, flow conservation and bandwidth constraint could not be satisfied. This is a typical phenomenon for problems with non-strictly convex primal objective functions. By adding a small strictly convex regularization term to the primal objective function, the problem can be solved [4]. Thus, we add a quadratic regularization term $\epsilon * (r^2_{maxmin} + \sum_{(i,j)\in L} r^2_{ij})$ to the objective with $\epsilon > 0$.

First, we rewrite the optimization problem for fairness (P1) as follows:

$$\begin{aligned} min: \quad & -r_{min} \\ s.t. \quad & \sum_{j\in O^i} r_{ij} + \sum_{j\in N^i}\sum_{l\in O^j} r_{jl} \le B^i \quad \forall i \in V \\ & \sum_{j\in O^i} r_{ij} = \sum_{l\in I^i} r_{li} + r_{min} \quad \forall i \in V' \\ & (N-1) * r_{min} = \sum_{i\in I^0} r_{i0} \quad Sink \\ & r_{min} \ge 0, \qquad \vec{r} \ge 0 \end{aligned}$$

Introduce dual variables λ_i and μ_i for the constraints, the Lagrange function of the primal P1 is:

$$\begin{aligned} & L(r_{min}, \vec{r}, \vec{\lambda}, \vec{\mu}) \\ = \quad & -r_{min} + \epsilon * r^2_{min} + \epsilon * \sum_{(i,j)\in L} r^2_{ij} \\ + \quad & \sum_{i\in V} \lambda_i (\sum_{j\in O^i} r_{ij} + \sum_{j\in N^i}\sum_{l\in O^j} r_{jl} - B^i) \\ + \quad & \sum_{i\in V'} \mu_i (\sum_{j\in O^i} r_{ij} - \sum_{l\in I^i} r_{li} - r_{min}) \\ + \quad & \mu_0((N-1) * r_{min} - \sum_{i\in I^0} r_{i0}) \end{aligned}$$

with $r_{min} \ge 0, \vec{\lambda} \ge 0$, and $\vec{r} \in X$. Primal variables r_{min}, and $\vec{r}$ can be separated in above Lagrange function. To obtain the optimal value at iteration k, for each primal variable, we solve the following problem:

$$\begin{aligned} r^{(k)}_{min} \quad = \quad & \arg\min_{r_{min}\ge 0} \{\epsilon * r^2_{min} + (-1 - \sum_{i\in V'} \mu^{(k)}_i \\ + \quad & (N-1) * \mu^{(k)}_0) * r_{min}\} \\ r^{(k)}_{ij} \quad = \quad & \arg\min_{r_{ij}\in X} \{\epsilon * r^2_{ij} + (\lambda^{(k)}_i + \sum_{l\in N^i} \lambda^{(k)}_l \\ + \quad & \mu^{(k)}_i - \mu^{(k)}_j) * r_{ij}\} \end{aligned}$$

ϵ can impact the final result of Max-Min rate and the speed of convergence. If ϵ is too large, it may change the optimization problem. If ϵ is too small, the convergence speed will be slow. In order to calculate r_{min}, the value of $\vec{\mu}$ is needed. Thus, this method cannot provide a fully distributed way to calculate Max-Min value for the network. There should be a centralized server to collect the value of μ on every sensor at each iteration. In [3], Madan *et al.* propose a way to design fully distributed algorithms in the context of optimizing network lifetime. By learning from their method, we reformulate our problem as shown in the following part.

4.2 Fully Distributed Algorithm for Fairness

In this new formulation, each sensor i solves the following optimization problem:

$$
\textbf{(P1')} \quad \begin{aligned}
min: &\quad -r^i_{src} \\
s.t. &\quad \sum_{j\in O^i} r_{ij} + \sum_{l\in N^i}\sum_{j\in O^l} r_{lj} \le B^i \quad \forall i \in V' \\
&\quad \sum_{j\in O^i} r_{ij} = \sum_{l\in I^i} r_{li} + r^i_{src} \quad \forall i \in V' \\
&\quad r^i_{src} = r^j_{src} \quad \forall j \in N^i \\
&\quad \vec{r_{ij}} \ge 0, \qquad r^i_{src} \ge 0
\end{aligned}
$$

We also need to consider the bandwidth constraint on the sink $\sum_{i\in I^0} r_{i0} \le B^0$. By adding the quadratic regularization term $\epsilon * (\sum_{i\in V'} (r^i_{src})^2 + \sum_{(i,j)\in L} r^2_{ij})$ to make the objective function a strictly convex function. Sum up all the optimization problems on all nodes in the network, the Lagrange of the global optimization problem is given by:

$$
\begin{aligned}
& L(\vec{r_{src}}, \vec{r}, \vec{\lambda}, \vec{\mu}, \vec{\nu}) \\
= & -\sum_{i\in V'} r^i_{src} + \epsilon * (\sum_{i\in V'} (r^i_{src})^2 + \sum_{(i,j)\in L} r^2_{ij}) \\
+ & \sum_{i\in V} \lambda_i (\sum_{j\in O^i} r_{ij} + \sum_{l\in N^i}\sum_{j\in O^l} r_{lj} - B^i) \\
+ & \sum_{i\in V'} \mu_i (\sum_{j\in O^i} r_{ij} - \sum_{l\in I^i} r_{li} - r^i_{src}) \\
+ & \sum_{i\in V'}\sum_{j\in N^i} \nu_{ij} (r^i_{src} - r^j_{src})
\end{aligned}
$$

with $\vec{\lambda} \ge 0$ and $\vec{r} \in X$. We apply sub-gradient methods to solve the dual problem. For dual variables, there are

$$
\begin{aligned}
\frac{\partial D}{\partial \lambda_i} &= \sum_{j\in O^i} r_{ij} + \sum_{l\in N^i}\sum_{j\in O^l} r_{lj} - B^i \\
\frac{\partial D}{\partial \mu_i} &= \sum_{j\in O^i} r_{ij} - \sum_{l\in I^i} r_{li} - r^i_{src} \qquad (4) \\
\frac{\partial D}{\partial \nu_{ij}} &= r^i_{src} - r^j_{src}
\end{aligned}
$$

At each iteration k, dual variables are updated by

$$
\begin{aligned}
\lambda^{(k)}_i &= [(\lambda^{(k-1)}_i - \alpha^{(k)} \frac{\partial D}{\partial \lambda_i})]^+ \\
\mu^{(k)}_i &= \mu^{(k-1)}_i - \alpha^{(k)} \frac{\partial D}{\partial \mu_i} \qquad (5) \\
\nu^{(k)}_{ij} &= \nu^{(k-1)}_{ij} - \alpha^{(k)} \frac{\partial D}{\partial \nu_{ij}}
\end{aligned}
$$

Step size $\alpha^{(k)} \to 0$ with $k \to \infty$. The Lagrange dual function is separable in $\vec{r}$. We have the sequence of the primal iterations as follows:

$$
\begin{aligned}
r^{i(k)}_{src} &= \arg\min_{r^i_{src}\ge 0} \{\epsilon * (r^i_{src})^2 \\
&+ (-1 - \mu^{(k)}_i + \sum_{j\in N^i}^{j\ne 0} (\nu^{(k)}_{ij} - \nu^{(k)}_{ji})) * r_{src}\} \qquad (6) \\
r^{(k)}_{ij} &= \arg\min_{r_{ij}\in X} \{\epsilon * r^2_{ij} + (\lambda^{(k)}_i \\
&+ \sum_{l\in N^i} \lambda^{(k)}_l + \mu^{(k)}_i - \mu^{(k)}_j) * r_{ij}\} \qquad (7)
\end{aligned}
$$

The fully distributed Algorithm 1 is described by

Algorithm 1 Fully Distributed Max-Min Algorithm

1: **Initialization**
2: set the value of δ and ϵ, set the domain X
3: initialize primal and dual variables: $\vec{r_{src}}, \vec{r_{ij}}, \vec{\lambda}, \vec{\mu}, \vec{\nu}$
4: $k \leftarrow 1$ and step size $\alpha \leftarrow C_1$, initialize $D(0), D(1)$
5: **while** $|D(k) - D(k-1)| \ge \delta$ and $k \le T$ **do**
6: solve $r^{i(k)}_{src} = \arg\min_{r^i_{src}\ge 0}\{\epsilon * (r^i_{src})^2 + (-1 - \mu^{(k)}_i + \sum^{j\ne 0}_{j\in N^i} (\nu^{(k)}_{ij} - \nu^{(k)}_{ji})) * r^i_{src}\}$
7: solve $r^{(k)}_{ij} = \arg\min_{r_{ij}\in X}\{\epsilon * r^2_{ij} + (\lambda_i + \sum_{k\in N^i} \lambda_k + \mu_i - \mu_j) * r_{ij}\}$
8: compute Lagrange Dual $D(k)$
9: compute sub-gradients of dual variables
10: $\frac{\partial D}{\partial \lambda_i} = \sum_{j\in O^i} r_{ij} + \sum_{l\in N^i}\sum_{j\in O^l} r_{lj} - B^i$
11: $\frac{\partial D}{\partial \mu_i} = \sum_{j\in O^i} r_{ij} - \sum_{l\in I^i} r_{li} - r^i_{src}$
12: $\frac{\partial D}{\partial \nu_{ij}} = r^i_{src} - r^j_{src}$
13: update $\alpha^{(k)} = \frac{C_1 * C_2}{k + C_2}$
14: compute new prices according to (5)
15: k++
16: **end while**
17: **if** Flow conservation and bandwidth constraints are satisfied **then**
18: **return** $\vec{r_{src}}, \vec{r_{ij}}$
19: **else**
20: modify ϵ, **goto** line 3
21: **end if**

Step size is updated as $\alpha^{(k)} = \frac{C_1 * C_2}{k + C_2}$, where C_1, C_2 are constants and satisfying $\lim_{k\to\infty} \alpha^k = 0$, $\sum_{k=1}^{\infty} \alpha^k = \infty$. When we generalize this algorithm to different network instances, it is hard to find a common ϵ. Thus, we set an initial value to ϵ and check flow conservation and bandwidth constraints at the end of the **while-loop**. If any of the constraints is violated, ϵ will be changed to scale down the flows.

By using the similar method, we also designed a fully distributed algorithm for the sum-rate problem.

But as we mentioned before, the sum-rate problem becomes easier after the max-min rate problem is solved. Also due to the space of the paper, we won't repeat the deduction of the algorithm for sum-rate here.

4.3 Performance Evaluation

We evaluate our algorithms for networks with 20 nodes. Figure 6 and Figure 7 shows the convergence speed of partially distributed algorithm and fully distributed algorithm (Alg. 1). We find that the fully distributed algorithm can converge faster than the partially distributed algorithm. Figure 8 compares the max-min rate allocated for each sensor by different algorithms. In the fully distributed algorithm, each sensor computes a max-min rate by its local information. These max-min values generated by Alg. 1 are very close to the optimal Max-Min. Minimal additional refinements are required to scale the final solutions in order to ensure that all constraints are satisfied feasibly.

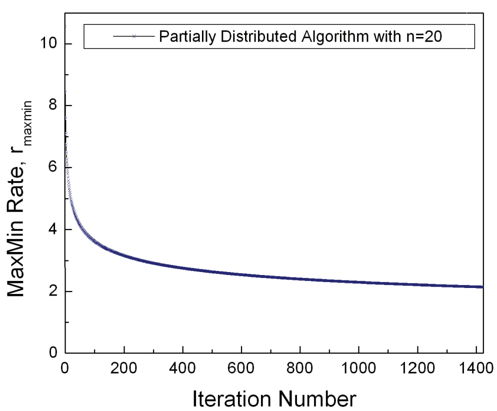

Figure 6: Convergence speed of partially distributed Max-Min Algorithm.

5 Conclusion and Future Work

In this paper, we formulate LP problems to obtain max-min fairness and sum-rate efficiency. Then we have studied the tradeoffs between fairness and efficiency in wireless sensor networks. In a large-scale low-rate network, fairness exhibits great importance. Our study shows that we can improve the fairness by allowing freedom in routing path-selection. Therefore, to achieve a fair rate allocation, we focus on designing a pricing-based fully distributed algorithm for the joint rate and routing control of a wireless network.

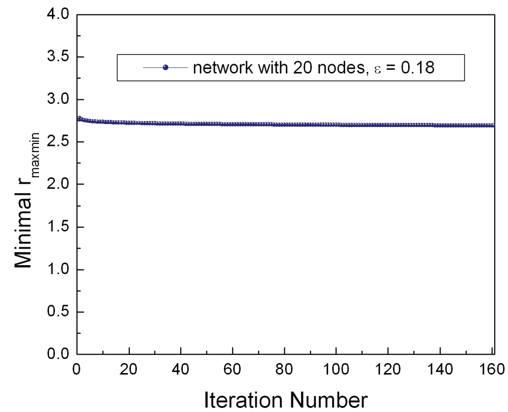

Figure 7: Convergence speed of fully distributed Max-Min Algorithm.

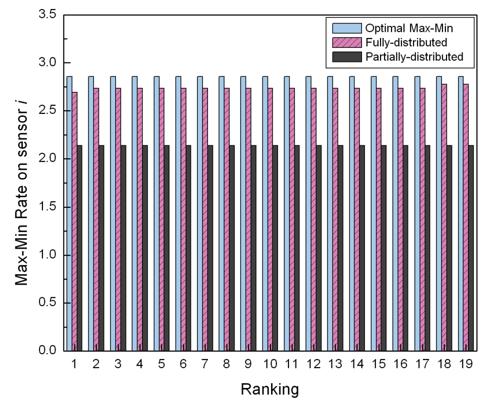

Figure 8: Compare the max-min rate obtained by different algorithms. This plot is for network with 20 nodes. We sort these nodes by the max-min values they generated. x-axis shows the ranking.

We believe that these algorithms show great promise for future development, and plan to work on extending them towards implementing a proof-of-concept on a real test-bed. In the future, we also want to obtain a distributed algorithm for a general formulation which combines fairness and efficiency together.

References

[1] A. Sridharan and B. Krishnamachari, *Maximizing Network Utilization with Max-Min Fairness in Wireless Sensor Networks*, 5th Intl. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), April 2007.

[2] A. Tang, J. Wang and S. Low, *Is Fair Allocation Always Inefficient*, in Proceedings of IEEE Infocom 2004.

[3] R. Madan and S. Lall, *Distributed Algorithms for Maximum Lifetime Routing in Wireless Sensor Networks* , IEEE Transactions on Wireless Communications, V. 5, No. 8, p. 2185–2193, 2006.

[4] L. Xiao, M. Johansson, and S. Boyd, *Simultaneous routing and resource allocation via dual decomposition* , IEEE Transactions on Communications, Vol. 52, No. 7, pages 1136-1144, July 2004.

[5] A. Woo and D.E. Culler. A transmission control scheme for media access in sensor networks. Proceedings of the 7th annual international conference on Mobile computing and netwoking, pages 221-235, 2001.

[6] C.Y. Wan, S.B. Eisenman, and A.T. Campbell. CODA: Congestion detection and avoidance in sensor networks. The First ACM Conference on Embedded Networked Sensor Systems, 2003.

[7] B. Hull, K. Jamieson, H. Balakrishnan, *Mitigating congestion in wireless sensor networks* , in Proceedings of ACM SenSys, pp.134147, Nov. 2004.

[8] C.T. Ee, R. Bajcsy, *Congestion control and fairness for many-to-one routing in sensor networks* , in Proceedings of ACM SenSys, 2004.

[9] S. Rangwala, R. Gummadi, R. Govindan, K. Psounis, *Interference-Aware Fair Rate Control in Wireless Sensor Networks* , in Proceedings of ACM SIGCOMM Symposium on Network Architectures and Protocols, Pisa, Italy, September 2006.

[10] J. Paek and R. Govindan. RCRT: rate-controlled reliable transport for wireless sensor networks. Sensys, 2007

[11] Y.T. Hou, Y. Shi, and H.D. Sherali, *Rate Allocation in Wireless Sensor Networks with Network Lifetime Requirement* , in Proceedings of ACM MobiHoc, pp. 67-77, Roppongi Hills, Tokyo, Japan, May 24-26, 2004.

[12] F.P. Kelly, A.K. Maulloo D.K.H. Tan, *Rate Control for Communication Networks: Shadow Prices, Proportional Fairness and Stability* , Journal of the Operational Research Society, 1998.

[13] H. Luss and D.R. Smith, *Resource allocation among competing activities: a lexicographic minimax approach* , Operations Research Letters, vol.5 no. 5, pp 227-231, Nov. 1986.

[14] Raymond R.-F. Liao, Andrew T. Campbell, *A Utility-Based Approach for Quantitative Adaptation in Wireless Packet Networks* , Wireless Networks, 2001.

[15] C. Curescu, S. Nadjm-Tehrani, *Price/utility-based optimization of resource allocation in ad hoc networks* , IEEE Secon, 2005.

[16] L. Tassiulas, S. Sarkar, *Maxmin Fair Scheduling in Wireless Networks* , in Proceedings of Infocom 2002, pp. 763-772, New York, USA, June 2002.

[17] V. Jacobson, *Congestion Avoidance and Control* , ACM Computer Communication Review; Proceedings of the Sigcomm '88 Symposium in Stanford, CA, August, + 1988, 18, 4:314.329, 1988.

[18] Kun Tan , Feng Jiang , Qian Zhang , Sherman Shen, *Congestion Control in Multi-hop Wireless Networks* , IEEE 2005.

[19] M. Chiang, *Balancing Transport and Physical Layers in Wireless Multihop Networks: Jointly Optimal Congestion Control and Power Control* , IEEE Journal of Selected Areas in Communications, vol. 23, no. 1, pp. 104-116, January 2005

[20] B. Johansson, P. Soldatti, M. Johansson, *Mathematical Decomposition Techniques for Distributed Cross-Layer Optimization of Data Networks* , IEEE JSAC, vol. 24, no. 8, August 2006.

[21] A. Sridharan and B. Krishnamachari, *Explicit and Precise Rate Control for Wireless Sensor Networks*, ACM Sensys 2009.