Ollivier-Ricci curvature on multi-channel interference constrained wireless networks

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***Abstract*—This paper investigates the Olivier-Ricci curvature (ORC) characteristics of a low-power wireless network deployment. We quantitatively analyze how the OCR varies among different channels of a IEEE 802.15.4-based network heavily influenced by external interference. And, finally, verify the correlation between the topology curvature of network graphs and the performance of time-scheduled protocols. Results show that the Ollivier-Ricci curvature has a direct connection with the number of timeslots required for data collection in multi-hop multi-channel networks. One could leverage such results to determine the performance of wireless networks based on the topological characteristic of its connectivity graph.**

I. Introduction

The geometric approach to networks has been used to explain the phenomenon of congestion as a result of least-cost routing on a Gromov hyperbolic graph. It has been shown that negatively curved graphs imply networks with larger routes, more queue occupancy and restricted capacity region [1]. In wireless networks the external interference and limited link capacities add extra limitations and increases the congestion. In this context, throughput optimal protocols based on the weighting-scheduling-forwarding structure of Back-Pressure have been proposed, such as Heat-Diffusion (HD) and Backpressure Collection Protocol (BCP). It has been proved that there is a clear relation between curvature in the Ollivier-Ricci sense and the capacity region of HD protocol [2]. In general the more negative the Ollivier-Ricci curvature, the higher the queue occupancy and, consequently, the smaller the capacity of such networks.

Backpressure-based protocols require frequent updates on each link weight in order to calculate a globally optimal scheduling matrix to be employed in forwarding stage. Even though this approach exhibits good performance in scenarios with high levels of external interference and mobility, the stochastic nature of such protocols makes them unsuitable for time-sensitive systems. Medium scheduling is one of the challenges that need to be coped by the Industrial Internet of Things (IIoT) and its applications [3].

The most common way of solving this problem is still through static time-frequency allocations, particularly in low-power wireless networks. The most recent IEEE 802.15.4 standard specifies the use of time-frequency scheduling by means of the Timeslotted Channel Hopping (TSCH) protocol. TSCH has been employed in real industrial deployments for the past years and has been demonstrated to be very effective in providing dependable wireless networks in face of external interference [4].

In this paper we investigate how topological features of real wireless network impact the performance of TSCH-based networks. We utilize real traces from a low-power wireless testbed and analyze how the network curvature varies along different channels and what is the relation between time-frequency scheduling and the Ollivier-Ricci curvature (ORC). As main result we show that when comparing the network curvature calculated over graphs generated on all 16 different channels, the more external interference that is present the more negative is the curvature of a particular channel. Considering commonly used algorithms to generate time-frequency schedules for data collection over TSCH-based networks we can verify that the network curvature is also linked with the size of the schedule and with the upper-bound on number of transmissions necessarily to collect data in such networks.

This paper is organized as follows. Section II introduces how Ollivier-Ricci curvature (ORC) is linked to congestion in wireless networks and describes the target scenario utilized in the analysis. Section III shows the results and compare ORC of wireless networks with protocol-related characteristics, such as packet delivery ratio, schedule size, etc. Finally, Section **??** concludes the analysis and summarizes the main results.

II. Ollivier-Ricci Curvature in real multi-channel wireless networks

The *Ollivier-Ricci curvature* is a graph version of the well know Ricci curvature. It has been used to anticipate congestion in interference constrained wireless networks employing Heat Diffusion protocol [1]. Ollivier-Ricci curvature has also been employed in diverse applications, from market fragility and risk analysis [5] to Hamiltonian Monte Carlo formulation [6], modeling robustness of cancer networks [7], etc.

A wireless sensor network (WSN) consists of a large number of low-power and low-cost devices. The standard IEEE 802.15.4 [8] provides specification for physical and medium access layers and is arguably one of the most used in WSN. Nodes can choose between 16 available channels to work on if the 2.4 GHz physical layer is employed. Since most part of these 16 channels overlap with channels from other technologies, such as Wi-Fi and Bluetooth, the quality in real deployments varies largely.

In wireless networks the quality of the channels strongly influences the overall performance of protocols. A network may be easily disrupted if a single channel is chosen and it suffers large interference for long time. More robust solutions, such as the Timeslotted Channel Hopping (TSCH) protocol, rely on frequency diversity to smooth the effect of external interference and multi-path fading. In these multi-channel protocol an agile blind sequence hopping is usually employed, where all channel are equally used for data transmission. Quantifying the quality of different channels in a particular deployment and understanding how such they affect the performance of protocols such as TSCH is still a problem under investigation. A mathematical tool such as Ollivier-Ricci may consist of a useful tool to help network designers understand and better configure wireless networks.

In this paper we present a first study on the use of ORC as way of quantifying the quality of multi-channel protocols, more specifically TSCH-based scheduled WSN. We use for this study traces obtained from a real deployment of low-power nodes. The testbed consists of 40 TelosB motes spread in a working environment [9]. The nodes are programmed with a firmware that continuously performs sequences of broadcast transmissions in order to verify the quality of all links. The nodes are synchronized and a centralized computer controls the sequence of nodes that transmit packets, so that two nodes do not collide and the packets loss are solely due to external factors, such as interference or multi-path fading. All nodes transmit 100 packets in sequence, iterating over all 16 channels. The experiment is repeated every 15 minutes for a 24-hour period. At the end we have a total of 96 snapshots of all link qualities in the network. Each snapshot consists of 16 NxN matrices with the packet delivery ratio (PDR) measurement. PDR is always between 0 and 100.

For each snapshot there are 16 weighted graphs, corresponding to all 16 channels available. Each of these 16 graphs G have the 40 nodes as vertices and the links as edges. The weights are the PDR for each link. We consider those graphs and calculate the ORC based on the works [1], [2]. Even though the OCR is calculated for each pair of nodes, we consider the average over all edges on the graph.

III. Results

We first intend to analyze the correlation between Ollivier-Ricci curvature and the quality of links over all 16 different channels. Figure 1 shows a scatter-plot of average ORC versus average packet reception rate (PRR). In this section the averages are always calculated over all 96 snapshots from the traces detailed in Section II. In 802.15.4 nomenclature, the channels are number from 11 to 26.

We can notice that channels with high PRR - greater than 50% - have positive curvature, while channels with PRR lower than 50% have negative curvature. Channels 25 and 26 have the highest curvature. These channel do not overlap with Wi-Fi spectrum and, thus, suffer less external interference.

Figure 2 shows another perspective of the same results. It is possible to see that channels that overlap the most with Wi-Fi spectrum: 12-13 and 16-18 show the lowest ORC values. While channel 15, 20, 25 and 26 have the highest ORC.

Figure 3 shows the variance of ORC, over all 96 snapshots, versus the variance of PRR. It also show a correlation between the variances, but not so clear as the average values. Channels with low variance in PRR tend to have higher variance of ORC, while channels with higher variance in PRR tend to have lower variance in ORC.

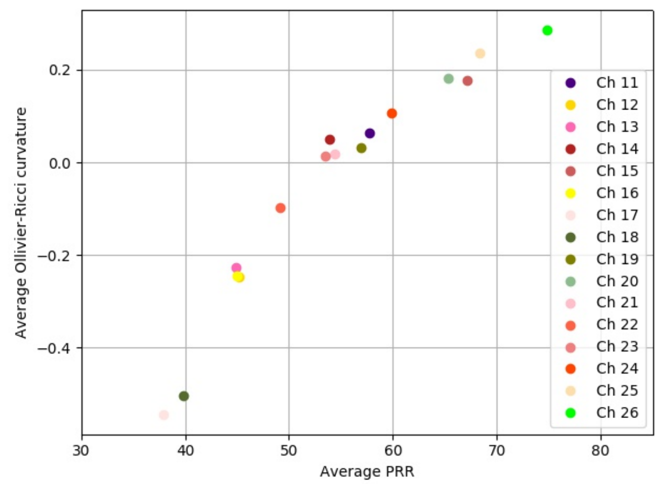

Fig. 1. Average Ollivier-Ricci curvature versus packet reception rate (PRR).

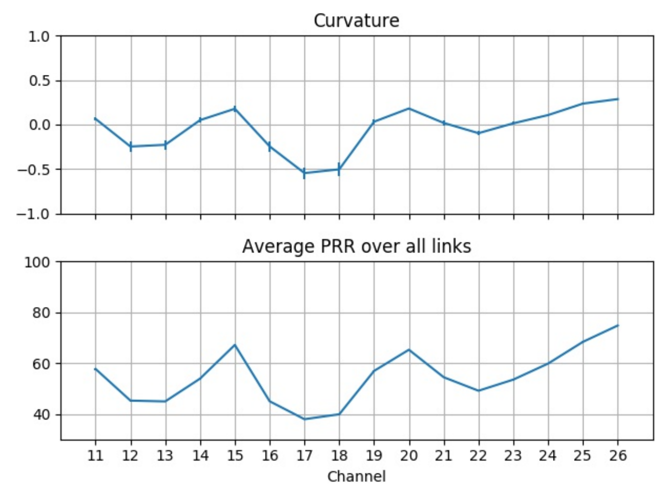

Fig. 2. Average Ollivier-Ricci curvature and average PRR across all 16 channels.

Overall it is clear that there is a strong relationship between link quality - here represented by the Packet Reception Ratio - and the OCR. The graphs shown so far consider the average values calculated over a long period of time (24 hours). The following analysis considers the same metrics (PRR and ORC) over time.

Figures 4, 5 and 6 compare the ORC and PRR values over time for three different channels: 12, 17 and 26.

Figure 4 shows a high correlation of PRR and ORC over time. The same can be noticed in channel 17 (Figure 4) but with lower intensity. In general considering all 16 channels it is possible to verify that when the PRR is above 60% the ORC is positive. Channels that are more stable and have high values of PRR, such as 20, 25 and 26 (Figure 6), show very low changes in the ORC values and they are always positive.

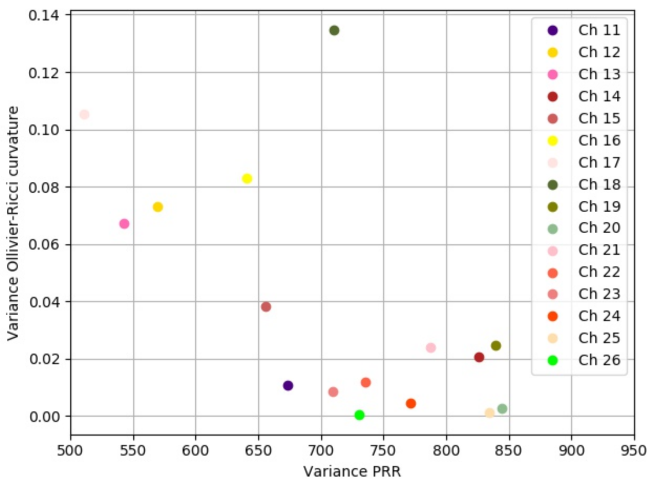

Fig. 3. Variance of Ollivier-Ricci curvature versus variance of PRR.

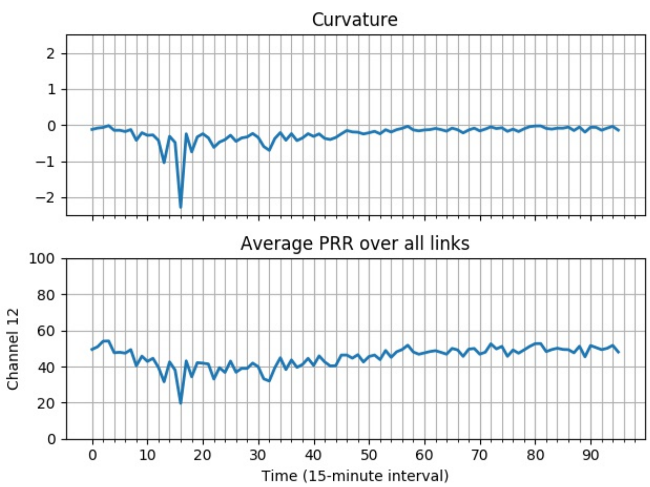

Fig. 4. Ollivier-Ricci curvature and PRR over time at channel 12.

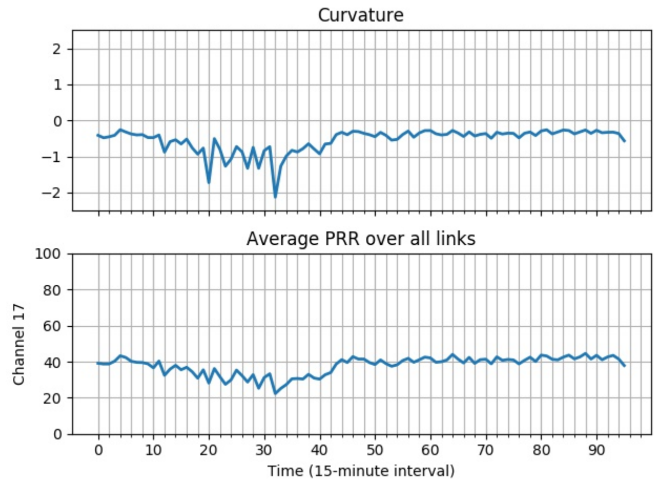

Fig. 5. Ollivier-Ricci curvature and PRR over time at channel 17.

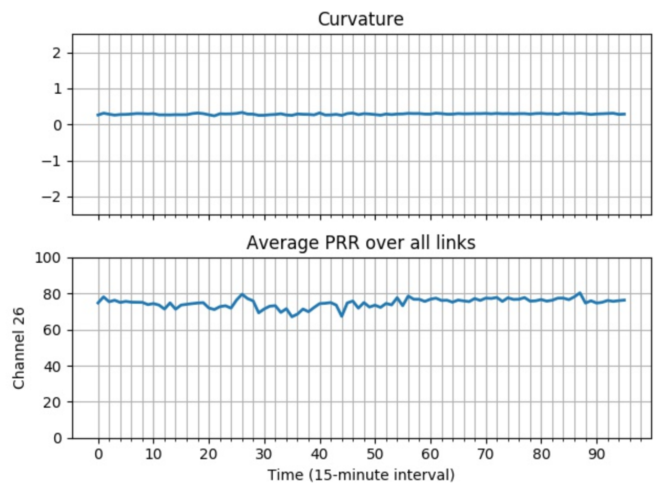

Fig. 6. Ollivier-Ricci curvature and PRR over time at channel 26.

Our last analysis compares the ORC with the expected performance of a TDMA-based networking protocol. For each of the connectivity graphs we run the algorithms that compose MCC (Multi-channel Collection) protocol [10]. MCC creates a minimum time-frequency schedule for data collection. It considers the links that form a balanced routing tree and minimizes the number of time slots required for receiving one packet from each sensor node.

We used all 40 nodes as possible sinks in the routing tree and calculated the average size of the schedule. Besides, since some of the selected links may be worse than other, i.e. have lower PRR, we also multiply the average schedule size by the average end-to-end ETX (expected transmission count) calculated for all nodes. The final metric obtained (average schedule size times average end-to-end ETX) represents the average number of time slots necessary to collect one packet from each sensor node at a sink.

The results from this last analysis are shown in Fig. 7. They clearly demonstrate the connection between topological curvature of the network graph and the performance of data collection protocols running on different channels. We can verify that channels less affected by interference, such as 26, 25 and 20 require less than 60 timeslots. On the other hand channels heavily affected, such as 17 and 18, require twice the amount of timeslots.

IV. Conclusion

We show in the paper an analysis of the correlation between the Ollivier-Ricci curvature of network graphs of real multi-channel wireless networks. Measuring the curvature and the quality of links over different channels we conclude that the topological characteristics have a strong connection with network performance. Moreover, network curvature can be seen as a way to estimate the performance of time-scheduled protocols and, this way, can be used as a tool to better

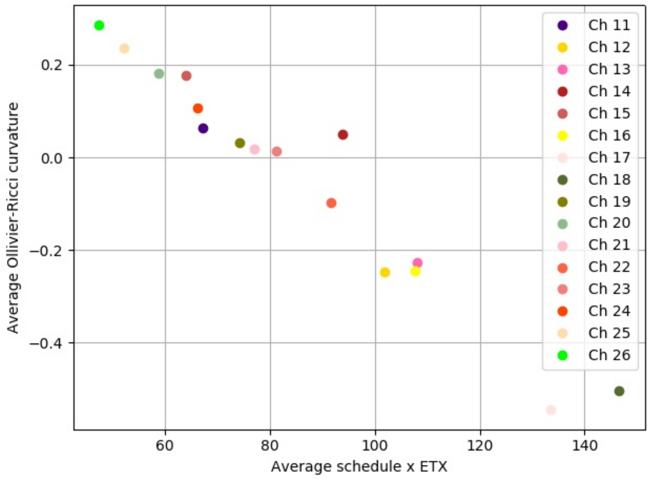

Fig. 7. Average Ollivier-Ricci curvature versus schedule size times ETX. x-axis represents the average number of timeslots required for data collection from all sensors.

design and improve such protocols in multi-hop multi-channel networks.

Acknowledgment

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