Energy Routing on the Future Grid: A Stochastic Network Optimization Approach

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***Abstract*—Population expansion and broad deployment of wind and solar renewable power generation has highlighted concerns over the long-standing strategy for grid deployment, expansion and upgrade. Due to their stochastic and often volatile nature, these renewable sources are difficult to integrate into the grid in its current power-on-demand paradigm. In this work, we propose a novel stochastic framework, leveraging distributed storage, that alleviates many of the problems of the current grid. Our proposed energy routing algorithm is distributed, agile to failures, and provably maximizes the carrying capacity of the existing power-line resources. We evaluate the performance of our proposed solution using analytical performance guarantees and sample simulation results. We hope the the result of our work provides a strong motivation for further development and application of large scale distributed storage.**

***Index Terms*—Grid integration, Stochastic networks, Energy routing, Stability, Storage, Self Healing, Renewable Power, Storage Planning, Distributed storage**

I. Introduction

With the percentage of power being generated by renewable sources rapidly growing, driven by both the policy-makers [1], [2], [3] and by advances in research globally[1], we are faced with the mounting problem of efficiently integrating these diverse sources into the grid. As these renewable sources are often stochastic and volatile in nature they cannot simply be added to the existing grid efficiently as it is not *smart enough*. The reason being that many renewables (wind farms, solar power) may ramp up or down unpredictably and faster than can be compensated for without idling of traditional generator resources [4].

The grid currently operates under a delivery-on-demand paradigm leveraging predictive optimization approaches. This assumes that parameters of demand and production processes are well known. With the introduction of renewable sources, production capacities become far more volatile and require substantial over-provisioning or idling of traditional generators. We believe that renewable integration strongly motivates a stochastic approach to the problem of power distribution.

Upgrading the grid is an enormous undertaking, and one which will have implications for power delivery for many decades. It may not be in our best interest to discount the possibility for large scale (i.e. ubiquitous) in-grid storage without thorough investigation of the potential benefits. We begin our investigation into a stochastic solution by defining the goals of the smart grid energy routing algorithm. We would like the solution to be autonomous yet supporting manual over-ride, self-healing in outages, capable of maximizing the networks' energy delivery capability, and reliable with low outage probability. Further, to be capable of pricing energy transfers and to minimize this cost would be beneficial for electrical operators.

In this paper we define the novel concept of a *depletion token network*, in which the grid is transformed into a network of distributed storage depletion queues. Using this transformed model, we provide a stochastic network framework for energy routing. By utilizing inter-disciplinary techniques from wireless communication network theory [11], we provably achieve the maximum steady-state energy distribution sustainable over an arbitrary power grid interconnect.

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[1]Global solar power capacity grew 44% in 2009. [Reuters, reported 30 March 2010]

Using utility-penalty optimization extensions to this stochastic framework ([6], [8], [9], [10]), we further discuss how sources can be assigned differing generation prices and the stochastic network can be made to minimize the cost of servicing the energy demand. We analytically show that there is a direct tradeoff between in-grid storage volume and the optimality of the cost of supply. We validate our theoretical findings using simulation results. The findings, albeit based on idealized assumptions, provide a strong motivation for further research into developing affordable energy storage mechanisms and provide a robust theoretical framework for future extensions that may more accurately reflect grid realities, such as energy losses due to transfer or storage inefficiencies.

The remainder of the paper is organized as follow: In Section II we define our terminology and provide a brief background on queuing theory for the readers' convenience. The concept of depletion token networks is described in Section III. Analytical performance guarantees of the proposed framework is presented in Section IV. Section V discusses the result of a simple simulation model, intended as proof of concept. The paper is concluded in Section VI, where future research directions are also highlighted.

II. Background on Network Model

In this section we will present a brief background on queuing theory, for readers' convenience. We try to keep the notation consistent with current communication network literature [6].

Consider a general network with $\mathcal{N}$ nodes, with each node holding a queue (storage device) for storing tokens. Let $U_n(t)$ represent the number of tokens stored in node n's queue at time t. Let nodes be connected to one another by a set of links $\mathcal{L}$, through which the tokens can be transmitted from one node to the other. A link between node i to another node j, is labeled by its corresponding ordered node pair (i, j) (where $i, j \in \mathcal{N}$). Note that link (i, j) is distinct from link (j, i). Let $\mu(t) = (\mu_{ij}(t))$ represent the matrix of transmission rates over each link (i, j) during slot t (in units of tokens/slot). By convention, we define $\mu_{ij}(t) = 0$ for all time t whenever a physical link (i, j) does not exist in the network. The capacity of the link (i, j), is represented by C_{ij} (in units of tokens/time), to describe the maximum number tokens that can flow through that link at any time. Therefore, $\mu_{ij}(t) \le C_{ij}$ for all t. Assume nodes could potentially have exogenous arrivals and allow $A_n(t)$ to represent the amount of exogenous tokens allowed to enter the queue at node n at time t. The network is assumed to operate in slotted time with slots normalized to integral units, so that slot boundaries occur at times $t \in 0, 1, 2, \ldots$. Hence, slot t refers to the time interval $[t, t+1)$. If we assume that only the tokens currently stored in node n at the beginning of slot t can be transmitted during that slot, the slot-to-slot dynamics of the queue backlog $U_n(t)$ satisfies the following equality:

$$U_n(t+1) = \max\left[U_n(t) - \sum_i \mu_{ni}(t), 0\right] + A_n(t) + \sum_j \mu_{jn}(t) \quad (1)$$

In each time slot, a control algorithm schedules inter-node transfers ($\vec{\mu}$). Recent work ([9], [10], [8], [6]) describes the Quadtratic Lyapunov Algorithm, which provides performance guarantees under prescribed control actions.

III. Smart Grid to Token Network Transformation

In its direct application, the Quadratic Lyapunov Algorithm maintains queue stability by making control decisions that provably minimize an upper bound of the queue drift. While this goal is sensible within a packet network, direct application of these techniques to energy storage in a smart grid would place the batteries frequently at risk of depletion. We therefore substantially modify the meaning of the queue network considered for smart grids.

In this work, we chose not to route blocks of energy across the infrastructure, but instead to route the energy holes in the form of Distributed Storage (DS) depletion tokens. Figure 1 depicts a simple microgrid with consumers, distributed storage facilities and a wind generator. When consumers deplete slightly the distributed storage resource providing the consumer with energy, tokens are injected into the token network. The tokens represent depletion levels of the distributed storage facilities, and are stochastically operated on by the Quadratic Lyapunov Algorithm. The algorithm naturally stabilizes the queues (depletion tokens) in the microgrid by routing energy towards the depleted DS resources.

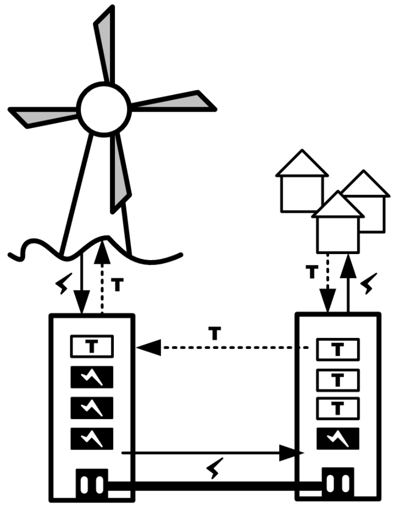

Fig. 1: A small generator / storage / consumer grid.

In order to capture financial drivers involved in these energy transfers, we will introduce link usage costs which charge for transport between distributed storage centers. These can be used to minimize the cost of energy by sourcing from least cost generators or to minimize charges incurred by transfers in cross-provider domains.

To describe the grid-to-token-network transformation, we will work in describing the transformation of Figure 1 to the token network of Figure 2.

A. *Smart Grid Links*

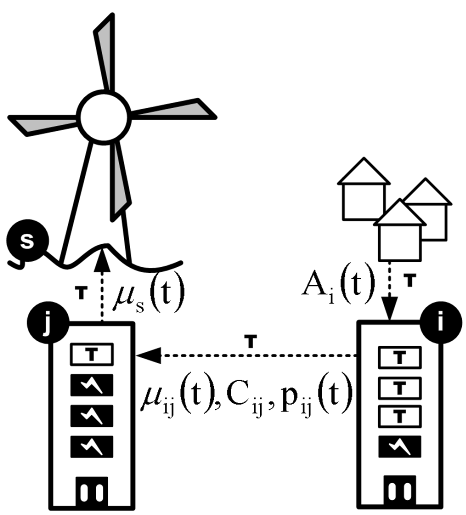

Fig. 2: A small generator / storage / consumer grid.

In Figure 2, the smart grid contains a link connecting distributed storage facility j to facility i, with capacity $C_{ij}(t)$ and cost per unit transferred $p_{ij}(t)$. In our token network then, the forwarding of a token from i to j results in grid control that forwards a unit of energy from j to i in the reverse path. The token network is therefore a reverse-path based graph. This technique is similar to that which has been applied to multi-rate Multicast by Bui *et al*. in packet routing networks, in which they refer to these tokens as shadow traffic [5]. Importantly, each transfer of a token in the token network is matched by an energy transfer within the smart grid.

B. *Consumers*

We define a consumer as any entity which consumes energy from some distributed storage unit, without itself storing that energy for transfer to other resource in the smart grid. For simplicity, consumers are assumed to be associated with a single distributed storage resource in the smart grid. This assumption may be broken if source admission controllers are introduced, but for brevity we omit them in this work. Multiple consumers may draw from the same DS.

In Figure 2, consumers are associated with distributed storage resource i. As users consume energy reserves they inject tokens into the network. These token arrivals are termed exogenous (i.e. external, not a result of node-to-node transfers inside of the grid) and are notated $A_i(t)$. The user energy consumption processes, in order to satisfy the QLA analysis assumptions, need only have finite first and second moments.

C. *Generators*

Generation facilities are network sinks for DS *depletion tokens*. In existing QLA literature from wireless packet networks [9], [10], [8], [6], the sink's receiving capacity is assumed to be limited only by the transfer capability of its receiving channel. In our framework, we pair each generator with an energy reservoir. Depletion tokens, arriving at the reservoir, can be dispatched by the generator. We restrict the per-timeslot generator capacity by modulating the link capacity between the energy reservoir and the associated generator. The underlying theory applies generally to reasonable generator capacity processes, including finite state Markov chains. Further, generator capacities are not required to be uncorrelated.

D. *Penalty Function*

We now define a system penalty function that is equal to the sum of the charges incurred by token transfers (and therefore energy transfers) across the smart grid. Let the cost incurred $p(t)$ by transfers in time slot t equal:

$$p(t) = \sum_{i,j} \mu_{ij}(t) \cdot p_{ij}(t) \tag{2}$$

Our goal will be to minimize the time average expected penalty incurred by our control actions.

E. Putting it all Together

Provided the penalty function defined above and solving for Lyapunov-drift minimizing control actions, as done in [9], [10], [8], [6], yields the following token network control algorithm.

Every distributed storage node computes per-neighbor weights w_{ij} as follows:

$$w_{ij} = (U_i(t) - U_j(t) - V \cdot p_{ij}(t)) \cdot C_{ij}(t) \tag{3}$$

If endpoint j describes a generator, then $U_j(t) = 0$. Then given these weights, a simple decision is made that determines token (and therefore energy) transfers.

Let j^* be the neighbor such that:

$$j^* = \arg\max_j w_{ij} \tag{4}$$

If distributed storage node i has no positive weight $w_{ij} \geq 0$ then no transfers out of i occur. Otherwise, the distributed storage facility transfers the maximum link capacity over the link represented by weight w_{ij^*} which is the maximal outbound weight.

IV. Performance Guarantees of QLA Application to Smart Grids

The question then is the following: are the proven performance bounds for QLA application in packet networks meaningful after transformation to route energy holes in smart grids? If the depletion token queues are stable then the time average injection rate must be lesser than the service rate. This then requires that the distributed storage resources are replenished at a time average rate that matches their consumer demand, and the system supports the load provided sufficient storage provisioning. We will now give a more rigorous description, from stochastic network optimization, of the relationship between stability and network capacity.

A. Capacity Region

Central to the analysis of performance guarantees for the Quadratic Lyapunov Algorithm is the definition of a network capacity region. Let the time average token injection rate sourced by resource n be:

$$\lambda_n = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t} A_n(\tau) \tag{5}$$

Note that by assumption that $A_n(t)$ has finite first and second moments, there exists such a λ_n for all n. Let the vector $\vec{\lambda}$ be composed of time average token injection rate for all nodes in the token network.

Definition A queueing network is *strongly stable* if the following holds:

$$\limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[U_n(\tau)] < \infty \text{ for all } n \tag{6}$$

Note that for a queuing network to be strongly stable, every queue must be strongly stable. This definition is then leveraged to provide a capacity region for the token network as follows.

Definition The *capacity region* Λ of token network $\mathcal{N}$ is the set of all token injection rate vectors $(\vec{\lambda})$ for which there exists some resource and routing control algorithm that guarantees *strong stability*.

Theorem IV.1 *(From [9], [10], [8], [6], [7]) As the token network service and arrival processes satisfy assumptions of prior work, the application of the distributed energy routing algorithm of section III-E operating over our transformed token network $\mathcal{N}$ guarantees strong stability for any token rate vector $\vec{\lambda} \in \Lambda$.*

B. Time Average Production Cost

Application of stochastic network optimization to this token network also affords us guarantees on the cost of system smart grid transfers, defined by our penalty function in section II. First, let us define the best achievable penalty rate.

Definition Let p^* be the minimum time-average token network penalty (payment for energy transfers per time slot) under an optimal control policy which guarantees strong stability.

Theorem IV.2 *(From [9], [10], [8], [6], [7]) As the token network service and arrival processes satisfy assumptions of prior work, the application of the distributed energy routing algorithm of section III-E operating over our transformed token network $\mathcal{N}$ guarantees:*

$$\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{i,j} \mathbb{E}\left[\mu_{ij}(t)\cdot p_{ij}(t)\right] \le p^* + \frac{B}{V} \quad (7)$$

That is, the control decisions result in a penalty that deviates in time average expectation from the optimal penalty rate by no more than $\frac{B}{V}$ for constant B which is a function of fixed network parameters.

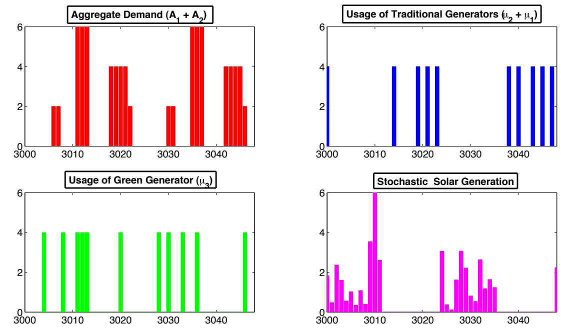

Fig. 4: All plots are taken over a 48 hour period during the steady-state, with $V = 50$. The y axis shows time in hours and the x axis represents units of energy.

V. Simulation Results

In this section, we simulate the algorithm of Section III-E for a simple grid consisting of 5 nodes, 2 of the nodes being consumers and 3 generators. One of the 3 generators is a renewable generator. The definition of consumers and generators are as per Section III, so the consumers could potentially be substations. The nodes are assumed to have storage available to them. This simulation setting has been intentionally chosen to be simple and demonstrate the proof of concept. The algorithm of Section III-E is capable of routing on multi-hop networks, so it easily supports more complex simulation settings.

The simulation network is presented in Figure 3. As can be seen, each consumer has access (through a dedicated link) to a traditional generator. In addition, the renewable generator, namely generator 3, can serve both consumers. The demand for the two consumers are chosen to be identical and periodic for simplicity, and the aggregate demand $(A_1(t) + A_2(t))$ is shown in Figure 4 for a 48 hour period. The green generator is assumed to generate power stochastically, during the first 12 hours of the day, according to an exponential distribution with mean 1.6 units of energy per hour and no power during the night. The traditional generators are assumed to have a deterministic generating capacity of 2 units of energy per hour each. We pick the capacities to be 2 units of energy per hour for all links connected to traditional sources and 1.6 for links connected to the green generator, i.e. $C_{11} = C_{22} = 2$ and $C_{23} = C_{13} = 1.6$.

To encourage the system to have a preference in utilizing the green source, we assign penalty functions to these sources with the links connecting green source charging a lower 1.6 units of penalty per token transferred and the links connected to traditional ones charging 2 units of penalty per token. The objective of the system is thus to stabilize the queues, while minimizing penalty. The green generator is therefore expected to be used more intensively than the traditional ones in the optimal setting. Looking at the aggregate demand plot and the power produced by the green generator, in Figure 4, we see that the green generator cannot support both users by itself without the help of the traditional sources. So our proposed algorithm finds the optimal solution to see how much power to draw from each source to achieve its objective while making sure the consumers do not face outage. As we can see from the plots, the system uses generator 3 when possible and whenever generator 3 goes out (during the night in this case) it automatically switches back to the traditional generators. As we see in the coming plots, this happens while the consumers' queue backlog is kept steady - so the consumers will not face any outages.

The instantaneous queue backlog of the consumers and generators, for $V = 10$ and $V = 50$, are shown in Figures 5 and 6 respectively. The queue backlog represents the instantaneous depletion level in the storage resource available to each node. As can be seen there is a transient period after which the curve reach a steady state. How long it takes to reach the steady-state depends on the size of V, with larger V causing longer delay before reaching the steady state. Once we have reached the steady-state though, the fluctuations are fairly small. For examples, the queues corresponding to consumers only fluctuate by 6 units, empirically indicating

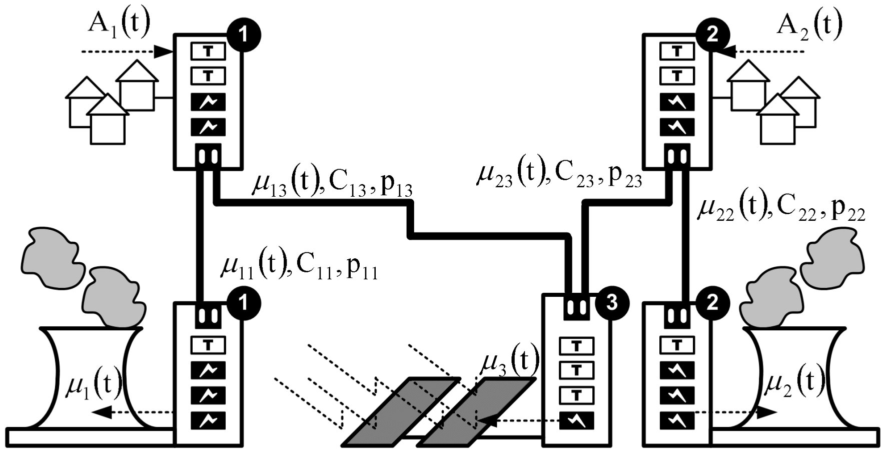

Fig. 3: Simulation setup, with two consumers, two traditional generators and one solar generator.

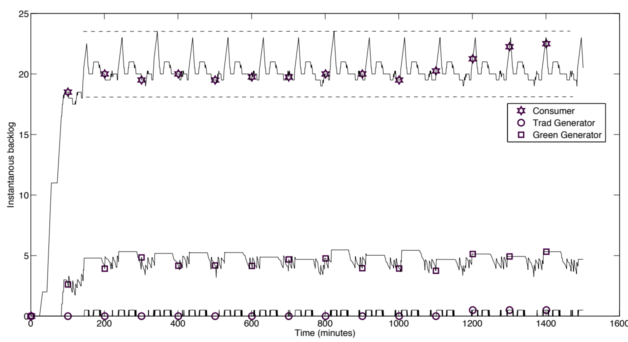

Fig. 5: Instantanous values of queue backlog vs Time, where time slots are of 15 min duration, and $V = 10$

that we could support this grid with a battery size capable of storing 6 units of energy, which is about two hours of energy for our example grid. So if consumers are equipped with batteries capable of storing two hours of power we can use the theory to make tight probabilistic guarantees that we will never be in an outage in this case.

The larger value of V, looking at equation (3), increases the weight of the penalty term and leads to the algorithm trying to minimize its usage of higher penalty links even further. This would lead to a more optimal solution with respect to the penalty term but at the expense of slightly higher fluctuation rates around the steady-state and the transient period taking longer. Table I shows how increasing the V values affects the amount of contribution of sources (in terms of their aggravate average rate) in the solution.

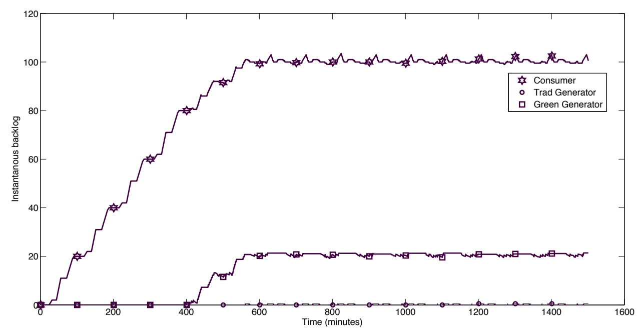

Fig. 6: Instantanous values of queue backlog vs Time, where time slots are of 15 min duration, and $V = 50$.

V	μ_1	μ_2	μ_3
10	0.499	0.499	0.668
50	0.423	0.423	0.822

TABLE I: Table showing the corresponding transfer rates of generators for different Vs.

VI. Conclusion and Future Work

Leveraging distributed storage, we presented a stochastic framework for fast, distributed energy switching in the future electric grid, enabling the efficient integration and dispatching of renewable sources. In addition to proposing an analytical model and reviewing theoretical performance guarantees, we presented simulation results that demonstrate the theoretical findings. By utilizing advancements in wireless communication networks, our model offers reliable and predictable power delivery while being autonomous, self-healing, and capable of maximizing the networks energy delivery capability while minimizing the costs incurred by the utilities.

While we chose to minimize cost, alternative utility or penalty functions are possible. The framework does currently support some forms of electrical loss modeling and minimization. Specifically, we could associate transfer losses to all links in the smart grid by introducing new queue update equations and a new penalty function. In this scenario, transferring one token results in inflation of the token count arriving to the link destination (due to efficiency losses in the reverse-path energy transfer). The assumptions of the stochastic optimization framework are not violated by this action. If we want to additionally model energy losses in the distributed storage facilities, as a fraction of the energized portion, the current framework's arrival process assumptions are violated as they become correlated with the depletion queue backlog. In-storage losses remains a topic for future investigation.

Our work can also be considered as a new and notable application of stochastic network optimization and routing techniques. We hope our results, albeit idealized at this stage, provide enough motivation for further research in this area and in design and development of new techniques in the field of storage and network optimization. The proposed framework brings about many interesting research directions that can be tackled, below are a few examples that the authors are currently investigating:

- Comprehensive simulation models using real data under more realistic settings
- Finite capacity distributed storage facilities
- New constraints through virtual queue processes in order to support requirements that at least some minimum percentage of total generation be from renewable sources
- Support for energy loss during storage as a percent of per-timeslot storage backlog

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BIOGRAPHIES

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