Lecture on 3/20/12 EE 597

To show $\pi_1 p_{11} - \pi_1^2 > 0$

$$\Rightarrow \pi_1 \left(\underbrace{p_{11} - \pi_1} \right) > 0$$

$\to$ if

$$\pi_1 = \frac{p_{01}}{p_{01} + p_{10}}$$

$p_{11} > p_{01}$ then

$\underbrace{p_{01} < \pi_1 < p_{11}}$

$\cancel{p_{11} - \frac{p_{01}}{p_{01} + p_{10}}}$

$p_{11}^{(+)} \to \pi_1$ decreasing.

$= \cancel{p_{11} < p_{01}}$.

$$\frac{p_{11}(p_{01} + p_{10}) - p_{01}}{p_{01} + p_{10}}$$

$$= \frac{p_{11} \cdot p_{01} + p_{11}(1 - p_{11}) - p_{01}}{p_{01} + (1 - p_{11})}$$

$E[X(t+1) \cdot X(t)] = ?$ $p_{11} \cdot \pi_1$

$E[X(t+1)]\underbrace{E[X(t)]} = ?$ π_1^2

$\downarrow$ π_1

$\downarrow$ $\pi_1 \cdot 1 + \pi_0 \cdot 0 = \pi_1$

$X(t)$	$X(t+1)$	Prob.	$X(t) \cdot X(t+1)$
0	0	$\pi_0 \cdot p_{00}$	0
0	1	$\pi_0 \cdot p_{01}$	0
1	0	$\pi_1 \cdot p_{10}$	0
1	1	$\pi_1 \cdot p_{11}$	1

$E[X(t)\, X(t+1)] = \pi_1\, p_{11}$

$E[X(t) \mid X(0) = 0)$.

$$\frac{p_{11} \cdot p_{01} + p_{11} - p_{11}^2 - p_{01}}{p_{01} + 1 - p_{11}}$$

$$= \frac{p_{11}(p_{01} - p_{11}) + (p_{11} - p_{01})}{p_{01} + 1 - p_{11}}$$

$$\to \frac{\boxed{(p_{11} - p_{01})}\underbrace{(1 - p_{11})}_{p_{10}}}{\underbrace{p_{01} + 1 - p_{11}}_{>0}}$$

>0

known to be >0

If $p_{11} - p_{01} > 0$

then this expression is > 0

$$\min \left(\log(1+P_1 g_1/n_1) - T_1\right)^2 + \left(\log\left(1+\frac{P_2 g_2}{n_2}\right) - T_2\right)^2$$

$$\text{s.t.} \quad P_1 + P_2 < P_T.$$

· Here: we need not use all the power.

specifically:

$$P_1^* : \quad \log\left(1+\frac{P_1^* g_1}{n_1}\right) = T_1$$

$$1 + \frac{P_1^* g_1}{n_1} = 2^{T_1}$$

$$P_1^* = \frac{(2^{T_1} - 1)\, n_1}{g_1}$$

$$\text{Ily} \quad P_2^* = (2^{T_2} - 1)\frac{n_2}{g_2}.$$

If $P_1^* + P_2^* < P_T$

then solution is:

$$P_1 = P_1^*$$
$$P_2 = P_2^*$$

$g_i / n_i = \alpha_i$

$$L(P_1, P_2, \lambda) = \left(T_1 - \log(1 + P_1\alpha_1)\right)^2 + \left(T_2 - \log(1+P_2\alpha_2)\right)^2 + \lambda\left(P_T - P_1 - P_2\right)$$

$$\frac{\partial L}{\partial P_i} = 0 \Rightarrow -\frac{2\left(T_i - \log(1+P_i\alpha_i)\right)}{(1+P_i\alpha_i)} \cdot \alpha_i = \lambda^* \Rightarrow \frac{\Delta i}{P_i} = \frac{\lambda^*}{2}$$

$$\frac{\cancel{2}\left(T_1 - \log(1+P_1\alpha_1)\right)\cdot\alpha_1}{1+P_1\alpha_1} = \frac{\cancel{2}\left(T_2 - \log(1+P_2\alpha_2)\right)\alpha_2}{1+P_2\alpha_2}$$

$$\left(P_2 = P_T - P_1\right)$$

$$\Delta_1 = T_1 - R_1$$

$$\Delta_2 = T_2 - R_2.$$

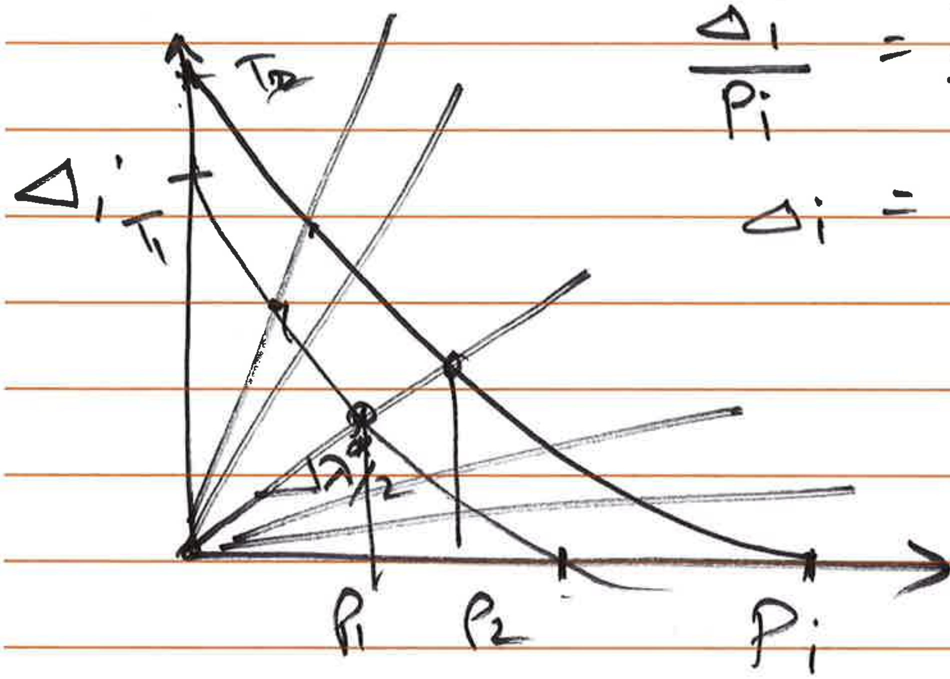
$$\frac{\Delta_1(P_1)\cdot \cancel{\alpha_1}}{\cancel{1+} P_1 \cancel{\alpha_1}} = \frac{\Delta_2(P_2)\cdot \cancel{\alpha_2}}{\cancel{1+} P_2 \cancel{\alpha_2}}$$

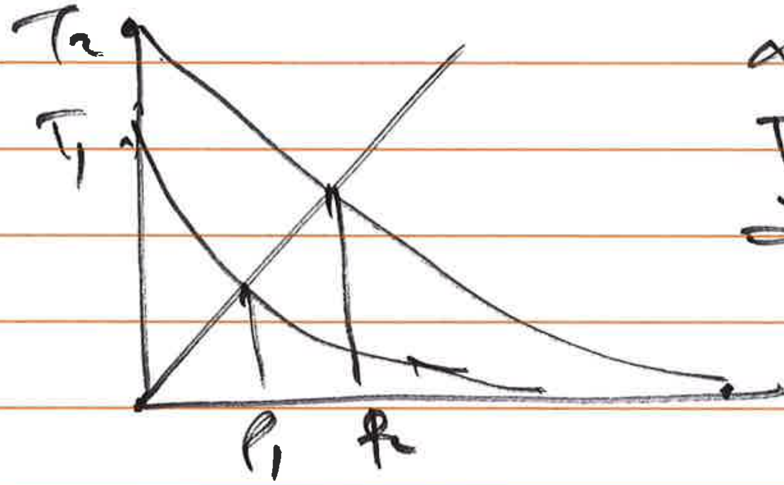
high SNR approx:

$$\left\{ \begin{array}{l} \dfrac{\Delta_1(P_1)}{P_1} = \dfrac{\Delta_2(P_2)}{P_2} \\[2ex] P_1 + P_2 = P_T \end{array} \right.$$

$$\frac{\Delta_i}{P_i} = \frac{\lambda^{\alpha}}{2} \Rightarrow \Delta_i = \frac{\lambda^{\alpha}}{2} P_i$$

$$\Delta_i = T_i - \log(1 + P_i \alpha_i)$$





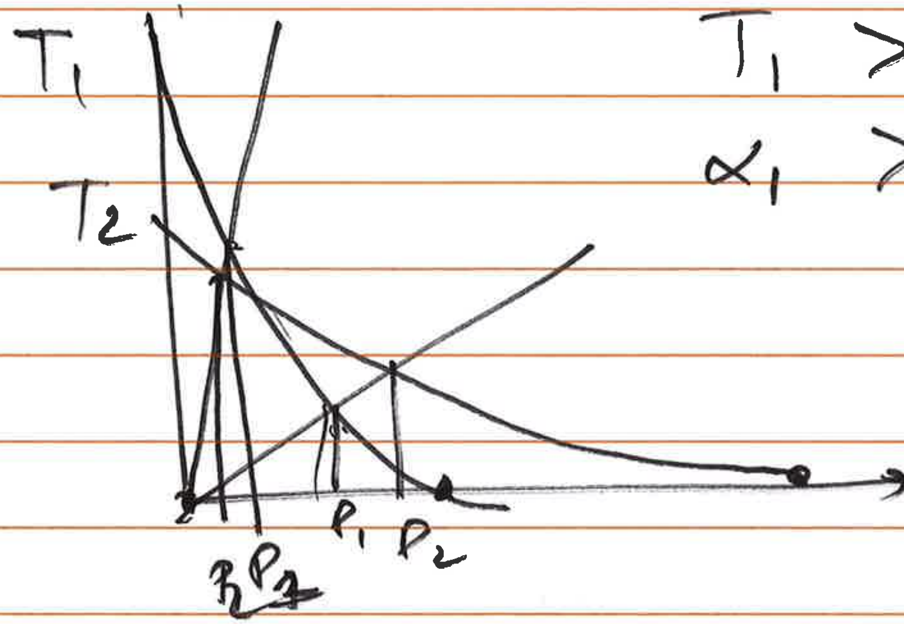
$\alpha_1 = \alpha_2$

$T_2 > T_1$

$\Rightarrow P_2 > P_1$

$T_1 > T_2$

$\alpha_1 > \alpha_2$



when P_T small, give more power to ch. 1

when P_T large " to ch. 2