EE 597

Jan 17, 2012 Lecture 3

expected /Average probability of error

$$= \int_0^{\infty} P_b(\gamma) \cdot f(\gamma)\, d\gamma$$

fading distribution

"instaneous" probability of error for a given SNR

e.g. AWGN / BPSK

$$P_b(\gamma) = Q\left(\sqrt{2\gamma_b}\right)$$

for BPSK over a Rayleigh fading channel

the received power & SNR distributions are equivalent, assuming constant noise power

$$\text{Prob}\left[P_r < x \right]$$

$$\text{Prob}\left[\gamma < \theta \right] = \text{Prob}\left[\frac{P_r}{N} < \theta \right]$$

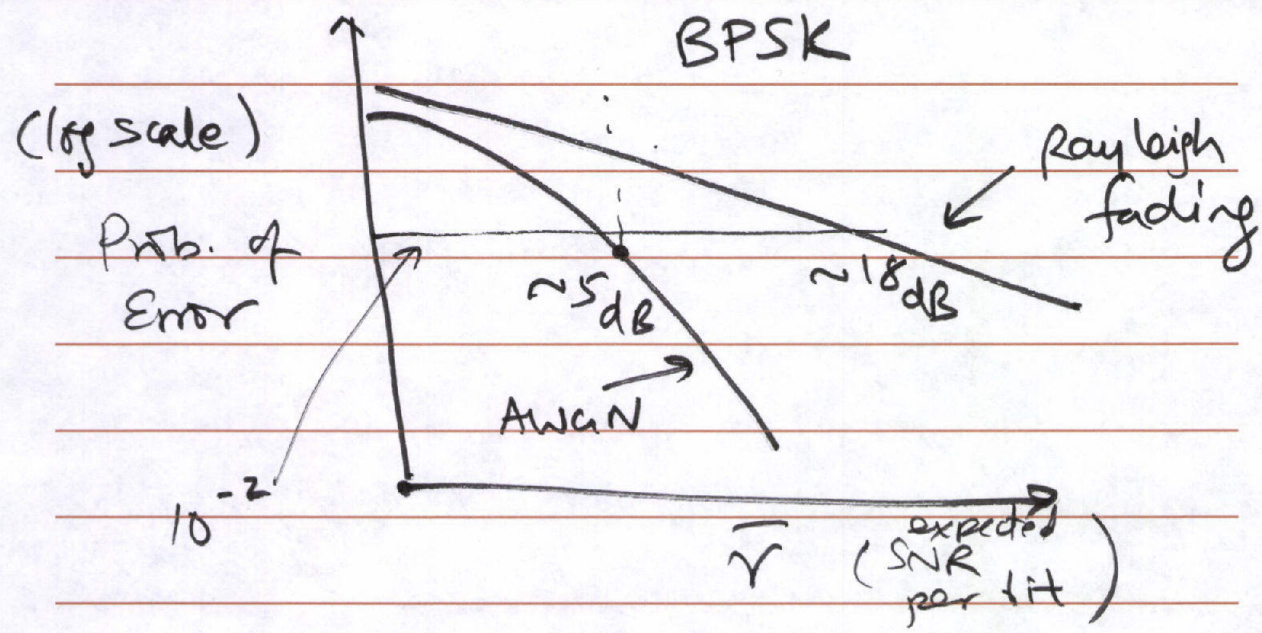
$$= \text{Prob}\left[P_r < \underbrace{\theta \cdot N}_{x} \right]$$

performance of BPSK over Rayleigh:

$$\text{Expected error probability} = \int_0^\infty Q(\sqrt{2\gamma}) \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma$$

$$= \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right] \approx \frac{1}{4\bar{\gamma}}$$

$\bar{\gamma} \leftarrow$ average SNR per bit.

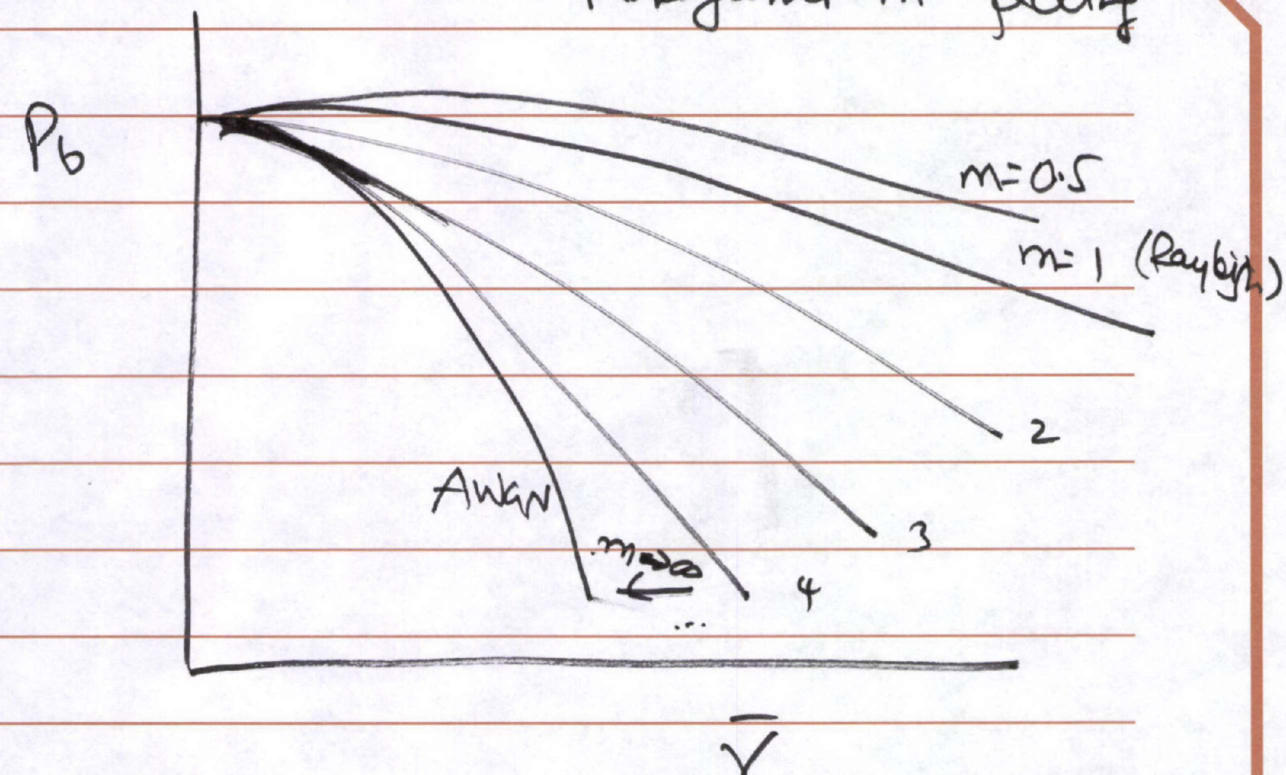


Two measures of link performance for fading channels:

- Outage Probability: Prob. that SNR drops below a threshold (depends on modn scheme)
- average probability of error: Expectation of the error probability obtained by averaging over SNR

common lesson here is that when SNR becomes a random variable due to fading, we need to inject more power in order to maintain the same error performance as with no fading.

Nakagami-m fading



Dynamics of the fading Environment

The fading distribution by itself does not indicate temporal quality of the link.

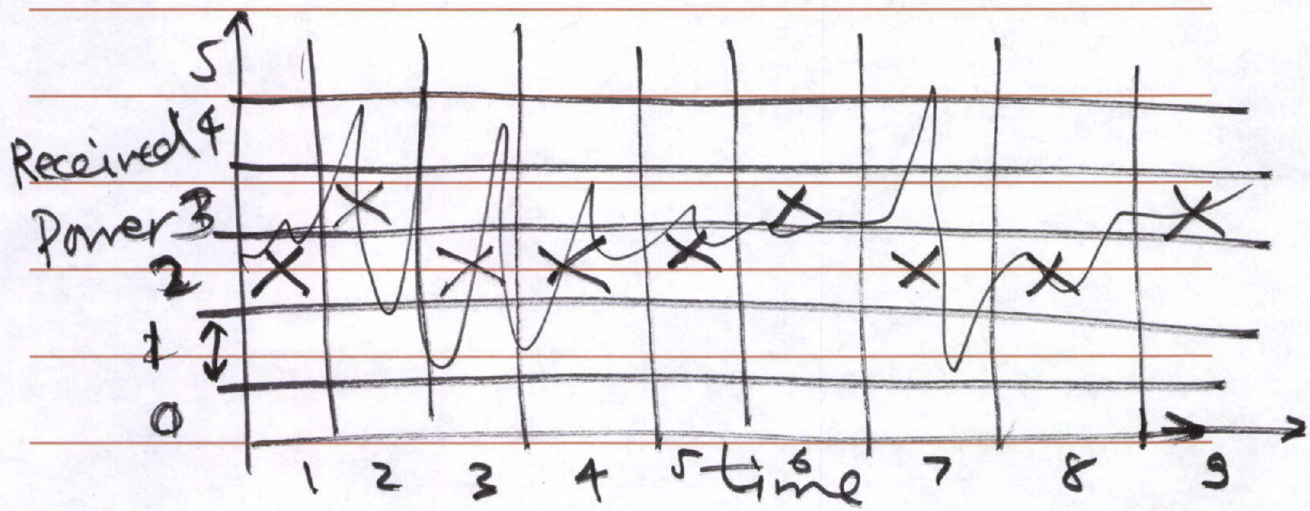
Depending on the environment, the link may face slow or fast fading.

In case of a slow fading environment, any given link does not change its signal strength (received power/SNR) over time, or changes very slowly. In this case the ~~ ~~ expected prob. of error may be higher or lower than the experienced link quality.

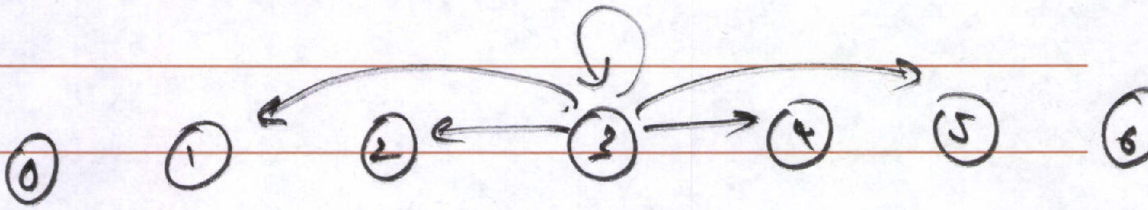
In a fast fading environment, the expected prob. of error & the time average probability of error will be close to each other.

In other words, we experience ergodicity in time-varying/fast fading conditions.

Markov model of a fading Channel



Discrete time finite state model of the channel that is Markovian.



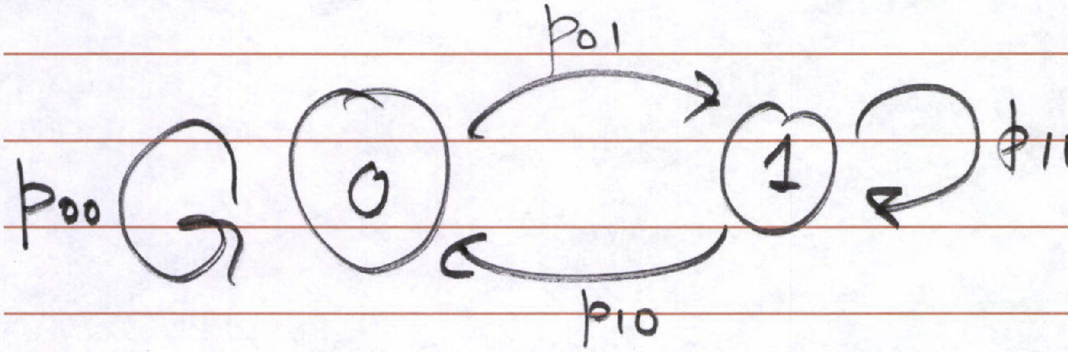
$$P_{2-3} = \text{Prob}(X_{t+1}=3 \mid X_t=2)$$
$$= \frac{\text{Prob}(X_{t+1}=3, X_t=2)}{\text{Prob}(X_t=2)}$$

The steady distribution of the ~~this~~ Markov chain represents the fading distribution.

Simplest Markov Channel Model

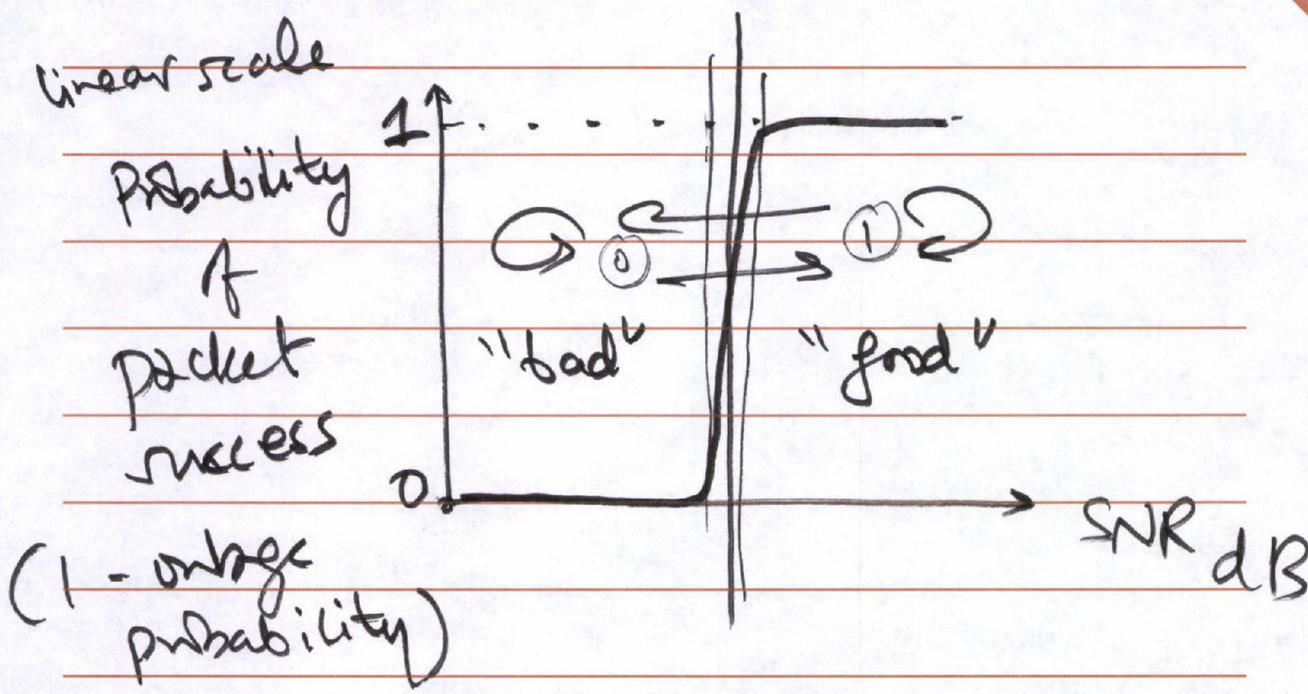
— 2 states

A "good" (1) & "bad" (0) state



Gilbert-Elliot model (p_{01}, p_{11})

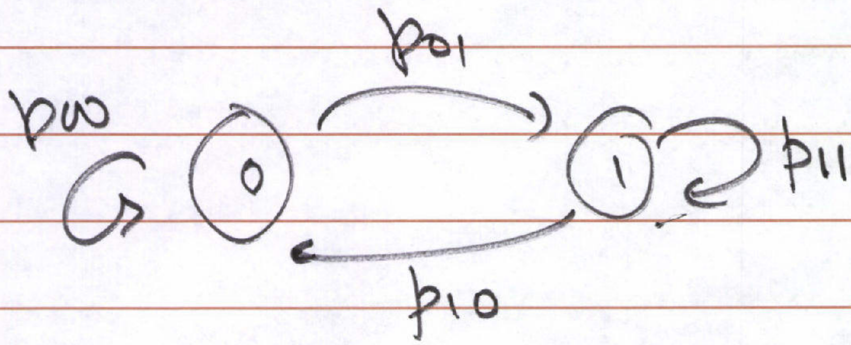
2-parameter model



PRR — packet reception rate

An even simpler model is the <u>2-state IID model</u>: At each time the probability that the channel is "good" is p, independent of past observations.

This is a 1-parameter model.



$$\pi_1 \cdot p_{10} = \pi_0 \cdot p_{01}$$

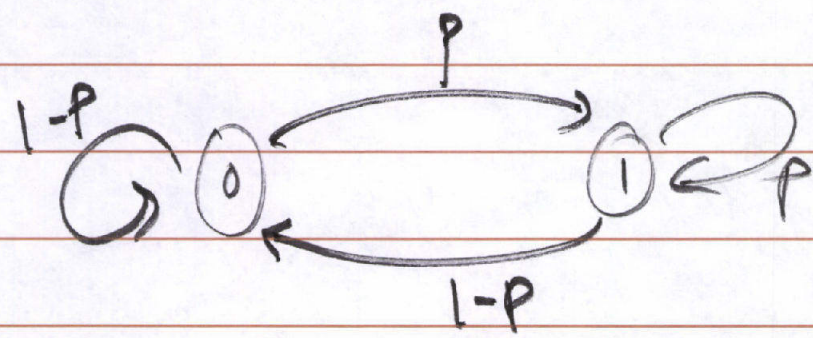
$$\pi_0 + \pi_1 = 1$$

$$\pi_0 p_{01} - (1 - \pi_0) p_{10} = 0$$

$$\pi_0 [p_{01} + p_{10}] - p_{10} = 0$$

$$\pi_0 = \frac{p_{10}}{p_{01} + p_{10}}; \quad \pi_1 = \frac{p_{01}}{p_{01} + p_{10}}$$

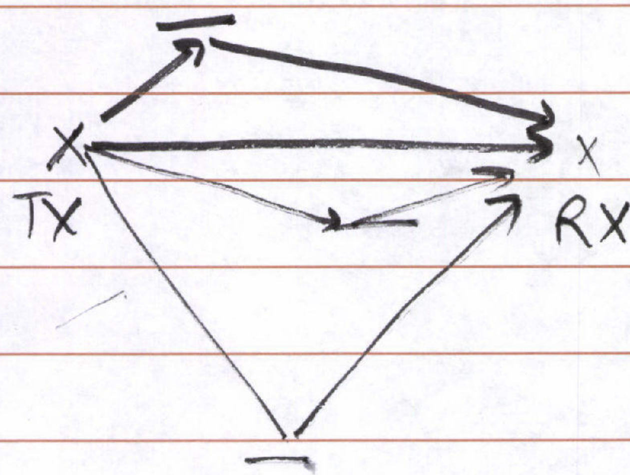
this is the IID random model



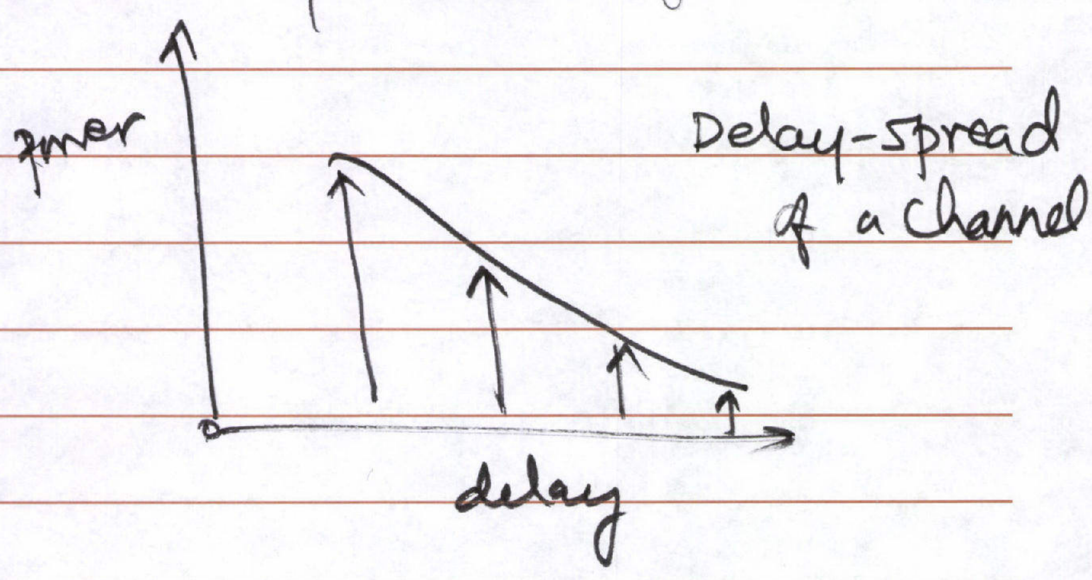
$$\pi_1 = \frac{p}{p+1-p} = p$$

$$\pi_0 = \frac{1-p}{p+1-p} = 1-p.$$

Aspects of fading Channels.



Power-delay profile of a channel.



characterizes the farthest delay
components with significant power

Typical values of the delay spread σ vary from 10's of ns (low power / small scale) to 10's of μs (high power / large scale).

Large delay spreads introduce inter-symbol interference, in high-rate data links.

Two solutions:

- Equalization
- OFDM

Large delay spread corresponds to a ~~smaller~~ narrower frequence response characteristic of the channel
∴ they are referred to as Frequency selective fading channels. (vs. Flat fading).